$(x-(2+3i))(x-(2-3i))$

$z = 2+3i$ & $2-3i = \bar{z}$ are a conjugate pair.

$(x-2-3i)(x-2+3i)$

$= x^2 - 2x + 3ix - 2x + 4 - 6i - 3ix + 6i - 9i^2$

$9+4=13$

$= x^2 - 4x + 13$

$-9i^2 = 9$

$z = a+bi$

$\bar{z} = a-bi$

$i^2 = -1$

$i = \sqrt{-1}$

$z\bar{z} = (a+bi)(a-bi)$

$= a^2 - abi + abi - b^2i^2$

$(a+bi)(a-bi) = a^2 + b^2$ quick twitch.

$(a-b)(a+b) = a^2 - b^2$

$(3+2i)(3-2i) = 3^2 + 2^2 = 13$

$x^2 - 4x + 13 = 0 \Rightarrow x = 2 \pm 3i$

because we see factors $x-(2+3i)$, $x-(2-3i)$

$(x-\sqrt{3})(x+\sqrt{3})$

$= x^2 - (\sqrt{3})^2 = x^2 - 3$

Notice that when zeros occur in conjugate pairs, that the resulting polynomial has real coefficients

$x^2 - 4x + 13 = 0$

Conjugate Pairs Theorem: If you have nonreal zeros AND real coefficients, then the zeros occur in conjugate pairs.

Build a polynomial with real coefficients that has $x = 2, 2+3i$ as zeros. Leave in factored form.

$$(x-2)(x-(2+3i))(x-(2-3i))$$

By C.P.T.

See?

$(x-2)(x^2-4x+13)$

$= x^3 - 4x^2 + 13x$
$\quad - 2x^2 + 8x - 26$

$x^3 - 6x^2 + 21x - 26$

↑ ↑ ↑ ↑

Break it Down

Divide by $x-2$:

2	1	-6	21	-26
		2	-8	26
	1	-4	13	0

Sweet!

(x^2, x)

This says

$x^3 - 6x^2 + 21x - 26$

$= (x-2)(x^2-4x+13)$

$x^2 - 4x + 13$

depressed polynomial

$x^4 - 5x^3 - 13x^2 + 107x - 390$

rational zeros:

$\pm 1, \pm 2, \pm 3, \pm 5, \pm 13$
$\pm 6, \pm 10, \pm 26,$
$\pm 15, \pm 39, \pm 30, \pm 78, \pm 390$
$\pm 65,$

2 | 390
3 | 195
5 | 65
13

x−6:

6	1	−5	−13	107	−390
		6	6	−42	390
	1	1	−7	65	0

x+5

Sweet!

This work says

$x^4 - 5x^3 - 13x^2 + 107x - 390$

$= (x-6)(x^3 + x^2 - 7x + 65) + 0$

& $x^3 + x^2 - 7x + 65$ is the depressed polynomial.

$3 \cdot 65$ (×6) = 390

Splitting off x−6 reduced a 4th-degree question to a 3rd degree

Dividing by $x-(-5)$ $x^3+x^2-7x+65$

$$\begin{array}{r|rrrr} -5 & 1 & 1 & -7 & 65 \\ & & -5 & 20 & -65 \\ \hline & 1 & -4 & 13 & 0 \end{array}$$ Sweet!

This says $(x-6)(x+5)(x^2-4x+13)$

$a=1,\ b=-4,\ c=13$

$b^2-4ac = (-4)^2-4(1)(13)$

$= 16-52$

$= -36$

$x = \dfrac{4 \pm \sqrt{-36}}{2(1)} = \dfrac{4 \pm 6i}{2} = \dfrac{2 \pm 3i}{1} = 2 \pm 3i$

$x = 6, -5, 2 \pm 3i$

Factored Form:

$(x-6)(x+5)(x-(2+3i))(x-(2-3i))$

We have split it into linear factors.

Fundamental Theorem of Agebra says we can ALWAYS split a polynomial of degree n into n linear factors.

(Complex Numbers comprise an algebraically closed field)

Degree n $\Rightarrow$ n complex zeros & we can split it!

No such thing as an irreducible polynomial, if you're operating in Complex # field

Reals aren't closed.

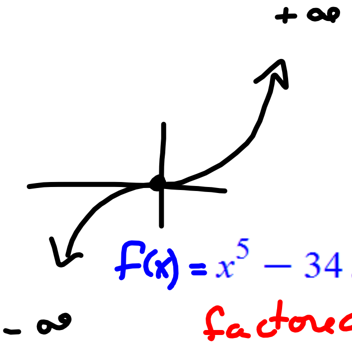
$f(x) = x^2 + 3$ is irreducible over the real number field.

