Today's questions taken from Fall '17 Test 3

① $x=1, m=3$; $3+i, m=2$; $x=2, m=1$

$(x-1)^3(x-(3+i))^2(x-(3-i))^2(x-2)$

CPT

$(x-1)^3(x-3-i)^2(x-3+i)^2(x-2)$

② $P(x) = 7x^5 - 2x^4 + 11x^3 + x^2 - 173x - 4$

Want $P(3)$

| | | | | | | |
|---|---|---|---|---|---|---|
| 3 | 7 | -2 | 11 | 1 | -173 | -4 |
| | | 21 | 57 | 204 | 615 | 1326 |
| | 7 | 19 | 68 | 205 | 442 | $\boxed{1322 = P(3)}$ |

$68 \times 3 = 204$

$442 \times 3 = 1326$

③ $P(x) = (x-3)(7x^4 + 19x^3 + 68x^2 + 205x + 442) + 1322$

See why $P(3) = 1322$?

④ $f(x) = (x+2)^2(x-1)^3(x-7)(x+5) = x^7 + \dots + 140$

ⓐ $f(x) \le 0$ : $\le 0$

=0

Y | No | N | |

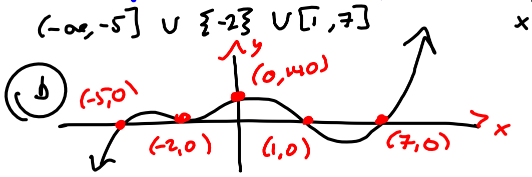
Y Y Y Y Y N

-5 -2 1 7

$(-\infty, -5] \cup \{-2\} \cup [1, 7]$

$x^7$

ⓑ




④ $f(x) = (x+2)^2(x-1)^3(x-7)(x+5)$

ⓒ $\sqrt{\dfrac{(x+2)^2(x-7)}{(x-1)^3(x+5)}} = R(x)$

Want $\mathcal{D}(R(x))$

Need $\dfrac{(x+2)^2(x-7)}{(x-1)^3(x+5)} \ge 0$ and $(x-1)^3(x+5) \ne 0$

4a

− + − − +

-5 -2 1 7

=0 =0 =0 =0

4c ≥0

− + − − +

-5 -2 1 7

$\cancel{=0}$ =0 $\cancel{=0}$ =0

N N Y Y N N N Y Y

$\mathcal{D}(R) = (-5, -2] \cup [7, \infty)$

(5) $f(x) = 4x^5 + 16x^4 + 25x^3 + 23x^2 - 16x - 52$

(a) Descartes: 1 pos. root.

$$f(-x) = \underbrace{-4x^5 + 16x^4}_{1}\underbrace{\;}_{2}\underbrace{- 25x^3 + 23x^2}_{3} + \underbrace{16x - 52}_{4}$$

4, 2, or 0 neg. roots.

(b) $\frac{p}{q}$ : $\frac{52}{4}$

$2\,|\,52$, $2\,|\,26$, $13$ $\quad$ $2\,|\,4$, $2$

$\pm 1, \pm 2, \pm 4, \pm 13, \pm 26, \pm 52,$

$\pm \frac{1}{2}, \cancel{\pm \frac{2}{2}}, \cancel{\pm \frac{4}{2}}, \pm \frac{13}{2}, \cancel{\pm \frac{26}{2}}, \cancel{\pm \frac{52}{2}}$

$\pm \frac{1}{4}, \cancel{\pm \frac{2}{4}}, \cancel{\pm \frac{4}{4}}, \pm \frac{13}{4}, \cancel{\pm \frac{26}{4}}, \cancel{\pm \frac{52}{4}}$

(c) Bonus: Return to later.

