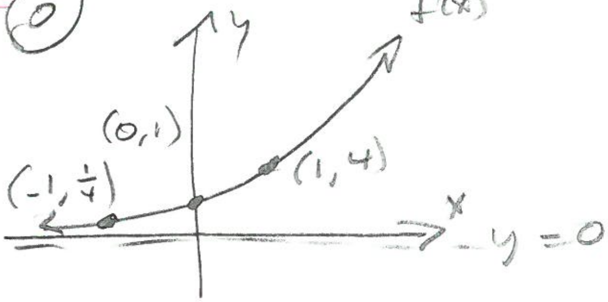
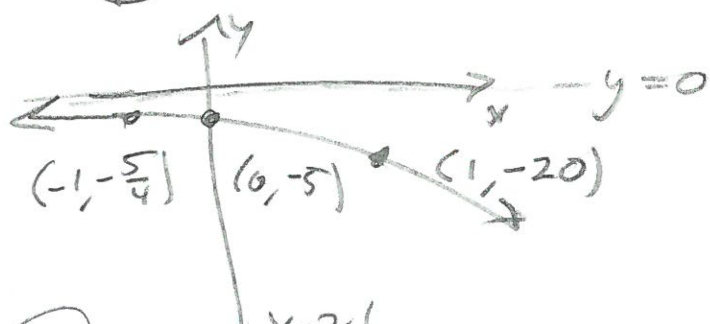
121 TEST 4

