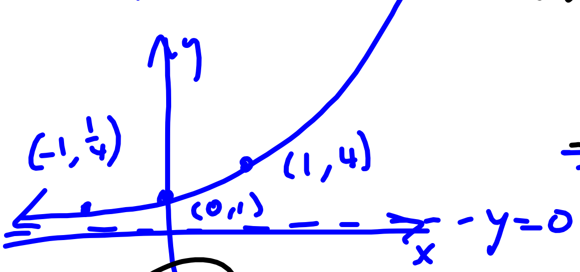
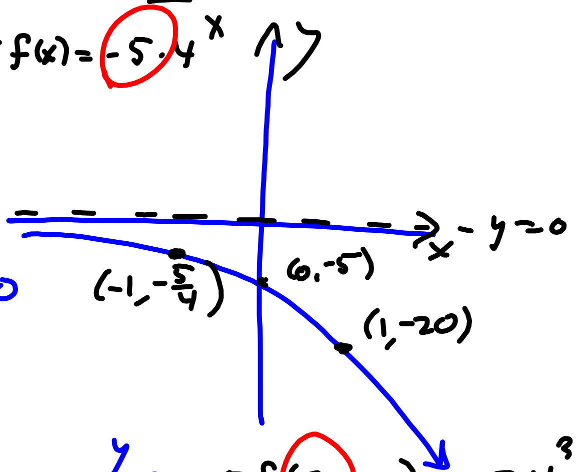
$g(x) = -5 \cdot 4^{3x-21} + 2$

