(2) $D = \{1, 2, 3, 4\}$

$R = \{3, 4, 5\}$

$f(1) = f(4) = 3 \Rightarrow$ Not 1-to-1.

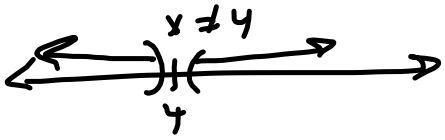
$(1, 3) \neq (4, 3)$

③ $f(x) = \frac{1}{x-4}$ & $g(x) = \sqrt{x+5}$

⇒ ② $\mathcal{D}(f) = \mathbb{R} \setminus \{4\} =$

Need $x - 4 \neq 0$

$x \neq 4$



$= (-\infty, 4) \cup (4, \infty)$

⑤ $\mathcal{D}(g)$: Need $x + 5 \geq 0$

$\mathcal{D}(g) = \{x \mid x \geq -5\} = [-5, \infty)$

$=$

ⓒ $\frac{f}{g} = \frac{\frac{1}{x-4}}{\sqrt{x+5}} = \frac{f(x)}{g(x)}$

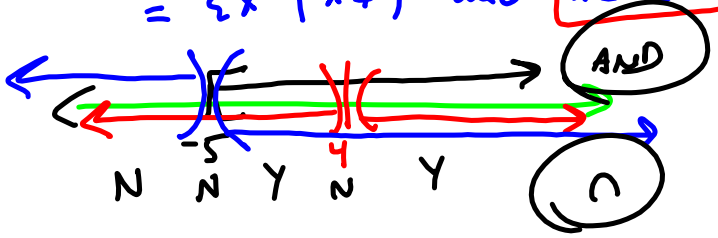
ⓓ $\mathcal{D}\left(\frac{f}{g}\right) = \{x \mid x \in \mathcal{D}(f) \text{ and } x \in \mathcal{D}(g) \text{ and } g(x) \neq 0\}$

$= \mathcal{D}(f) \cap \mathcal{D}(g) \cap \{x \mid g(x) \neq 0\}$

$= \{x \mid f(x) \exists \text{ & } g(x) \exists \text{ and } g(x) \neq 0\}$

$= \{x \mid x \neq 4 \text{ and } x \geq -5 \text{ and } x \neq -5\}$

$x > -5$



$g(x) \neq 0$

$\sqrt{x+5} \neq 0$

$x + 5 \neq 0$

$x \neq -5$

$= (-5, 4) \cup (4, \infty)$

f(x) ∃

∃ means "there is"
or "there exists"
or "exists"

$f(x) = \frac{1}{x-4}$

$f(\text{☺}) = \frac{1}{\text{☺}-4}$

$f(\Delta) = \frac{1}{\Delta-4}$

Composition $f \circ g$ (NOT $f \cdot g$)

$(f \circ g)(x) = f(g(x)) = \frac{1}{g(x)-4} = \frac{1}{\sqrt{x+5}-4}$

$\mathcal{D}(f \circ g) = \{x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f)\}$ Better

~~$f(g(x)) = \{x \mid g(x) \neq \text{and } f(g(x)) \neq\}$ weaker.~~

$= \{x \mid x \geq -5 \text{ and } g(x) \neq 4\}$

Scratch:

