The messiest kind of exponential equation involves different bases.

$5 \cdot 3^x = 7 \cdot \pi^x$

Until you've done 100 of these, show this separate step.

$\log(AB) = \log(A) + \log(B)$

$\ln(5 \cdot 3^x) = \ln(7 \cdot \pi^x)$

$\ln(5) + \ln(3^x) = \ln(7) + \ln(\pi^x)$

$\log(C^D) = D\log(C)$

$\ln(5) + x\ln(3) = \ln(7) + x\ln(\pi)$

$a = \ln(5)$, $b = \ln(3)$, $c = \ln(7)$, $d = \ln(\pi)$

$a + xb = c + xd$

$bx - dx = c - a$

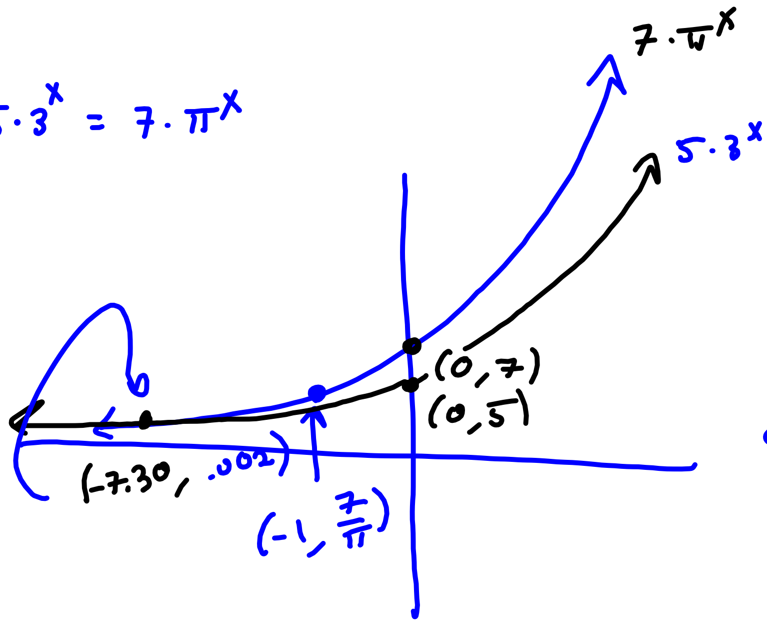
$x(b-d) = c - a$

$x = \dfrac{c-a}{b-d}$

$= \dfrac{\ln(7) - \ln(5)}{\ln(3) - \ln(\pi)}$

≈ -7.295962038

$5 \cdot 3^x = 7 \cdot \pi^x$



Grows faster to right shrinks faster to the left.

$\pi^x > 3^x$ eventually →

$\pi^x < 3^x$ eventually ←

0.001651624974

Building an exponential function.

Radioactive Millsium has a half-life of 50 years.

Model $A(t) = A_0 e^{kt}$

A = Amount present in future
t = time in years
k = (relative) growth rate
A_0 = "A nought" = Initial Amount

$A(0) = A_0 e^{k \cdot 0} = A_0 e^0 = A_0$

Radioactive Millsium has a half-life of 50 years.

NEED TO SEE

$A(50) = A_0 e^{k \cdot 50} = A_0 e^{50k} = \frac{1}{2} A_0$ THE $\frac{1}{2}$-life EQUATION.

Book sez memorize $e^{50k} = \frac{1}{2}$
They're full of nonsense.

$\frac{A_0 e^{50k}}{A_0} = \frac{\frac{1}{2}A_0}{A_0}$

$e^{50k} = \frac{1}{2}$ Need to see

$\ln(AB) = \ln(A) + \ln(B)$

$\ln\left(\frac{A}{B}\right)$
$= \ln(AB^{-1})$
$= \ln(A) + \ln(B^{-1})$
$= \ln(A) - \ln(B)$

$\ln(e^{50k}) = \ln\left(\frac{1}{2}\right)$ $= \ln(2^{-1})$

$50k = -\ln(2)$

$k = \frac{-\ln(2)}{50}$

Book sez
"Write equation in logarithmic form."

How much Radioactive Millsium is present after 50 years if there was 40g to start with?

20g (Logic)

$A_0 e^{50k} = 40 e^{50\left(-\frac{\ln(2)}{50}\right)}$

$= 40 e^{-\ln(2)} = 40 \cdot \frac{1}{e^{\ln(2)}} = \frac{40}{2} = 20$

How old is a sample with 15% of the original amount remaining?