$3(x-7)$

$f(x) = 4^x$

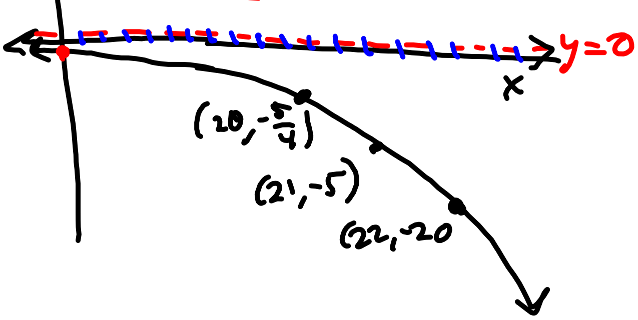



$-5f(x) = -5 \cdot 4^x$

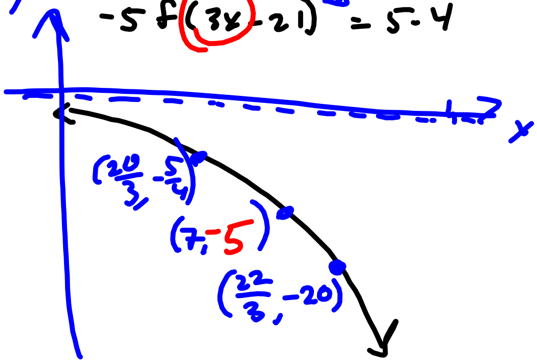



M1

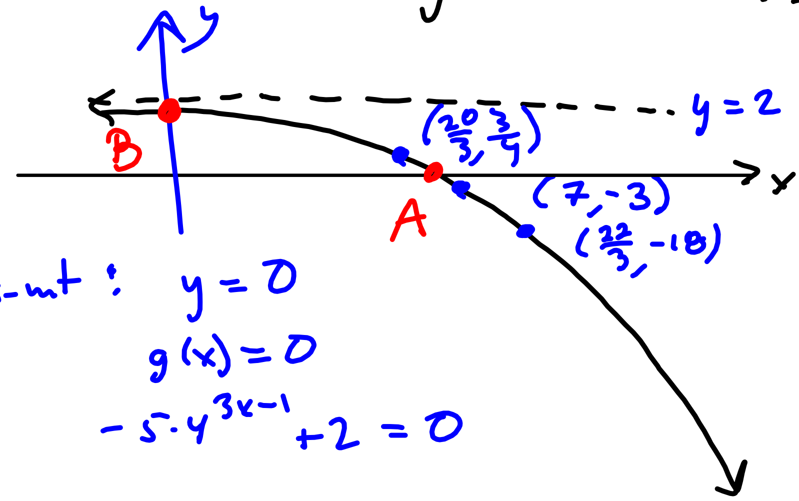
$-5f(x-21) = -5 \cdot 4^{x-21}$




$-5f(3x-21) = 5 \cdot 4^{3x-21}$




$-5f(3x-21) + 2 = g(x) = -5 \cdot 4^{3x-21} + 2$




up 2

$-\frac{5}{4} + 2 = \frac{-5+8}{4} = \frac{3}{4}$

x-int: $y = 0$

$g(x) = 0$

$-5 \cdot 4^{3x-1} + 2 = 0$

$$-5\cdot 4^{3x-21} = -2$$

$$4^{3x-21} = \frac{-2}{-5}$$

$$\log_4\left(4^{3x-21}\right) = \log_4\left(\tfrac{2}{5}\right)$$

$$3x-21 = \log_4\left(\tfrac{2}{5}\right)$$

$$3x = \log_4\left(\tfrac{2}{5}\right)+21$$

x-int $\quad x = \log_4\left(\tfrac{3}{5}\right)+21$

$$\boxed{A = \left(\frac{\log_4\left(\frac{3}{5}\right)+21}{3}, 0\right)}$$

y-int: $\quad x = 0$

$$g(0) = -5\cdot 4^{3(0)-21}+2$$

$$= -5\cdot 4^{-21}+2$$

$$-\frac{5}{4^{21}}+2$$

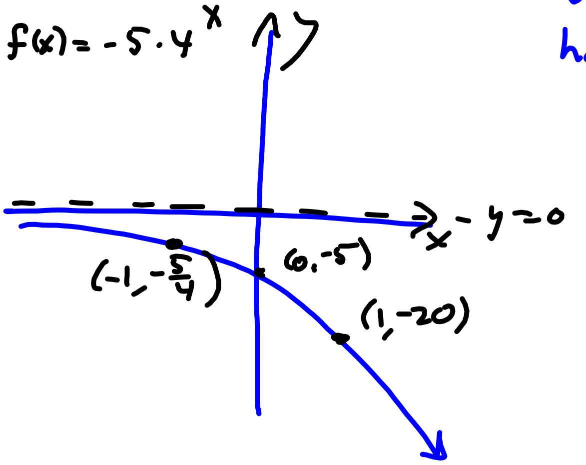
$$\boxed{\left(0, -5\cdot 4^{-21}+2\right) = B}$$

$3(x-7)$

$-5\cdot 4^{3x-21}+2$

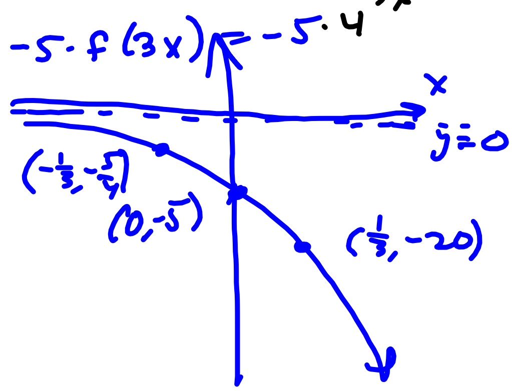
In trig, we prefer to do horizontal stretch before the horizontal shift.

$-5f(x) = -5\cdot 4^{x}$



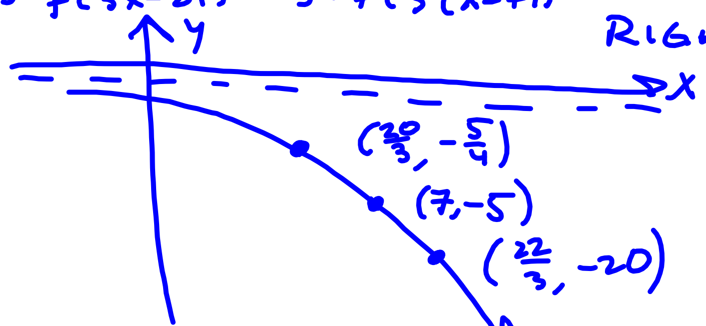

M2

$-5\cdot f(3x) = -5\cdot 4^{3x}$




$-5\cdot f(3x-21) = -5\cdot f(3(x-7))$

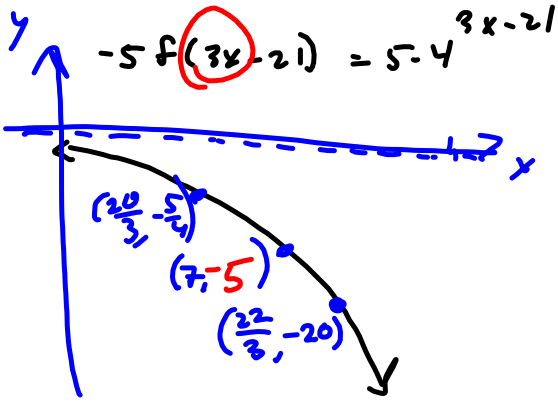
RIGHT 7




$-\frac{1}{3}+7 = \frac{-1+21}{3} = \frac{20}{3}$

$\frac{1}{3}+7 = \frac{22}{3}$

$-5f(3x-21) = 5\cdot 4^{3x-21}$



Find $g^{-1}(x)$

$$g(x)=y=-5\cdot 4^{3x-21}+2=y$$

$$-5\cdot 4^{3y-21}+2=x$$

$$-5\cdot 4^{3y-21}=x-2$$

$$4^{3y-21}=\frac{x-2}{-5}$$

$$\log_4\left(4^{3y-21}\right)=\log_4\left(\frac{x-2}{-5}\right)$$

$$3y-21=\log_4\left(\frac{2-x}{5}\right)$$

$$3y=\log_4\left(\frac{2-x}{5}\right)+21$$

$$y=\boxed{\frac{\log_4\left(\frac{2-x}{5}\right)+21}{3}=g^{-1}(x)}$$

$$g(g^{-1}(x))=x$$

Solve $g(x)=7 \notin R(g)$, stupid

$$-5\cdot 4^{3x-21}+2=7$$

But $g^{-1}$ un-does $g$

$$g^{-1}(g(x))=g^{-1}(7)$$

$$x=g^{-1}(7)$$

Solve $g(x)=0$

$$g^{-1}(g(x))=x=g^{-1}(0)$$

$$=\frac{\log_4\left(\frac{2-0}{5}\right)+21}{3}=\frac{\log_4\left(\frac{2}{5}\right)+21}{3}$$