$g(x) \neq 4$

$\sqrt{x+5} \neq 4$

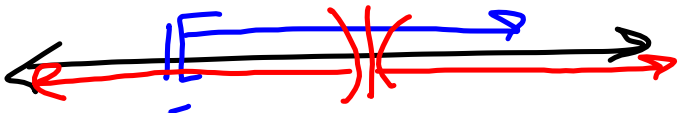
$(\sqrt{x+5})^2 \neq 4^2$

$x+5 \neq 16$

$x \neq 11$

$= \{x \mid x \geq -5 \text{ and } x \neq 11\}$

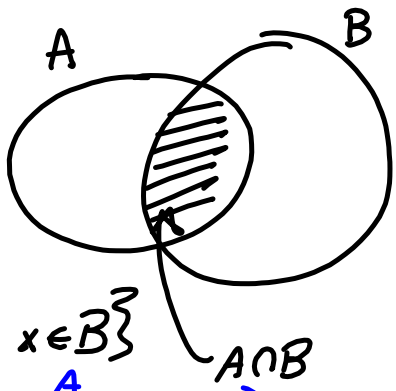
AND



$= [-5, 11) \cup (11, \infty)$

AND : $\cap$ Intersect

OR : $\cup$ Union



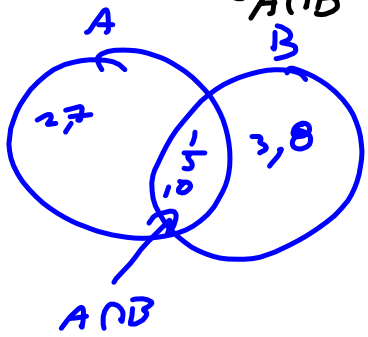
Formally:

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

$A = \{1, 2, 5, 7, 10\}$

$B = \{1, 3, 5, 8, 10\}$

$A \cap B = \{1, 5, 10\}$



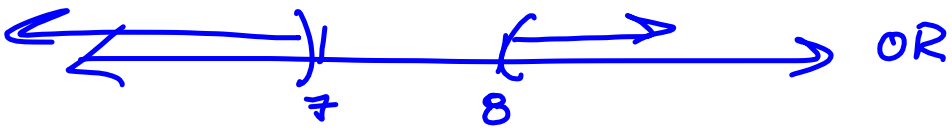
$$A \cup B = \{x \mid x \in A \text{ OR } x \in B\}$$

$$= \{1, 2, 3, 5, 7, 8, 10\}$$

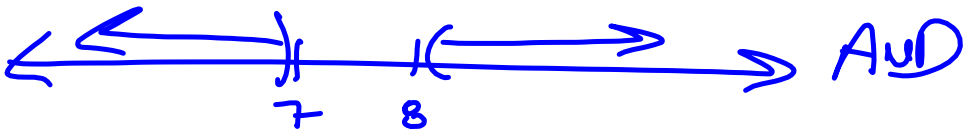
$$(-\infty, 7) \cup (8, \infty) = \{x \mid x < 7 \text{ OR } x > 8\}$$

$$(-\infty, 7) \cap (8, \infty) = \{x \mid x < 7 \text{ AND } x > 8\}$$

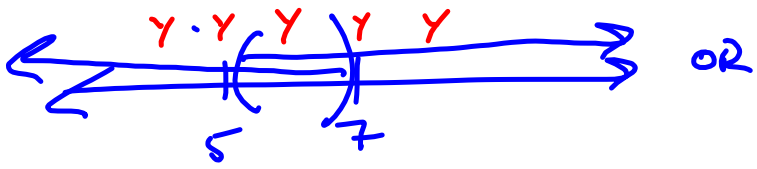
$$= \emptyset$$



$= (-\infty, 7) \cup (8, \infty)$

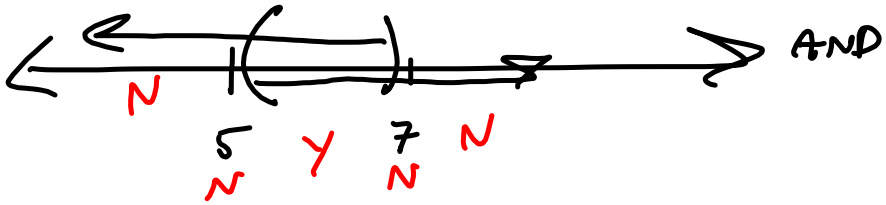


$\{x \mid x < 7 \text{ OR } x > 5\}$



$= (-\infty, \infty)$

$\{x \mid x < 7 \text{ AND } x > 5\}$



$= (5, 7)$