5) $f(x) = 4x^5 + 16x^4 + 25x^3 + 23x^2 - 16x - 52$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 4 | 16 | 25 | 23 | -16 | -52 | |
| | | 4 | 20 | 45 | 68 | 52 | |
| 1 | 4 | 20 | 45 | 68 | 52 | 0 | Sweet! |
| | | 4 | 24 | 69 | 137 | | |
| | 4 | 24 | 69 | 137 | Naw! | | |

But Descartes said Exactly one. So this is stupid

| | | | | | | |
|---|---|---|---|---|---|---|
| -1 | 4 | 20 | 45 | 68 | 52 | |
| | | -4 | -16 | -29 | | |
| | 4 | 16 | 29 | Naw | | |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| -2 | 4 | 20 | 45 | 68 | 52 | | |
| | | -8 | -24 | -42 | -52 | | |
| -2 | 4 | 12 | 21 | 26 | 0 | Sweet! | |
| | | -8 | -8 | -26 | | | |
| | 4 | 4 | 13 | 0 | Sweet! | | |
| | $x^2$ | $x^1$ | c | r | | | |

$4x^2 + 4x + 13 = 0$

$a = 4,\ b = 4,\ c = 13$

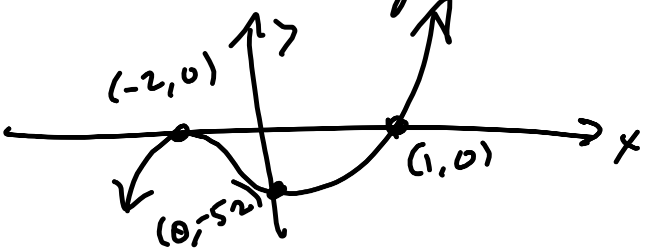
$b^2 - 4ac = 4^2 - 4(4)(13) < 0$

Over the real-number field,

$$\boxed{f(x) = (x-1)(x+2)^2(4x^2+4x+13)}$$

$x = 1,\ m = 1;\ x = -2;\ m = 2$

Great spot for a quick sketch.



Over the real-number field,

$$f(x) = (x-1)(x+2)^2\underline{(4x^2+4x+13)}$$

7. $4x^2+4x+13=0 \ldots$

$$b^2-4ac = 4^2-4(4)(13)$$
$$= 16-16(13)$$
$$= -16(12) = -192$$

$2 \mid 192$
$2 \mid 96$
$2 \mid 48$
$2 \mid 24$
$2 \mid 12$
$2 \mid 6$
$3$

$$-192 = i\cdot 8\sqrt{3} = 8i\sqrt{3}$$

$$x = \frac{-4 \pm 8i\sqrt{3}}{2(4)} = \boxed{\frac{-1 \pm 2i\sqrt{3}}{2} = x}$$

$$\frac{\cancel{4}(-1 \pm 2i\sqrt{3})}{2(\cancel{4})}$$

$$\boxed{f(x) = 4(x-1)(x+2)^2\left(x-\left(\frac{-1+2i\sqrt{3}}{2}\right)\right)\left(x-\left(\frac{-1-2i\sqrt{3}}{2}\right)\right)}$$