$A_0 e^{kt} = .15 A_0$

$e^{kt} = .15$

$\ln(e^{kt}) = \ln(.15)$

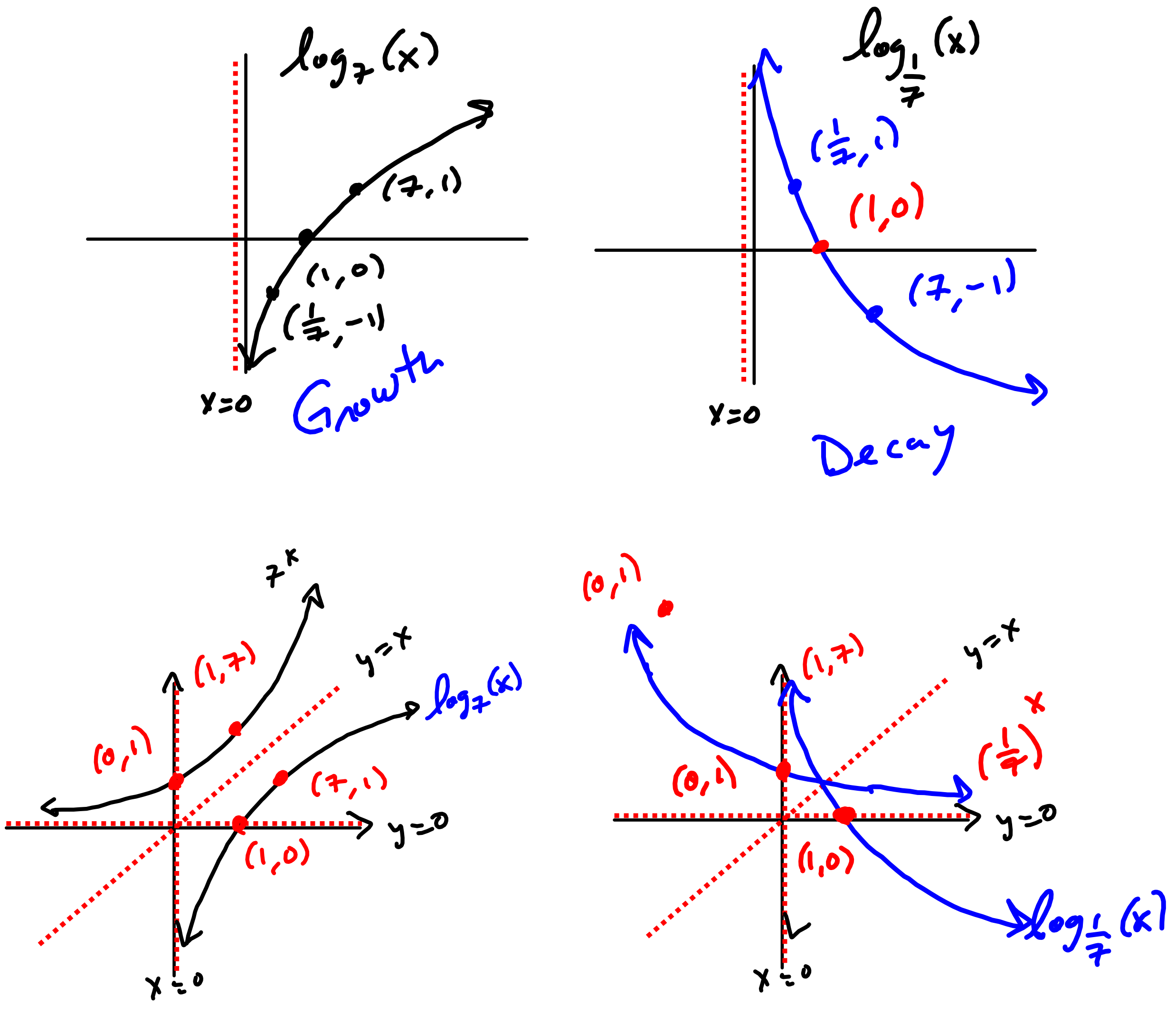
$kt = \ln(.15)$

$t = \frac{\ln(.15)}{k} = \frac{\ln(.15)}{\frac{-\ln(2)}{50}}$

$= (\ln(.15))\left(\frac{50}{-\ln(2)}\right)$

$= \frac{-50\ln(.15)}{\ln(2)} \approx$ 136.8482797

t	Amt
50	$\frac{1}{2} = .5$
100	$\frac{1}{4} = .25$ ← 15% = .15
150	$\frac{1}{8} = .125$



To graph an inverse function, reflect across the line $y = x$.

If $f(x) = 7^x$, $f^{-1}(x) = \log_7(x)$