1 20pts $f(x) = 4^x$, $g(x) = -5 \cdot 4^{3x-21} + 2$

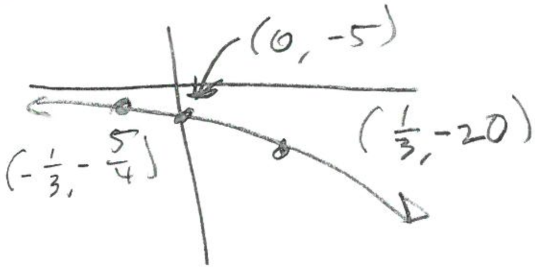
0



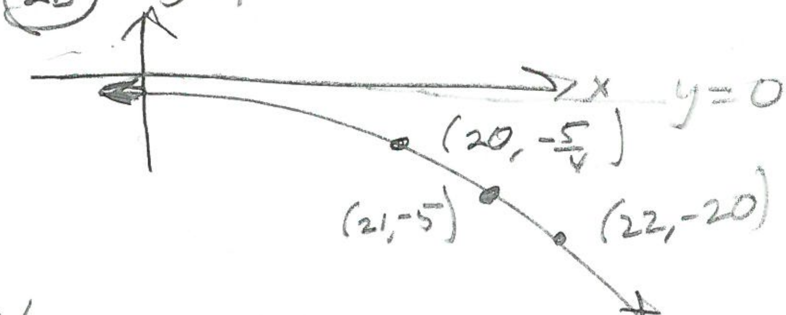
1 $-5 \cdot 4^x$



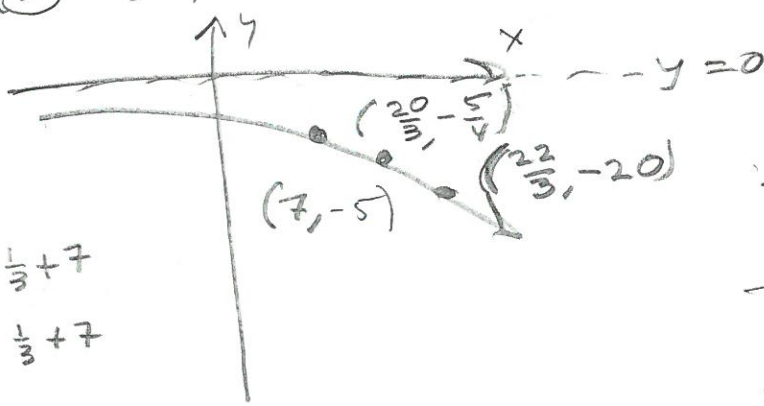
2a $-5 \cdot 4^{3x}$



2b $-5 \cdot 4^{x-21}$



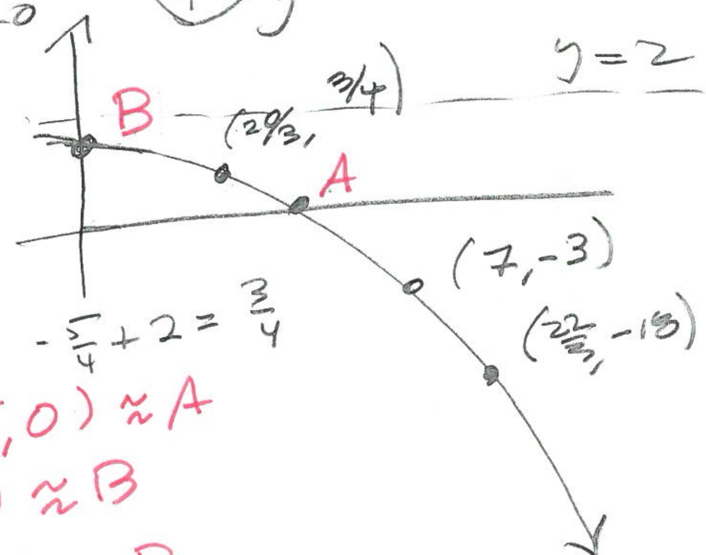
3 $-5 \cdot 4^{3(x-7)} = -5 \cdot 4^{3x-21}$



$-\frac{1}{3} + 7$

$\frac{1}{3} + 7$

4 $g(x)$!



$-\frac{5}{4} + 2 = \frac{3}{4}$

$\cancel{B} \approx (6.7796786 5, 0) \approx A$

$\cancel{A} \approx (0, 2.00000) \approx B$

EXACT : $\cancel{A} = (0, 5 \cdot 4^{-21} + 2) = B$

$\cancel{B} = (\frac{1}{3} \log_4(\frac{2}{5}) + 7, 0) = A$

121 J4

2. x-int:

10pts

$g(x) = 0$

$-5 \cdot 4^{3x-21} + 2 = 0$

$-5 \cdot 4^{3x-21} = -2$

$4^{3x-21} = \frac{-2}{-5} = \frac{2}{5}$

$\log_4(\quad) = \log_4(\quad)$

$3x - 21 = \log_4\left(\frac{2}{5}\right)$

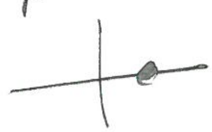
$3x = \log_4\left(\frac{2}{5}\right) + 21$

$x = \frac{\log_4\left(\frac{2}{5}\right) + 21}{3}$

$\boxed{A = \left(\frac{1}{3}\log_4\left(\frac{2}{5}\right) + 7, 0\right)}$

$\approx (6.779678651, 0)$

positive



y-int:

$g(0) = 5 \cdot 4^{-21} + 2$

$= \frac{5}{4^{21}} + 2$

$\boxed{B = (0, 5 \cdot 4^{-21} + 2)}$

$\approx (0, 2.0000)$

121 T4

(3) $-5 \cdot 4^{3y-21} + 2 = x$

5pts

$-5 \cdot 4^{3y-21} = x - 2$

$4^{3y-21} = \frac{x-2}{-5}$

$\log_4(\text{left}) = \log_4(\text{right})$

$3y - 21 = \log_4\left(\frac{x-2}{-5}\right)$

$3y = \log_4\left(\frac{x-2}{-5}\right) + 21$

$$\boxed{y = \frac{1}{3}\log_4\left(\frac{x-2}{-5}\right) + 7 = g^{-1}(x)}$$

A21 T4

(4) $f(x) = \sqrt{x+5}$ & $g(x) = x^2 - 3x - 5$

(a) $\boxed{\mathcal{D}(f) = [-5, \infty)} = \{x \mid x \geq -5\}$ 5pts

(from Need $x+5 \geq 0$
$x \geq -5$)

(b) $\boxed{\mathcal{D}(g) = (-\infty, \infty)} = \mathbb{R}$ 5pts

(g is polynomial)