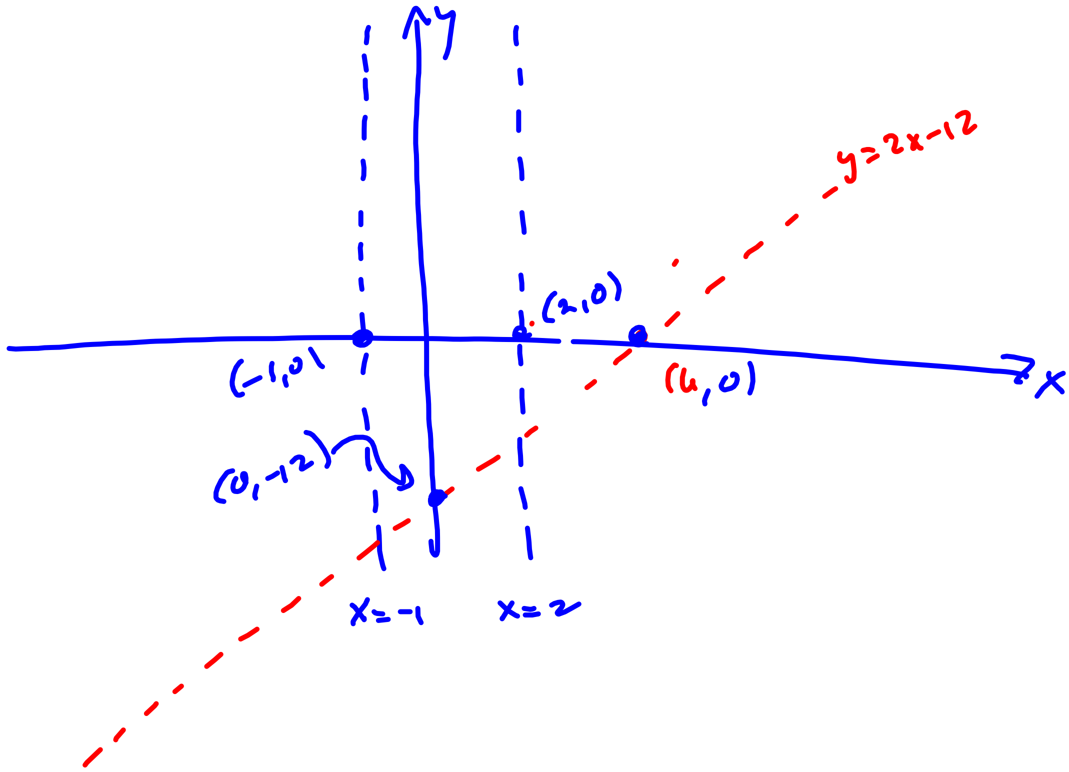
(8) $\frac{2x^3-14x^2+23x-10}{x^2-x-2} = \frac{\qquad}{(x-2)(x+1)}$

$\mathcal{D} = \mathbb{R} \setminus \{-1, 2\}$

$x=-1,\ x=2$

O.A.

$$\begin{array}{r} 2x - 12 \\ x^2-x-2 \overline{\big)\, 2x^3-14x^2+23x-10} \\ -(2x^3-2x^2-4x) \\ \hline -12x^2 \end{array}$$



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$$\frac{3x^2-14x+15}{x^2-5x-14} = \frac{(3x-5)(x-3)}{(x-7)(x+2)}$$

$D = \mathbb{R} \setminus \{-2, 7\}$

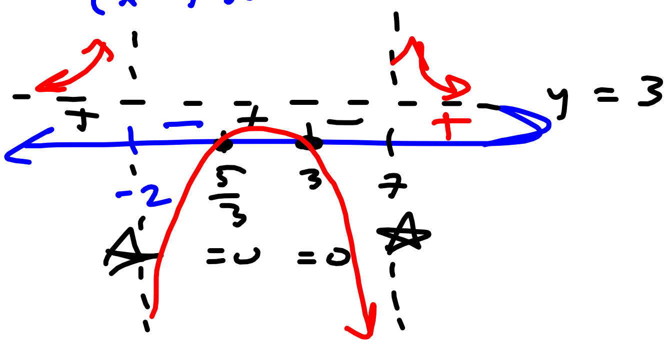
No holes

$\boxed{x = -2,\ x = 7}$

$\boxed{\text{x-int: } \left(\tfrac{5}{3}, 0\right),\ (3, 0)}$

$\boxed{\text{y-int: } \left(0, -\tfrac{15}{14}\right)}$

$\boxed{\text{H.A.: } y = 3}$




$-2,\ 5/3,\ 3,\ 7$

$A = \left(0, -\tfrac{15}{14}\right)$

$B = \left(\tfrac{5}{3}, 0\right)$

$C = (3, 0)$