§3.2 #55 $f(x) = 18x^3 - 21x^2 + 10x - 2$

p's: -2

q's: 18

Find possible rational zeros.

$$\begin{array}{r|l} 2 & 18 \\ 3 & 9 \\ & 3 \end{array}$$

$$\frac{p}{q} = \pm 1, \pm 2, \pm \frac{1}{2}, \cancel{\pm \frac{2}{2}}, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{1}{6}, \cancel{\pm \frac{2}{6}},$$

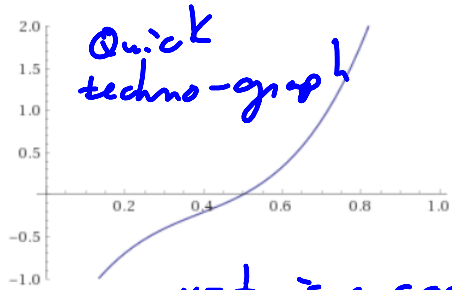
$$\pm \frac{1}{9}, \pm \frac{2}{9}, \pm \frac{1}{18}, \cancel{\pm \frac{2}{18} = \frac{1}{9}}$$

In the sequel, when they ask you to FIND all zeros (& Steve'll ask you to "split the polynomial into linear factors.)

I'm dividing $f(x)$ by $x-\frac{1}{2}$

$\frac{1}{2}$	18	−21	10	−2
		9	−6	2
	18	−12	4	0
	x^2	x	c	r

Quick techno-graph



$x=\frac{1}{2}$ is a good guess

This says...

$$f(x)=\left(x-\tfrac{1}{2}\right)\left(18x^2-12x+4\right)$$

Solve $18x^2-12x+4=0$

$\Rightarrow \quad 9x^2-6x+2=0$

$$b^2-4ac=(-6)^2-4(9)(2)$$
$$=36-72$$
$$=-36$$

Magic #: $9\cdot 2=18$

$-6=-5-1$	5
$=-4-2$	8
$=-3-3$	9

$2 \,|\, 36$
$2 \,|\, 18$
$3 \,|\, 9$
3

$2^5=32$
$2^6=64$
$2^7=128$
$2^8=256$

& $\boxed{\sqrt{36}=2\cdot 3\sqrt{1}=6}$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{6\pm 6i}{2(9)}=\frac{6(1\pm i)}{2\cdot 3\cdot 3}=\frac{1\pm i}{3}\Rightarrow$$

$$f(x)=18\left(x-\tfrac{1}{2}\right)\left(x-\left(\tfrac{1+i}{3}\right)\right)\left(x-\left(\tfrac{1-i}{3}\right)\right)$$

$= 18x^3 +$ smaller stuff

#21 3.2

$f(x) = 24x^3 - 26x^2 + 9x - 1$

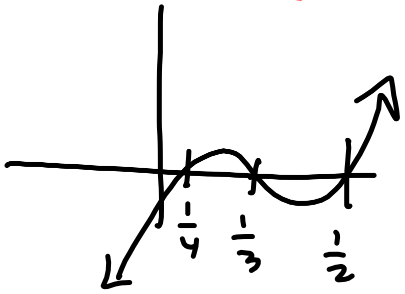
Find all real and imaginary zeros.

p's : 1

q's: 24

$2 \lfloor 24$
$2 \lfloor 12$
$2 \lfloor 6$
3

$\pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm \frac{1}{8}, \pm \frac{1}{3}, \pm \frac{1}{6}, \pm \frac{1}{12}, \pm \frac{1}{24}$



from grapher/tech

https://www.wolframalpha.com/

$\frac{1}{2}$	24	-26	9	-1
		12	-7	1
	24	-14	2	0

sweet!

Depressed polynomial is quadratic.