(c) $\boxed{\left(\frac{f}{g}\right)(x) = \dfrac{\sqrt{x+5}}{x^2-3x-5}}$ 5pts

(d) $\mathcal{D}\left(\frac{f}{g}\right) = \{x \mid x \in \mathcal{D}(f)$ and $x \in \mathcal{D}(g)$ & $g(x) \neq 0\}$

$= \{x \mid x \geq -5$ and $x \in \mathbb{R}$ and $x^2 - 3x - 5 \neq 0\}$

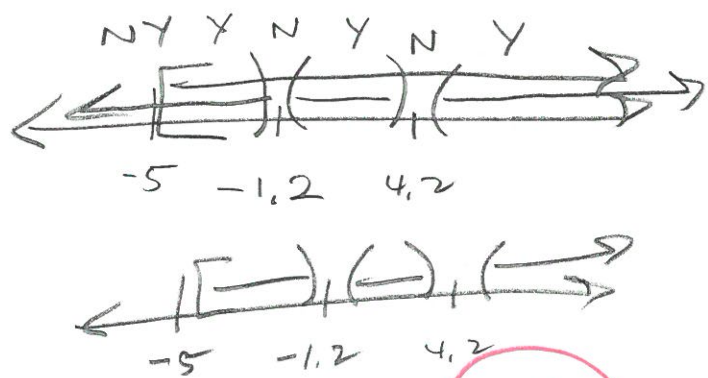
$x^2 - 3x - 5 = 0$

$x^2 - 3x + \left(\frac{3}{2}\right)^2 = 5 + \frac{9}{4}$

$\left(x - \frac{3}{2}\right)^2 = \frac{29}{4}$

$x - \frac{3}{2} = \pm \frac{\sqrt{29}}{2}$

$x = \dfrac{3 \pm \sqrt{29}}{2} \notin \mathcal{D}\left(\frac{f}{g}\right)$



$[-5, -1) \cup ($ 5pts

$\boxed{\left[-5, \frac{3-\sqrt{29}}{2}\right) \cup \left(\frac{3-\sqrt{29}}{2}, \frac{3+\sqrt{29}}{2}\right) \cup \left(\frac{3+\sqrt{29}}{2}, \infty\right)}$

121 T4

(4) (e) $(f \circ g)(x) = f(g(x))$

$= \sqrt{x^2 - 3x - 5 + 5}$

$= \boxed{\sqrt{x^2 - 3x} = f \circ g}$

(f) $\mathcal{D}(f \circ g) = \{x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f)\}$

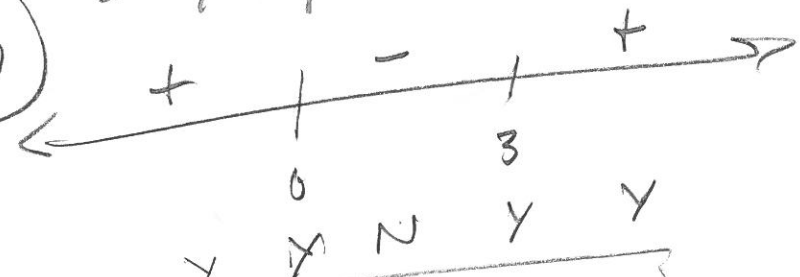
$= \{x \mid x \in \mathbb{R} \text{ and } x^2 - 3x - 5 \geq -5\}$

