§5.3 #43

$y = \log_2(x+2)$

$y = 3 - \log_2(x)$

Solve $\log_2(x+2) = 3 - \log_2(x)$

$\log_2(x+2) + \log_2(x) = 3$

$\log_2((x+2)x) = 3$

$2^{\log_2((x+2)x)} = 2^3$

$x^2 + 2x = 8$

$x^2 + 2x - 8 = 0$

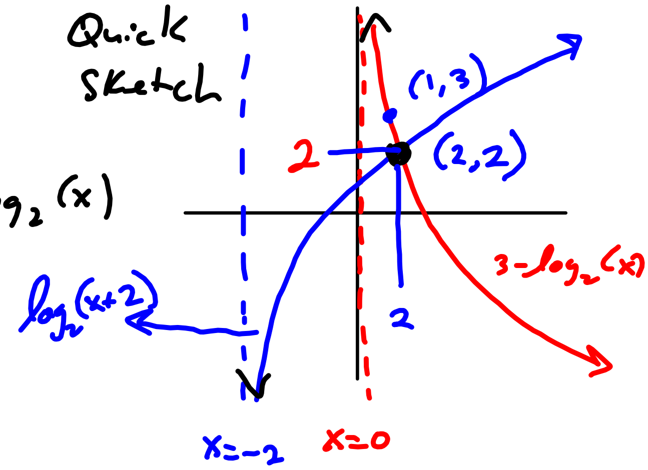
$(x+4)(x-2) = 0$

$x \in \{-4, 2\}$

$-4 \notin D$ (Problem)

$\boxed{x \in \{2\}}$

Quick Sketch



$\log_2(2+2) = 3 - \log_2(2)$?

$\log_2(4) = 3 - 1$

$2 = 3 - 1$? yes

Sketch the system

$5x + 3y = 15$

x	y
0	5
3	0

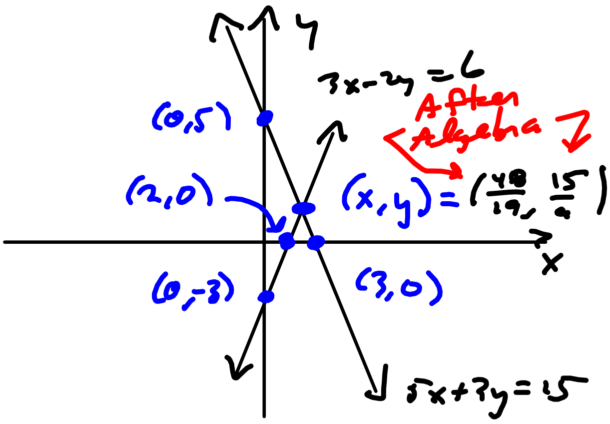
$3x - 2y = 6$

x	y
0	-3
2	0

$(x,y) \in QI$

$2 < x < 3$

$0 < y < 5$



Solve system:

$5x + 3y = 15$

$3x - 2y = 6$

LCM of 5 & 3 is 15

$-3(5x + 3y = 15)$

$5(3x - 2y = 6)$

$-3R1 \quad -15x - 9y = -45$

$5R2 \quad 15x - 10y = 30$

$-3R1 + 5R2 \quad 0 - 19y = -15$

$\boxed{y = \frac{15}{19}}$

New System:

$5x + 3y = 15$

$-19y = -15$

$y = \frac{15}{19}$

$5x + 3y = 15$

$5x + 3\left(\frac{15}{19}\right) = 15$

$5x + \frac{45}{19} = 15$ LCD = 19

$\frac{5x}{1} \cdot \frac{19}{19} + \frac{45}{19} = \frac{15}{1} \cdot \frac{19}{19}$

$\frac{95x + 45}{LCD} = \frac{285}{LCD}$

$95x + 45 = 285$

$95x = 240$

$x = \frac{240}{95} = \boxed{\frac{48}{19} = x}$

$(x, y) = \left(\frac{48}{19}, \frac{15}{19}\right)$

$$\begin{array}{r} 19 \\ \underline{5} \end{array} \qquad \begin{array}{r} {}^{1}190 \\ \underline{95} \\ 285 \end{array}$$

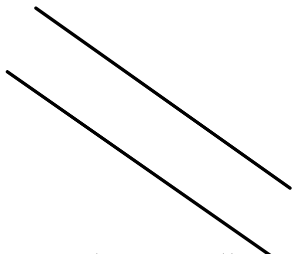
Reductio ad Absurdum

Reduce to Absurdity

Consider

Assume There IS a solution.

$$5x + 2y = 10$$
$$10x + 4y = 5$$
$$5x + 2y = \frac{5}{2}$$



These two lines are parallel!

