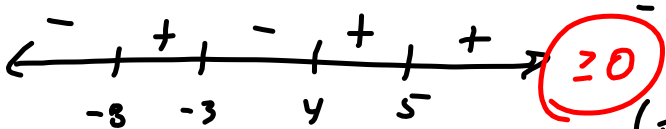
5. (5 pts) What is the domain of $\sqrt{\dfrac{(x+3)(x-5)^2}{(x-4)^3(x+8)}}$? ≥ 0

Need $\dfrac{(x+3)(x-5)^2}{(x-4)^3(x+8)} \geq 0$

$-3, 5, 4, -8$

$-8, -3, 4, 5$



$\dfrac{(316+3)(3+6-5)^2}{(316-4)^3(316+8)} \approx \dfrac{(316)(316)^2}{(316)^3(316)} = \dfrac{(+)(+)^2}{(+)^3(+)}$

☆ $=0$ ☆ $=0$

N N Y Y N N Y Y Y

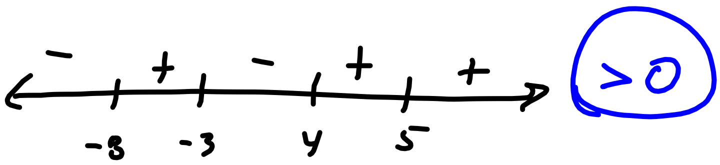
$\mathcal{D} = \boxed{(-8,-3] \cup (4,\infty)}$

$[4,\infty) = [4,5] \cup [5,\infty)$

$x > 5$

6. (5 pts) What is the domain of $\log_7\left(\dfrac{(x+3)(x-5)^2}{(x-4)^3(x+8)}\right)$?

$\dfrac{(x+3)(x-5)^2}{(x-4)^3(x+8)} > 0$



☆ $=0$ ☆ $=0$

N N Y N N N Y N Y

$\boxed{\mathcal{D} = (-8,3) \cup (4,5) \cup (5,\infty)}$

7. (10 pts) Let $f(x)=4^{4x+7}-6$. Find $f^{-1}(x)$.

$$y = 4^{x+7} - 6 = y$$

$4^{y+7} - 6 = x$ Solve for y.

$$4^{y+7} = x+6$$

$$\log_4\left(4^{y+7}\right) = \log_4(x+6)$$

$$y+7 = \log_4(x+6)$$

$$y = \boxed{\log_4(x+6) - 7 = f^{-1}(x)}$$

8. (10 pts) Solve $\ln(x-5)+\ln(x+2)=\ln(18)$.

$a^{b+c} = a^b a^c$

$\ln\left((x-5)(x+2)\right) = \ln(18)$

$e^{\ln((x-5)(x+2))} = e^{\ln(18)}$

$(x-5)(x+2) = 18$

$x^2-3x-10=18$

$x^2-3x-28=0$

$(x-7)(x+4)=0$

$\Rightarrow x \in \{-4, 7\} \Rightarrow \boxed{x \in \{7\}}$

$-4 \notin D$

x = -4 is extraneous

$\boxed{\ln(-4-5)\ \nexists}$

$\ln(2)+\ln(9)$

$=\ln(2\cdot 9) = \ln(18)$ ✓

$e^{LHS} = e^{RHS}$ to get rid of

$\log_e = \ln$

9. Suppose the half-life of C-14 is 5400 years. (It isn't, quite, but just suppose…).

   a. (10 pts) Derive the exponential decay model, $A(t) = A_0 e^{kt}$. The trick is to use the half-life to find the relative decay rate, k.

   b. (5 pts) How old is a sample of charcoal from a prehistoric fire pit, if 28% of the C-14 has decayed (i.e., 72% is left.)? Round to the nearest year in your final answer. If it makes it easier for you, use an initial

(a) $\frac{1}{2}$-life is 5400 yrs

$$\boxed{A_0 e^{5400k} = \frac{1}{2} A_0}$$

$$e^{5400k} = \frac{1}{2}$$

$$\ln\left(e^{5400k}\right) = \ln\left(\frac{1}{2}\right) = -\ln(2)$$

$$5400k = -\ln(2)$$

$$k = -\frac{\ln(2)}{5400} = \boxed{\frac{\ln\left(\frac{1}{2}\right)}{5400} = k}$$

$k = \frac{\ln\left(\frac{1}{2}\right)}{50}$

$A_0 e^{50k} = \frac{1}{2} A_0$

50-yr $\frac{1}{2}$-life.

$$A(t) = A_0 e^{-\frac{\ln(2)}{5400} t}$$

OR $\boxed{A(t) = A_0 e^{kt} \text{ is fine.}}$

(b) 72% remains. How old is it?