$f(x)=\sqrt{x+5}$ , $g(x)=x^2-3x-5$

$\mathcal{D}(f)$:

Need $x+5\ge 0$

$\boxed{\mathcal{D}(f)=\{x \mid x\ge -5\}}$

$\boxed{\mathcal{D}(g)=\mathbb{R}}$

$$\left(\frac{f}{g}\right)(x)=\frac{\sqrt{x+5}}{x^2-3x-5}$$

$$\mathcal{D}\left(\frac{f}{g}\right)=\{x \mid x\in\mathcal{D}(f) \text{ and } x\in\mathcal{D}(g) \text{ and } g(x)\ne 0\}$$

$$=\{x \mid x\ge -5 \text{ and } x\in\mathbb{R} \text{ and } x^2-3x-5\ne 0\}$$

$x^2-3x-5\ne 0$

$x^2-3x \ne 5$

$x^2-3x+\left(\frac{3}{2}\right)^2\ne \frac{9}{4}+5\cdot\frac{4}{4} = \frac{9}{4}+\frac{20}{4}$

$\left(x-\frac{3}{2}\right)^2\ne\frac{29}{4}$

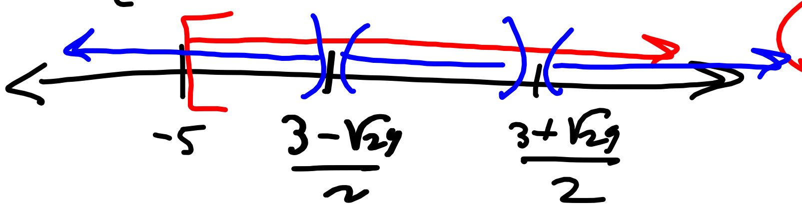
$x-\frac{3}{2}\ne\pm\sqrt{\frac{29}{4}}=\pm\frac{\sqrt{29}}{2}$

$x\ne\frac{3\pm\sqrt{29}}{2}$

10 pts

$$=\left\{x \mid x\ge -5 \text{ and } x\ne\frac{3\pm\sqrt{29}}{2}\right\}$$

AND




Interval Notation

5 pts

$$=\left[-5,\frac{3-\sqrt{29}}{2}\right)\cup\left(\frac{3-\sqrt{29}}{2},\frac{3+\sqrt{29}}{2}\right)\cup\left(\frac{3+\sqrt{29}}{2},\infty\right)$$

$$(f \circ g)(x) = f(g(x)) = \sqrt{g(x)+5}$$

$$= \sqrt{x^2-3x-5+5} = \sqrt{x^2-3x}$$

$$\mathcal{D}(f \circ g) = \text{FORMALLY} = \{x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f)\}$$

$$= \{x \mid x \in \mathbb{R} \text{ and } x^2-3x-5 \geq -5\}$$

$$= \{x \mid x^2-3x \geq 0\}$$

Need Intuitively $x^2-3x \geq 0$

$$x^2-3x \geq 0$$

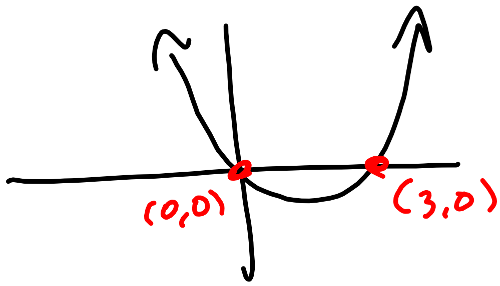
$$x(x-3) \geq 0$$

$+ \quad - \quad +$ ; $0 \quad 3$ ; $\geq 0$

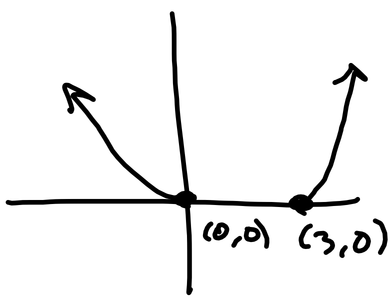
Y $\quad$ Y ($=0$) $\quad$ N $\quad$ Y ($=0$) $\quad$ Y

$$\boxed{\mathcal{D}(f \circ g) = (-\infty, -0] \cup [3, \infty)}$$

$x^2-3x$




$\sqrt{x^2-3x}$




$x^2-3x-5 \geq -5$

$\left(x-\frac{3}{2}\right)^2$

$\left(x-\frac{3}{2}\right)^2 = \frac{29}{4}$

$g(x) = \left(x-\frac{3}{2}\right)^2 - \frac{29}{4}$