Section 4.2 #80 Find a cubic polynomial with the following root:

$z = a+bi \Rightarrow \bar{z}$ = complex conjugate $= a-bi$.

Conjugate Pairs Theorem(s) for polynomial $P(x)$:

- If $P(x)$ has real coefficients then nonreal roots occur in conjugate pairs
- Special: If $P(x)$ has rational coefficents and $a+\sqrt{b}$ is a root, then so is $a-\sqrt{b}$ (to kill off the $\sqrt{b}$).

Factor Theorem: $x=c$ is a root of $P(x)$ $\Rightarrow$ $x-c$ is a factor.

So $-3+\sqrt{2}i$ is a root $\rightarrow$

$$P(x) = (x-c)(x-(-3+\sqrt{2}i))(x-(-3-\sqrt{2}i))$$

They want a cubic, so $x-c$ can have any "c."

I recommend $c=0$ (easiest).

So, $P(x) = x(x+3+\sqrt{2}i)(x+3-\sqrt{2}i)$

$(x+3+\sqrt{2}i)(x+3-\sqrt{2}i) =$

$x^2 + 3x - \sqrt{2}ix + 3x + 9 - 3\sqrt{2}i + \sqrt{2}ix + 3\sqrt{2}i - (\sqrt{2}i)(\sqrt{2}i)$

$x^2 + 6x + 9 - (\sqrt{2}i)^2$

$= x^2 + 6x + 9 - (-2)$

$= x^2 + 6x + 11$

So $P(x) = x^3 + 6x^2 + 11x$

(From $(x-c)(x^2+6x+11)$ when I chose $c=0$)

$(\sqrt{2}i)(\sqrt{2}i)$

$= \sqrt{2}\sqrt{2}\, i \cdot i$

$= i^2 \cdot 2$

$= -1 \cdot 2 = -2$

§4.2 #12 (from book)

5021863

$\frac{1}{3}x^2-5x+25=0$

$\Rightarrow x^2-15x+75=0$

$a=1,\ b=-15,\ c=75$

$D=b^2-4ac=(-15)^2-4(1)(75)$

$=225-300$

$=-75$

$5\sqrt{3}=\sqrt{75}$: 3 | 75, 5 | 25, 5

$\sqrt{-75}=5i\sqrt{3}$

$x=\frac{b\pm\sqrt{D}}{2a}=\frac{15\pm5i\sqrt{3}}{2(1)}=\frac{15\pm5i\sqrt{3}}{2}$

Complete-the-square method:

$x^2-15x+75=0$

$1x^2-15x+\left(\frac{15}{2}\right)^2=-75+\frac{225}{4}$

$\left(x-\frac{15}{2}\right)^2=\frac{-75}{4}$

$\sqrt{\left(x-\frac{15}{2}\right)^2}=\sqrt{\frac{-75}{4}}$

$\left|x-\frac{15}{2}\right|=\sqrt{\frac{-75}{4}}$ opt

$x-\frac{15}{2}=\pm\frac{5i\sqrt{3}}{2}$

$x=\frac{15\pm5i\sqrt{3}}{2}$

$\sqrt{x^2}=|x|$

$(\sqrt{x})^2=x$

$-75+\frac{225}{4}=\frac{-300+225}{4}$

$$\frac{\cancel{2}\pm 6\sqrt{2}}{\cancel{4}} = \frac{\cancel{2}(1\pm 3\sqrt{2})}{\cancel{2}(2)}$$

$$= \frac{1\pm 6\sqrt{2}}{2}$$

$$\frac{1\pm 3\sqrt{2}}{2}$$

$$\frac{3}{4} = \frac{2+1}{4} = \frac{1+1}{2} = 1$$

3	75
	25
	5

2, 3, 5, 7, 11, 13, 17, 19,
23, 29, 31, 37, 41, 43, 47

2	361574136
2	180780768
2	90393534
3	45196767
3	15065589
7	5021863
7	717409
7	102487
11	14641
11	1331
11	121
	11

So $\sqrt{361574136}$

$= 2\cdot 3\cdot 7\cdot 11\cdot 11\sqrt{2\cdot 7}$

$= 5082\sqrt{14}$

$f(x) = 3x^5 - 4x^4 + x^3 - 7x + 2$

Find $P(2)$: Divide $f(x)$ by $x-2$

$$\begin{array}{r|rrrrrr} 2 & 3 & -4 & 1 & 0 & -7 & 2 \\ & & 6 & 4 & 10 & 20 & 26 \\ \hline & 3 & 2 & 5 & 10 & 13 & \boxed{28 = P(2)} \end{array}$$

by Remainder Theorem.

