$$f(x) = 2x^5 \overbrace{+x^4 - 6x^3}^{1} \overbrace{+16x^2}^{2} \overbrace{+8x - 48}^{3}$$

① $2x^5$

② Descartes: 3 or 1 positive roots

$$f(-x) = \underbrace{-2x^5 + x^4}_{1} + 6x^3 \underbrace{+16x^2 - 8x}_{2} - 48$$

2 or 0 negative roots

③ Rational zeros

p's: $a_0 = -48$

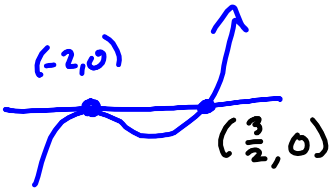
q's: $a_n = a_5 = 2$

$2 \,|\, 48$
$2 \,|\, 24$
$2 \,|\, 12$
$2 \,|\, 6$
3

$\pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm 48, \pm 3, \pm 6, \pm 12, \pm 24,$

$\pm \frac{1}{2}, \cancel{\pm \frac{2}{2}}, \cancel{\pm \frac{4}{2}}, \cancel{\pm \frac{8}{2}}, \cancel{\pm \frac{16}{2}}, \cancel{\pm \frac{48}{2}}, \pm \frac{3}{2}$

④



$x = -2, m = 2?$

$x = \frac{3}{2}, m = 1$

$$f(x) = 2x^5 + x^4 - 6x^3 + 16x^2 + 8x - 48$$

$x+2$

-2	2	1	-6	16	8	-48	
		-4	6	0	-32	48	
-2	2	-3	0	16	-24	0	sweet!
		-4	14	-28	24		
-2	2	-7	14	-12	0	sweet!	
		-4	22	-72			
	2	-11	36	Nope.			

$\frac{3}{2}$	2	-7	14	-12	
		3	-6	12	
	2	-4	8	0	sweet!

This gives $(x+2)^2(x-\frac{3}{2})(2x^2-4x+8)$

Analyze $2x^2-4x+8=0$

$\iff x^2-2x+4=0$

$a=1, b=-2, c=4$

$b^2-4ac=(-2)^2-4(1)(4)$

$=4-16=-12<0 \Rightarrow$

No real roots
(2 nonreal roots)

#4 $x=-2$, $m=2$

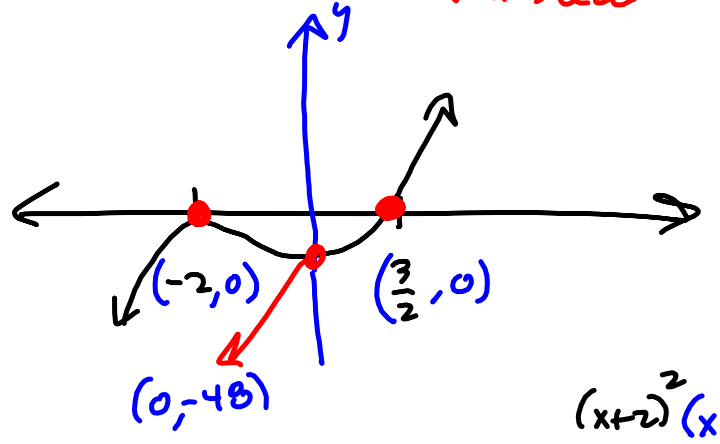
$x=\frac{3}{2}$, $m=1$

#5 $(x+2)^2(x-\frac{3}{2})(2x^2-4x+8)$

Irreducible over the real #. field.

5

6

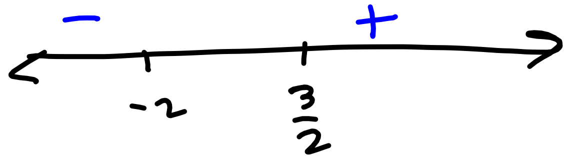


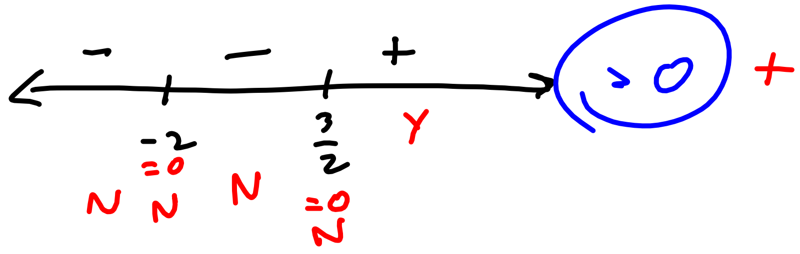
$(x+2)^2(x-\frac{3}{2})(2x^2-4x+8)$

Extra!