Solve for t.

$$A_0 e^{kt} = .72 A_0 \qquad 100 e^{kt} = 72$$

$$e^{kt} = .72$$

$$\ln\left(e^{kt}\right) = \ln(.72)$$

$$kt = \ln(.72)$$

$$t = \frac{\ln(.72)}{k} = \frac{\ln(.72)}{\frac{-\ln(2)}{5400}}$$

Invert & Multiply

$$= -\frac{5400 \ln(.72)}{\ln(2)} \approx 2559.228417$$

$$\boxed{\approx 2559 \text{ yrs}}$$

```
-5*4^-21+2
     2.000000000
5400*ln(.72)/ln(
2)
    -2559.228417
```

0 → 5400   100% → 50%

72% @ 2559 yrs.

B 1 (5 pts) Solve the absolute value inequality: $|-5x+8|-11>-2$

$|-5x+8|>9$

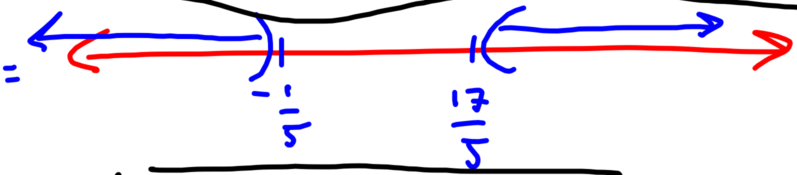
$-5x+8>9$ OR $-5x+8<-9$

$-5x>1$ $\qquad$ $-5x<-17$

$\frac{-5x}{-5}<\frac{1}{-5}$

$x\in \boxed{\{x \mid x<-\frac{1}{5} \text{ OR } x>\frac{-17}{-5}=\frac{17}{5}\}}$

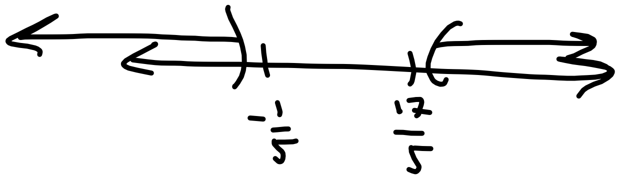
$=$



OR

$= \boxed{(-\infty,-\frac{1}{5})\cup(\frac{17}{5},\infty)}$

Note. Interpret: $\{x \mid x<-\frac{1}{5} \text{ AND } x>\frac{17}{5}\}$



AND

No overlap $\Rightarrow$ AND gives us $\emptyset$

B 2 (5 pts) Re-write $f(x)=5x^2-3x+1$ in the form $a(x-h)^2+k$.

$f(x) = 5x^2 - 3x + 1$

$= 5\left(x^2 - \frac{3}{5}x + \left(\frac{3}{10}\right)^2\right) + 1 - 5\left(\frac{3}{10}\right)^2$

$\frac{\frac{3}{5}}{\frac{2}{1}} = \frac{3}{5} \cdot \frac{1}{2} = \frac{3}{10}$

$1 - 5\left(\frac{9}{100}\right)$

$= 1 - \frac{9}{20} = \frac{20-9}{20} = \frac{11}{12}$

$\boxed{= 5\left(x - \frac{3}{10}\right)^2 + \frac{11}{20}}$

$f(x) = 5x^2 - 3x + 1$

$\frac{f(x)}{5} = x^2 - \frac{3}{5}x + \frac{1}{5}$

$= x^2 - \frac{3}{5}x + \left(\frac{3}{10}\right)^2 - \frac{9}{100} + \frac{1}{5}$