$$\frac{28}{3} = \frac{27}{3} + \frac{1}{3} = 9 + \frac{1}{3}$$

$$\frac{28}{3} = 9 + \frac{1}{3}$$

$$\frac{\text{Dividend}}{\text{Divisor}} = \text{Quotient} + \frac{\text{REMAINDER}}{\text{DIVISOR}}$$

$$28 = 3 \cdot 9 + \frac{3}{3}$$

$$\underbrace{\text{Dividend} = \text{Divisor} \cdot \text{Quotient} + \text{Remainder}}_{\text{Division Algorithm}}$$

$f(x) = 3x^5 - 4x^4 + x^3 - 7x + 2$

Find $P(2)$: Divide $f(x)$ by $x-2$

$$
\begin{array}{r|rrrrrr}
2 & 3 & -4 & 1 & 0 & -7 & 2 \\
 & & 6 & 4 & 10 & 20 & 26 \\
\hline
 & 3 & 2 & 5 & 10 & 13 & \boxed{28 = P(2)} \\
 & x^4 & x^3 & x^2 & x & c & r
\end{array}
$$

by Remainder Theorem.

This says $f(x) =$

$f(x) = (x-2)(3x^4 + 2x^3 + 5x^2 + 10x + 13) + 28$

So $f(2) = 28$, obviously

$28 = 3 \cdot 9 + 1$

Always watch the first video on a problem type, at least.

From the Test 3 in the Chapter 4 Videos:

7. Let $f(x) = 2x^3 - 7x^2 + 10x - 6$.

a. Use synthetic division to find $f(3)$.

Remainder Theorem !

b. Use synthetic division to show that $x = 1 + i$ is a solution of the equation $f(x) = 0$.

c. Find the linear factorization of f that is promised to us in the Fundamental Theorem of Algebra.

a)
$$\begin{array}{r|rrrr} 3 & 2 & -7 & 10 & -6 \\ & & 6 & -3 & 21 \\ \hline & 2 & -1 & 7 & \boxed{15 = f(3)} \end{array}$$

Dividing by $x-3$

b) $1+i$ is a root:

$$\begin{array}{r|rrrr} 1+i & 2 & -7 & 10 & -6 \\ & & 2+2i & -7-3i & 6 \\ \hline 1-i & 2 & -5+2i & 3-3i & 0 \\ & & 2-2i & -3+3i & \\ \hline & 2 & -3 & 0 & \end{array}$$

sweet!

c)

$2x-3 \longleftrightarrow 2, -3$ (x, c, r)

So, $f(x) = (x-(1+i))(x-(1-i))(2x-3)$

$(1+i)(-5+2i)$
$= -5 + 2i - 5i + 2i^2$
$= -5 - 3i - 2$
$= -7 - 3i$

$(1+i)(3-3i)$
$= 3(1+i)(1-i)$
$= 3(1^2 + 1^2)$
$= 6$

(exploiting $(a+bi)(a-bi) = a^2 + b^2$

1st Divide by $(x-(1+i))$:

$(x-(1+i))(2x^2 + (-5+2i)x + (3-3i))$

2nd Divide by $(x-(1-i))$:

$(x-(1+i))(x-(1-i))(2x-3)$

$z = 2(\cos\theta + i\sin\theta)$

$\Rightarrow \sqrt{z} = \sqrt{2}\left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right)$

is the 1st square root. (Principal).

The 2nd is obtained by adding $\frac{2\pi}{2} = \pi$ to the argument.

$$\sqrt{2}\left(\cos\left(\frac{\theta}{2}+\pi\right) + i\sin\left(\frac{\theta}{2}+\pi\right)\right)$$

$z = 5i = 5\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

$\sqrt{z} = \sqrt{5}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

$= \sqrt{5}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right) = \frac{\sqrt{5}}{\sqrt{2}} + \frac{\sqrt{5}}{\sqrt{2}}i$

Add $\frac{2\pi}{2}$ to arg. $= \frac{\sqrt{10}}{2} + \frac{\sqrt{10}}{2}i$

$\therefore \sqrt{5}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right) = -\frac{\sqrt{10}}{2} - \frac{\sqrt{10}}{2}i$