Solve $f(x) > 0$

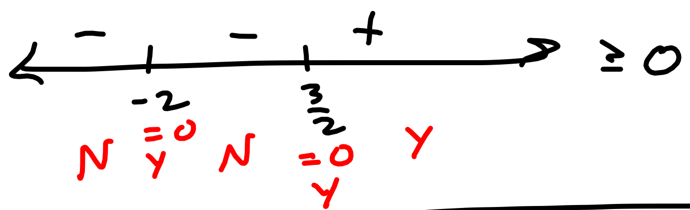
$= 2x^5 + \ldots$





$\Rightarrow x \in (\frac{3}{2}, \infty)$

what about $f(x) \geq 0$?



$\boxed{\Rightarrow x \in \{2\} \cup [\frac{3}{2}, \infty)}$

Compare & contrast with ">0" sitch.

$(x+2)^2\left(x-\frac{3}{2}\right)(2x^2-4x+8)$

7 Take it home:

x^2-2x+4

$a=1,\ b=-2,\ c=4$

$b^2-4ac=-12$

$$x=\frac{-(-2)\pm\sqrt{-12}}{2(1)}$$

$$=\frac{2\pm 2\sqrt{3}\,i}{2}$$

$$=1\pm\sqrt{3}\,i$$

$2\,|\,12$
$2\,|\,6$
3

$\sqrt{12}=2\sqrt{3}$

$$\Rightarrow f(x)=2(x+2)^2\left(x-\frac{3}{2}\right)\left(x-(1+\sqrt{3}i)\right)\left(x-(1-\sqrt{3}i)\right)$$

$$=2x^5+\cdots$$

Remember the leading coefficient

8

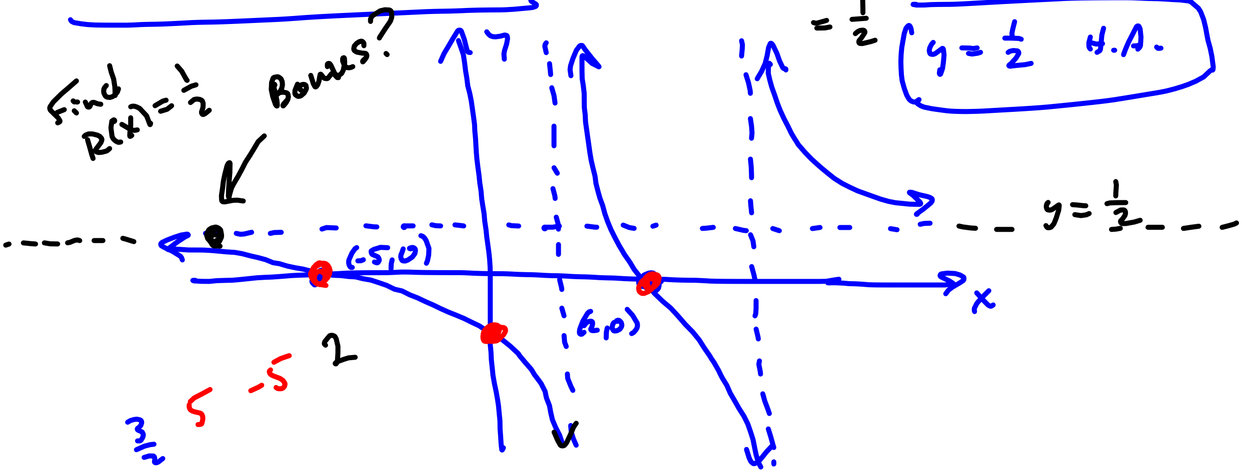
$$R(x) = \frac{x^2+3x-10}{2x^2-13x+15} = \frac{(x+5)(x-2)}{(2x-3)(x-5)}$$