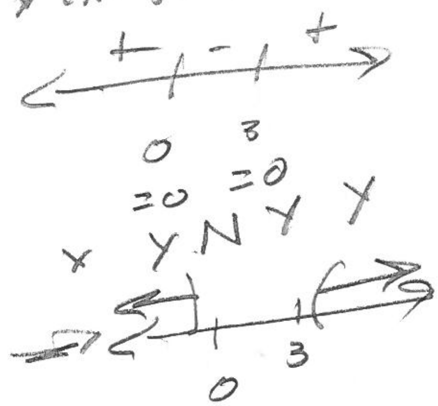
$= \{x \mid x^2 - 3x \geq 0\}$

$x^2 - 3x \geq 0$

$x(x-3) \geq 0$

≥ 0





$\boxed{\mathcal{D} = (-\infty, 0] \cup [3, \infty)}$

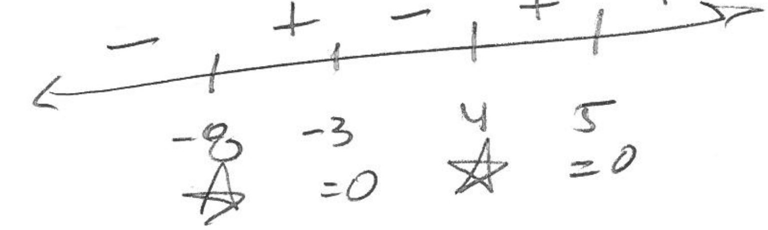
121 JY

(5) $g(x) = \sqrt{\dfrac{(x+3)(x-5)^2}{(x-4)^3(x+8)}} = \sqrt{\text{stuff}}$

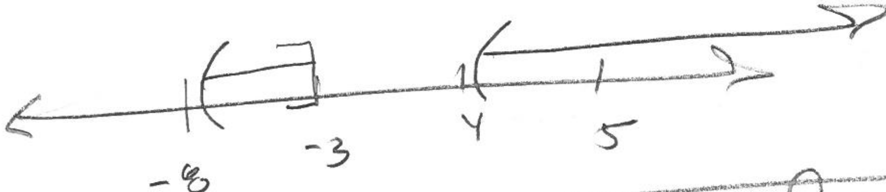
Need stuff ≥ 0

N N Y Y N N Y Y Y

$-$ $+$ $-$ $+$ $+$



≥ 0



$= \boxed{(-8, -3] \cup (4, \infty) = \mathcal{D}(f \circ g)}$

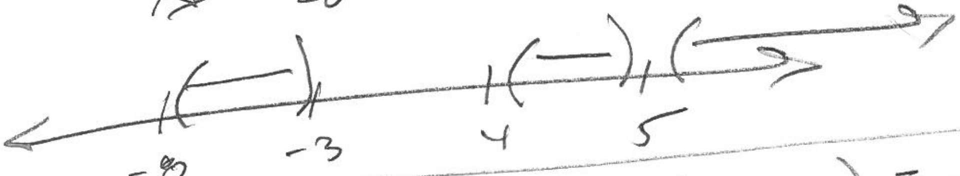
(6) $\mathcal{D}(\log_7 (\underset{\text{STUFF}}{\text{same}}))$ needs

stuff > 0

N N Y N N N Y N Y

$-$ $+$ $-$ $+$ $+$

> 0



$= \boxed{(-8, -3) \cup (4, 5) \cup (5, \infty) = \mathcal{D}.}$

121 T4

(7) $\ln(x-5) + \ln(x+2) = \ln(18)$

$$A = B$$

$$e^A = e^B$$

$$e^{\ln(x-5) + \ln(x+2)} = e^{\ln(18)}$$

$$e^{\ln(x-5)} e^{\ln(x+2)} = e^{\ln(18)}$$

$$(x-5)(x+2) = 18$$

$$x^2 - 3x - 10 = 18$$

$$x^2 - 3x - 28 = 0$$

$$(x-7)(x+4) = 0$$

$$x \in \{-4, 7\}$$

Extraneous

$$\boxed{x \in \{7\}}$$

$\mathcal{D}$: Need $x - 5 > 0 \Rightarrow x > 5$

&

$x + 2 > 0 \Rightarrow x > -2$

So Need $x > 5$ (stronger condition)

So $x = -4 \notin \mathcal{D}$.