$x = \pm\sqrt{3}\, i$ solve it.

$(\sqrt{3} i)^2 + 3 = \sqrt{3}^2 i^2 + 3 = -3 + 3 = 0$

$(-\sqrt{3} i)^2 + 3 = (-\sqrt{3})^2 i^2 + 3 = 3(-1) + 3 = 0$

x^5



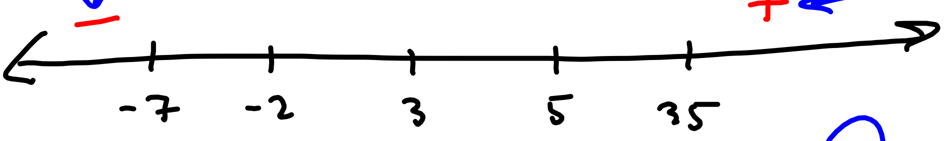
$f(x) = x^5 - 34x^4 - 78x^3 + 1528x^2 - 595x - 7350$

factored $f(x)$:

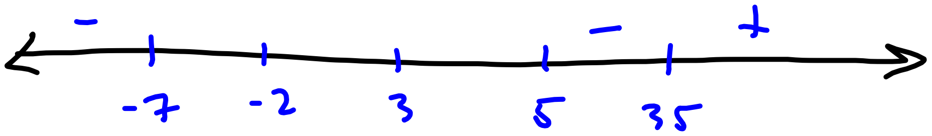
Solve $f(x) = (x+2)(x-3)(x-5)(x+7)(x-35) > 0$

x^5 controls the end behavior.

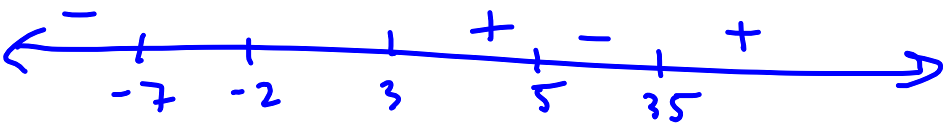
Because x^5:



Because $x-35$ changes sign @ $x=35$:

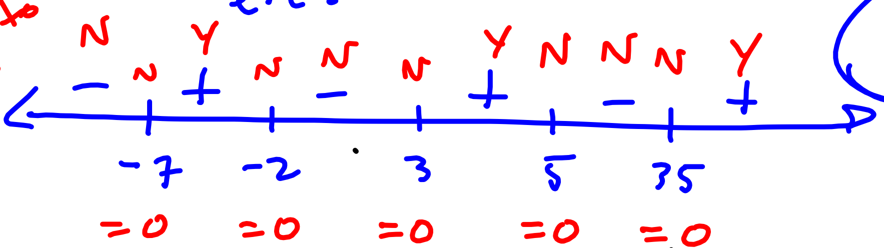


Because $x-5$ changes sign at $x=5$:



Sign Patterns to CRUCIAL TEST 3.

etc!



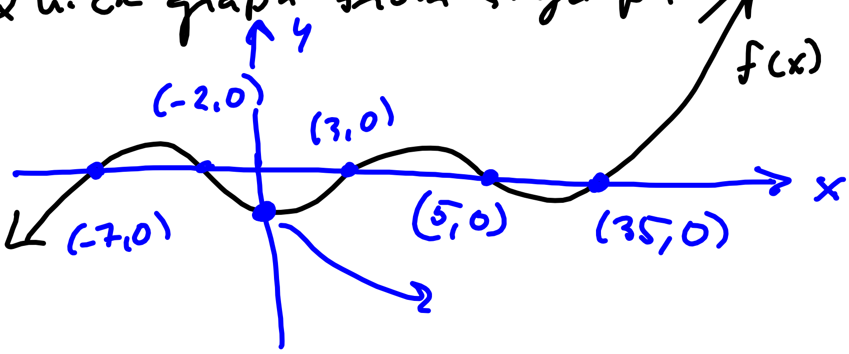
want > 0

$f(x) = (x+2)(x-3)(x-5)(x+7)(x-35) > 0$

→ $x \in (-7,-2) \cup (3,5) \cup (35,\infty)$

$(0,-7350)$

Quick graph from sign pattern



$f(x) = (x+2)(x-3)(x-5)(x+7)(x-35)$ →

$f(0) = (2)(-3)(-5)(7)(-35) = -7350$