$\frac{\frac{3}{5}}{2} = \frac{3}{10}$

$-\frac{9}{100} + \frac{1}{5} \cdot \frac{20}{20}$

$= \frac{-9+20}{100} = \frac{11}{100}$

$\frac{f(x)}{5} = \left(x - \frac{3}{10}\right)^2 + \frac{11}{100}$

$\frac{11}{100} \cdot \frac{5}{1} = \frac{11}{20}$

$\Rightarrow \boxed{f(x) = 5\left(x - \frac{3}{10}\right)^2 + \frac{11}{20}}$

B 3 (5 pts) Solve the exponential equation $3 \cdot (7.7)^x = 11 \cdot (2.1)^x$

$$3 \cdot (7.7)^x = 11 \cdot (2.1)^x$$

$$\ln\left(3(7.7)^x\right) = \ln\left(11(2.1)^x\right)$$

$$\ln(3) + \ln\left((7.7)^x\right) = \ln(11) + \ln\left((2.1)^x\right)$$

$$\ln(3) + \left(\ln(7.7)\right)x = \ln(11) + \left(\ln(2.1)\right)x$$

$$x \ln(7.7) \Rightarrow$$

$$a + bx = c + dx \text{ , where } a = \ln(3)$$
$$-a - dx = -a - dx \qquad b = \ln(7.7)$$
$$c = \ln(11)$$
$$bx - dx = c - a \qquad d = \ln(2.1)$$

$$(b-d)x = c - a$$

$$x = \frac{c-a}{b-d} = \boxed{\frac{\ln(11) - \ln(3)}{\ln(7.7) - \ln(2.1)}} \approx 1.764569342$$

$\Rightarrow x$ is exactly right.

```
  2.000000000
5400*ln(.72)/ln(
2)
     -2559.228417
(ln(11-ln(3))/(l
n(7.7)-ln(2.1))
      1.764569342
█
```

B 4 John can finish a job in 5 hours that it takes Bill 8 hours to finish. Suppose Bill shows up and starts working 2 hours before John shows up, and then they work together until the job is done. How many hours does each of the two end up working?

Let x = the # of hours John works

y = " " " " Bill works.

$y = x+2$

$\left(\frac{1}{5}\,\frac{\text{job}}{\cancel{\text{hr}}}\right)\left(x\,\cancel{\text{hrs}}\right) + \left(\frac{1}{8}\,\frac{\text{job}}{\cancel{\text{hr}}}\right)\left(x+2\,\cancel{\text{hrs}}\right) = 1\ \text{JOB}$

$\frac{1}{5}x + \frac{1}{8}(x+2) = 1$ LCD = $5 \cdot 8$

$\frac{1}{5}\cdot\frac{8}{8}$, $\frac{1}{8}\cdot\frac{5}{5}$

$\frac{8x + 5(x+2)}{40} = \frac{40}{40}$

$8x + 5x + 10 = 40$

$13x + 10 = 40$

$13x = 30$

$\boxed{x = \frac{30}{13}} \Rightarrow x+2 = \frac{30}{13} + \frac{2}{1}\cdot\frac{13}{13}$

$= \frac{30+26}{13} = \boxed{\frac{56}{13}\text{ hrs}} = x+2$

John works $\frac{30}{13}$ hrs

Bill works $\frac{56}{13}$ hrs

B 5 What is the future value of $5,000 in 10 years, if interest is 4%, compounded weekly? (52 weeks in a year.).

$$A = P(1+i)^n = P\left(1+\frac{r}{m}\right)^{mt}$$

$$A = 5000\left(1+\frac{.04}{52}\right)^{52\cdot 10} \approx \boxed{\$7457.98}$$

B 6 What is the present value of $5,000 in 10 years, if interest is 4%, compounded weekly?

$$P = A(1+i)^{-n} = A\left(1+\frac{r}{m}\right)^{-mt}$$

$$= 5000\left(1+\frac{.04}{52}\right)^{-52\cdot 10} \approx \boxed{\$3352.12}$$

```
5000*(1+.04/52)^
(52*10)
     7457.976607
5000*(1+.04/52)^
-(52*10)
     3352.115636
```