Check $x = 7$: $\ln(2) + \ln(9) = \ln((2)(9)) = \ln(18)$

121 T4

(8) $\frac{1}{2}$-life is 5400 yrs

$$A(5400) = A_0 e^{5400k} = \frac{1}{2}A_0$$

(a)

$$e^{5400k} = \frac{1}{2}$$

$$\ln(\ )$$

$$5400k = \ln\left(\tfrac{1}{2}\right)$$

$$\boxed{k = \frac{\ln\left(\frac{1}{2}\right)}{5400}}$$

$-1.28360589 \times 10^{-4}$

$.000128360589 \approx k$

$$\boxed{A(t) = A_0 e^{\frac{\ln\left(\frac{1}{2}\right)}{5400} t}}$$

(b) 28% gone $\Rightarrow$ 72% remains

$$A_0 e^{kt} = .72 A_0$$

$$e^{kt} = .72$$

$$\ln(e^{kt}) = \ln(.72)$$

$$kt = \ln(.72)$$

$$t = \frac{\ln(.72)}{k} = \frac{\ln(.72)}{\frac{\ln\left(\frac{1}{2}\right)}{5400}} = \frac{5400\ln(.72)}{\ln\left(\frac{1}{2}\right)}$$

$$\approx 2559.228417$$

$$\approx \boxed{2559 \text{ yrs}}$$

121 TY

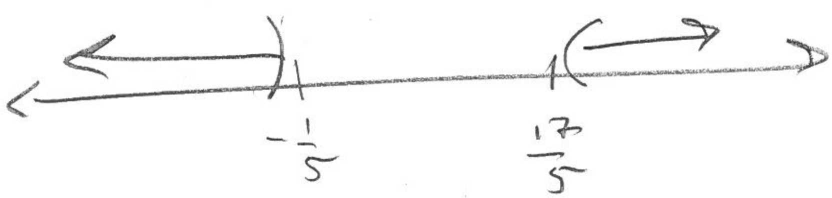
B1 $|-5x+8| - 11 > -2$

$|-5x+8| > 9$

$-5x+8 > 9$ or $-5x+8 < -9$

$-5x > 1$ $\quad$ $-5x < -17$

$\{ x \mid x < -\frac{1}{5}$ OR $x > \frac{17}{5} \}$

$=$ 

$= (-\infty, -\frac{1}{5}) \cup (\frac{17}{5}, \infty)$

B2 $f(x) = 5x^2 - 3x + 1$

$= 5\left(x^2 - \frac{3}{5}x + \left(\frac{3}{10}\right)^2\right) + 1 - 5\left(\frac{9}{100}\right)$

$= \boxed{5\left(x - \frac{3}{10}\right)^2 + \frac{11}{20}}$ $\quad$ $\boxed{(h,k) = \left(\frac{3}{10}, \frac{11}{20}\right)}$

$1 - 5\left(\frac{9}{100}\right) = 1 - \frac{9}{20} = \frac{20-9}{20} = \frac{11}{20}$

. 121 T4

(B3) $3(7.7)^x = 11 \cdot (2.1)^x$

$$\ln\left(3 \cdot (7.7)^x\right) = \ln\left(11 \cdot (2.1)^x\right)$$

$$\ln(3) + \ln\left((7.7)^x\right) = \ln(11) + \ln\left((2.1)^x\right)$$

$$a + x\ln(7.7) = b + x\ln(2.1)$$

$$a + xc = b + xd \qquad a = \ln(3)$$

$$a + cx = b + dx \qquad b = \ln(11)$$

$$cx - dx = b - a \qquad c = \ln(7.7)$$

$$x(c-d) = b - a \qquad d = \ln(2.1)$$

$$x = \frac{b-a}{c-d} = \boxed{\frac{\ln(11) - \ln(3)}{\ln(7.7) - \ln(2.1)} = x \approx} \; ! \; !$$

$$\boxed{\approx 1. = x} \; !$$

(B4) Let x = time (in hrs) that John works.