No Solution!

$-2R1 \quad -10x - 4y = -40$

$R2 \quad 10x + 4y = 5$

$-2R1 + R2 \quad 0 = -35$?! RIDIKELUS.

Conclude something absurd.

Therefore, there is no solution

A basketball team has 20 players.

How many ways can a starting 5 be chosen from the 20 players?

20 choose 5

$C(20,5) = \frac{20!}{5!\,15!} = \frac{\cancel{20}\cdot 19 \cdot \cancel{18} \cdot 17 \cdot 16}{\cancel{5\cdot 4}\cdot\cancel{3\cdot 2}} = 19\cdot 3\cdot 17\cdot 16 = \boxed{15504 = C(20,5)}$

There are 5 positions for the starting 5: Center, small forward, power forward, shooting guard, point guard. How many ways can you choose 5 players to make up a starting 5?

20 choose-and-arrange 5

$P(20,5) = 19\cdot 17\cdot 3\cdot 16\cdot \underline{5\cdot 4\cdot 3\cdot 2}$ because

$C(20,5)\cdot 5! = P(20,5)$

$= \frac{20!}{(20-5)!} = \frac{20!}{15!} = 20\cdot 19\cdot 18\cdot 17\cdot 16 = \boxed{1860480}$

$= P(20,5)$

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20 nCr 5
     15504.00000
20 nPr 5
     1860480.000
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$$\sum_{k=1}^{\infty} 5\left(\frac{2}{7}\right)^{k-1} = 5\left(\frac{1}{1-\frac{2}{7}}\right)$$

$$S' : \sum_{k=1}^{n} a\cdot r^{k-1} = a\left(\frac{1-r^n}{1-r}\right)$$

$$= a + ar + ar^2 + \dots + ar^{n-1}$$

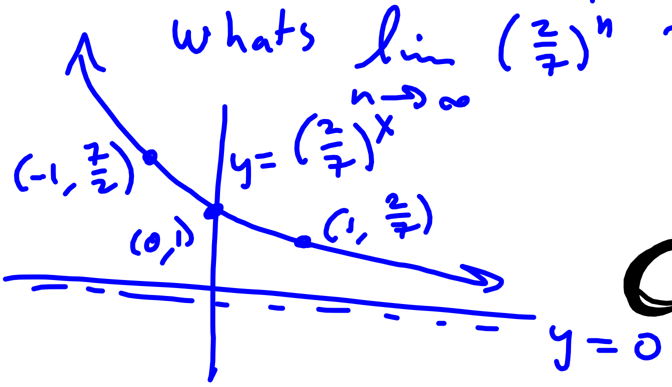
What happens if $r = \frac{2}{7}$?

$$S'_n = 5\left(\frac{1-\left(\frac{2}{7}\right)^n}{1-\frac{2}{7}}\right) \xrightarrow{n\to\infty} 5\left(\frac{1}{\frac{5}{7}}\right) = 5\cdot\frac{7}{5} \boxed{= 7}$$

what happens if $n \to \infty$?

This: $5\left(\frac{1-\left(\frac{2}{7}\right)^{\infty}}{1-\frac{2}{7}}\right) = 5\left(\frac{1-0}{1-\frac{2}{7}}\right)$

whats $\lim_{n\to\infty} \left(\frac{2}{7}\right)^n$? 0.



0

$$\sum_{k=1}^{\infty} a r^{k-1} = \begin{cases} \infty & \text{if } |r| \geq 1 \\ a\left(\frac{1}{1-r}\right) & \text{if } -1 < r < 1 \end{cases}$$

because $\left(\frac{5}{3}\right)^n \xrightarrow{n \to \infty} \infty$

and $\left(\frac{3}{5}\right)^n \xrightarrow{n \to \infty} 0$