$f(x) = (x - \frac{1}{2})(24x^2 - 14x + 2)$

Solve this = 0

Descartes' rule of signs

$f(x) = \underbrace{2x^5 - 7x^4}_{1}\underbrace{\;}_{2} + 9x^3 + \underbrace{8x^2 - x^1}_{3}\underbrace{\;+ 7}_{4}$

4, 2, 0 possible positive zeros

Horizontal Flip. $f(-x) = -2x^5 - 7x^4 \underbrace{- 9x^3 + 8x^2}_{1} + x + 7$

1 negative zero

3 sign changes

3 or 1 pos./neg. zeros

5 sign changes

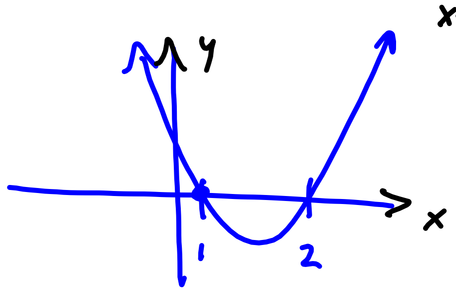
5, 3, 1 possible...

$(-x)^5 = ((-1)(x))^5 = (-1)^5 (x)^5 = -x^5$

$(-1)^5 = (-1)(-1)(-1)(-1)(-1) = -1$

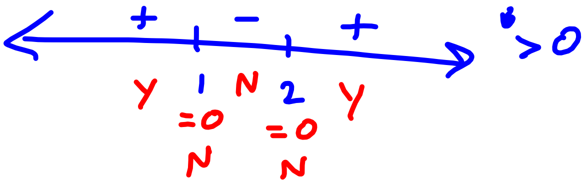
$x^2-3x+2 > 0$

$\Rightarrow (x-2)(x-1) > 0 \Rightarrow x \in (-\infty, 1) \overset{\text{OR}}{\cup} (2, \infty)$

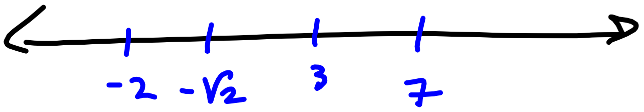


$+$ means > 0

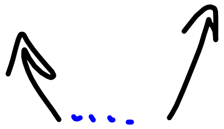
$-$ " < 0



$(x-3)(x+2)(x-7)(x+\sqrt{2}) \leq 0$

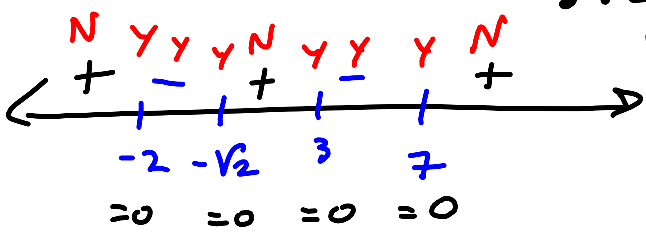


$(x-3)(x+2)(x-7)(x+\sqrt{2}) = x^4$ + smaller.



x^4 end behavior.

Sign pattern



≤ 0

$$(x-3)(x+2)(x-7)(x+\sqrt{2}) \le 0 \Rightarrow$$

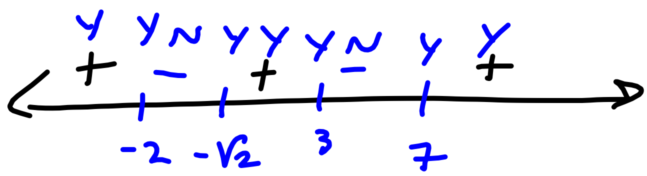
$$x \in [-2, -\sqrt{2}] \cup [3, 7]$$

$$\sqrt{(x-3)(x+2)(x-7)(x+\sqrt{2})}$$

Find Domain

Need the radicand ≥ 0

Need $(x-3)(x+2)(x-7)(x+\sqrt{2}) \ge 0$



$$\boxed{x \in (-\infty, -2] \cup [-\sqrt{2}, 3] \cup [7, \infty)}$$