Then $x+2$ = time (in hrs) that Bill works,

so $\frac{1}{5}x + \frac{1}{8}(x+2) = 1$

$$\frac{8x + 5(x+2)}{40} = \frac{40}{40}$$

$$8x + 5x + 10 = 40$$

$$13x = 30 \longrightarrow \boxed{x = \frac{30}{13} \;\&\; x+2 = \frac{56}{13}}$$

John Bill

.121 T4

B5 $5000\left(1+\frac{.04}{52}\right)^{52(10)} \approx 7457.976607$

$\approx \boxed{\$7457.98}$

52039.74 if no parens in exponent

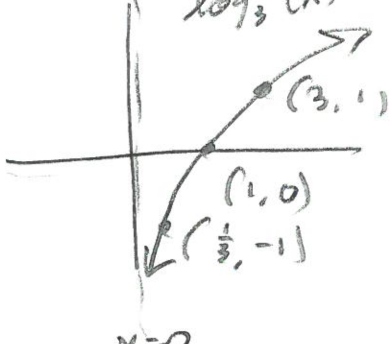
B6 $5000\left(1+\frac{.04}{52}\right)^{-(52)(10)} \approx 3352.115636$

$\approx \boxed{\$3352.12}$

$\frac{1}{3}+20=\frac{61}{3}$

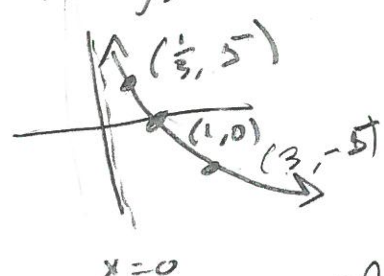
B7 $g(x) = -5\log_3(3x-21)+2$

$\log_3(x)$



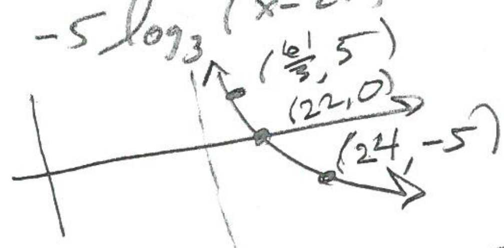
$x=0$

$-5\log_3(x)$



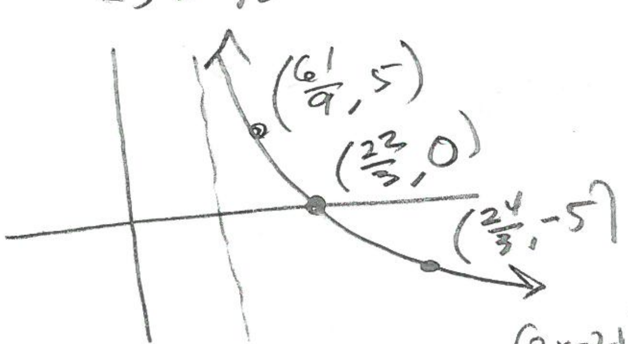
$x=0$

$-5\log_3(x-21)$



$x=21$

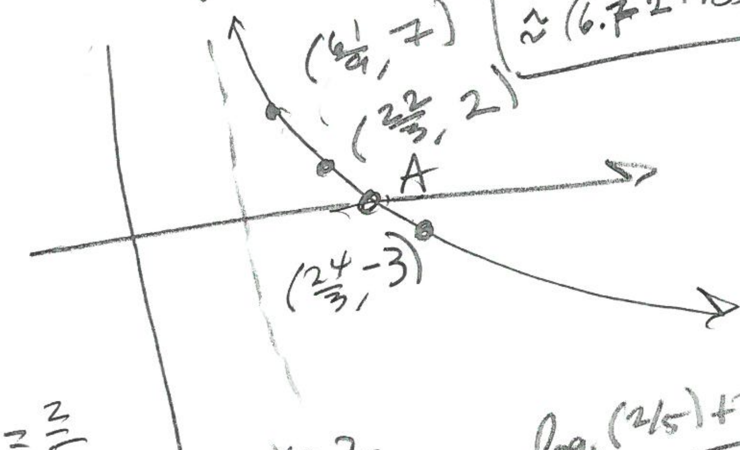
$-5\log_3(3x-21)$



$x=7$

$-5\log_3(3x-21)+2$

$A=\left(\log_3\left(\frac{2}{5}\right)+21, 0\right)$
$\approx (6.721985, 0)$



$x=2$

$\log_3(3x-21)=\frac{2}{5}$

$3x-21=\log_3\left(\frac{2}{5}\right)$

$3x=\log_3\left(\frac{2}{5}\right)+21 \Longrightarrow x=\frac{\log_3(2/5)+21}{3}$