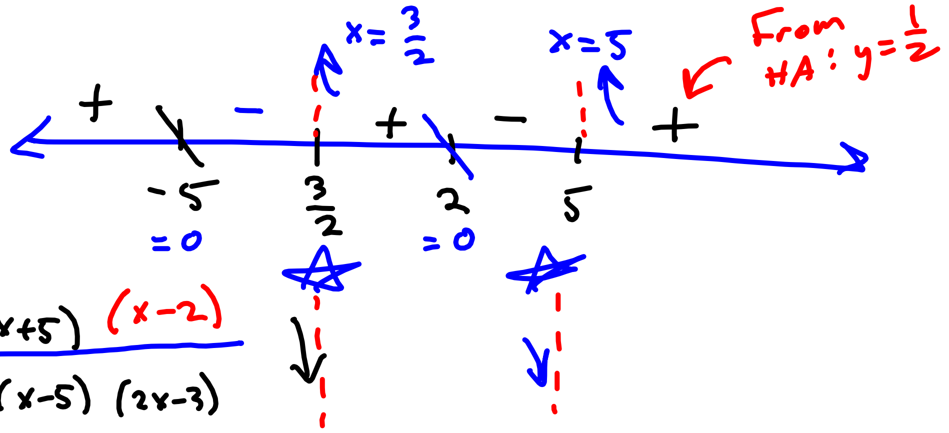
$$\mathcal{D}(R) = \mathbb{R} \setminus \left\{ \tfrac{3}{2}, 5 \right\} = \left(-\infty, \tfrac{3}{2}\right) \cup \left(\tfrac{3}{2}, 5\right) \cup (5, \infty)$$

$\boxed{\text{V.A.: } x = \tfrac{3}{2},\ x = 5}$

H.A.: $R(x) \xrightarrow{x \to \pm\infty} \frac{x^2}{2x^2} = \frac{1}{2}$ $\boxed{y = \tfrac{1}{2} \text{ H.A.}}$

Find $R(x) = \frac{1}{2}$ Bonus?





$$\frac{(x+5)(x-2)}{(x-5)(2x-3)}$$

$a = 2, b = -13, c = 15$

$b^2 - 4ac = (-13)^2 - 4(2)(15)$

$= 169 - 120$

$= 49$

$\sqrt{49} = 7$

$x = \frac{13 \pm 7}{2(2)}$

$\frac{20}{4} = 5$

$\frac{6}{4} = \frac{3}{2}$

$2(x-5)(x-\frac{3}{2}) = (x-5)(2x-3)$

$$\frac{\text{same degree}}{\text{same degree}} = \frac{x^2 + \cdots}{2x^2 + \cdots} \xrightarrow{x \to \pm\infty} \frac{x^2}{2x^2} = \frac{1}{2} = y$$

$$\frac{\text{Lower degree}}{\text{Higher degree}} = \underline{\text{Proper}} = \frac{273x^2 + \cdots}{1x^5 + \cdots}$$

$$\xrightarrow{x \to \pm\infty} 0 = y$$

$$\frac{\text{Higher degree}}{\text{Lower degree}}$$ O.A. (Oblique asymptote)

<u>Slant</u>

10 $Q(x) = \dfrac{x^3 - 7x^2 - 9x + 63}{x^2 - 6x + 5}$

$x^2(x-7) - 9(x-7)$

$= (x^2-9)(x-7)$

Long Division

$$\begin{array}{r} x - 1 \\ x^2 - 6x + 5 \overline{\big)\, x^3 - 7x^2 - 9x + 63} \\ -\left(x^3 - 6x^2 + 5x\right) \\ \hline -x^2 \end{array}$$

$\frac{-x^2}{x^2} = -1$

$\boxed{y = x - 1 \text{ is O.A.}}$

$x^2 - 6x + 5 = (x-5)(x-1)$

$(x^2-9)(x-7) = (x-3)(x+3)(x-7)$

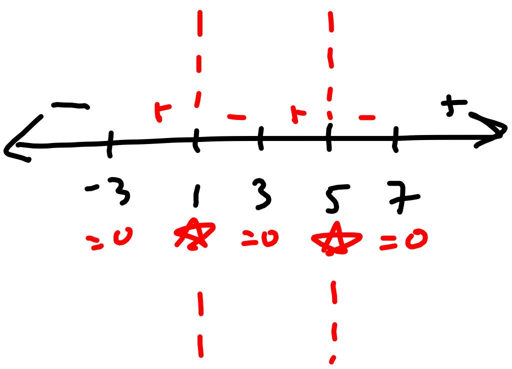
$$Q(x) = \frac{(x-3)(x+3)(x-7)}{(x-5)(x-1)}$$

$\mathcal{D}(Q) = \mathbb{R} \setminus \{1, 5\}$

$\boxed{\text{V.A.: } x=1,\ x=5}$

H.A.: NONE

$\boxed{\text{O.A.: } y = x-1}$



x-int: (3,0), (-3,0), (7,0)