MAT 121 | Test 3 – spring, 2017 | Name______________________________
70 Points | Polynomial and Rational Functions

You know how to format and write your work. I remind some of my hard workers that you *don't* have to write out the question

1. (10 pts) Form a polynomial in *factored form* and *minimal possible degree* that has real coefficients (after expanding) and will have the given zeros. Do *not* expand your polynomial. Leave it factored! If you run out of room, you're doing it wrong!

$x=-7$, multiplicity 3; $x=3-7i$, multiplicity 1; $x=5$, multiplicity 1.

Bonus (5 pts) What is the degree of your answer to #1?

2. (10 pts) Use synthetic division to find $P(1)$ if $P(x)=x^5-7x^4+12x^3+16x^2-37x-33$.

3. (5 pts) Represent the work you just did on the previous problem by writing $P(x)$ in the form $\mathit{Dividend} = \mathit{Divisor} \bullet \mathit{Quotient} + \mathit{Remainder}$.

4. Suppose $f(x)=(x+4)^2(x-1)(x-5)(x-9)=x^5-7x^4-45x^3+187x^2+584x-720$.

   I'm showing you both factored and expanded form to help you answer the following:

   a. (5 pts) Use a sign pattern to solve the inequality $f(x)<0$.

   b. (5 pts) Provide a rough sketch of f, using its zeros, their respective multiplicities and its end behavior. Include *x*- and *y*-intercepts. Your graph should be smooth. Un-exaggerate the vertical for a better-quality graph.

   c. (5 pts) What is the domain of $g(x)=\sqrt{\dfrac{(x-9)(x+4)^2}{(x-1)(x-5)}}$?

5. Let $P(x)=x^5-7x^4+12x^3+16x^2-37x-33$. You've seen it, already, in #2.

   a. (5 pts) Use Descartes' Rule of Signs to determine the possible number of positive and negative zeros of P.

   b. (5 pts) List all possible rational zeros of P.

6. (10 pts) Find the *real* zeros of P. Then factor P over the set of **real numbers**. This should involve an irreducible quadratic factor.

   (If things go haywire, come up with (BUILD!) a *plausible*-looking polynomial, in factored form, with the right number of real roots and 2 nonreal roots. The 2 nonreal roots will still be living inside the irreducible quadratic factor, so you'll have to make up a quadratic factor with nonreal zeros).

7. (5 pts) Find the remaining (nonreal) zeros of P and factor P over the set of **complex numbers**. This step requires breaking down the quadratic piece that's irreducible over the real numbers. The fundamental theorem tells us that *nothing* is irreducible over the complex numbers.

   (You can still get full credit for this one, even if things went haywire, in #6, if you work your build, correctly, and display your *plausible*-looking follow-up to your *plausible*-looking answer to #6. The more you know about what you're doing, the more points you'll earn.)

   And yes, "*plausible*" must *still* always be italicized.

8. (5 pts) You don't need to graph $R(x)=\dfrac{2x^3-3x^2-3x+2}{x^2-x-6}$, here, but I do want to see you graph its asymptotes. Hint: This function has no holes. Hint: Don't waste time trying to factor the numerator.

9. (10 pts) Sketch the graph of $F(x)=\dfrac{3x^2-7x-26}{x^2+2x-15}$. Show all asymptotes and intercepts.

ANSWER ANY THREE (3) OF THE FOLLOWING, FOR UP TO 15 BONUS POINTS!!!

B1 (5 pts) Solve both of the following absolute value inequalities.

a. $|2x-5|-2\geq 8$ **b.** $|2x-5|-2<8$

B2 (5 pts) Sketch the graph of $R(x)=\dfrac{2x^3-3x^2-3x+2}{x^2-x-6}$. Hints:

a. You already found $R(x)$'s asymptotes.

b. One of $R(x)$'s *x*-intercepts is $(2,0)$.

B3 (5 pts) Sketch the graph of $G(x)=\dfrac{3x^3-28x^2+23x+182}{x^3-5x^2-29x+105}$. Hint: $G(x)$ looks exactly like $F(x)$, from #9, except it has a hole.

B4 (5 pts) Re-write $f(x)=7x^2-3x-10$ in the form $f(x)=a(x-h)^2+k$. State its vertex.

B5 (5 pts) Sketch the graph of the piecewise-defined function $f(x)=\begin{cases}(x+1)^2 & \text{if } x<1\\ 2x-1 & \text{if } x\geq 1\end{cases}$.

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(1) $x=-7,\ m=3$; $x=3-7i,\ m=1$; $x=5,\ m=1$ 10pts

$f(x)=(x+7)^3(x-(3-7i))(x-(3+7i))(x-5)$

(B) $\boxed{\deg(f)=6}$ 5pts

(2)

$$\begin{array}{r|rrrrrr} 1 & 1 & -7 & 12 & 16 & -37 & -33 \\ & & 1 & -6 & 6 & 22 & -15 \\ \hline & 1 & -6 & 6 & 22 & -15 & \boxed{-48=P(1)} \end{array}$$

10pts

(3) $P(x)=(x-1)(x^4-6x^3+6x^2+22x-15)-48$ 5pts

(4) Next pg.

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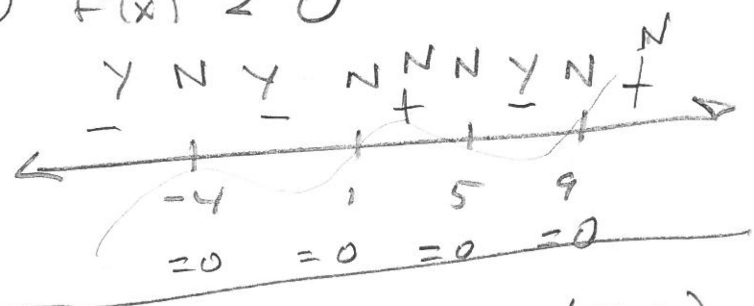
(4) $f(x) = (x+4)^2(x-1)(x-5)(x-9)$

$= x^5 - 7x^4 - 45x^3 + 187x^2 + 584x - 720$

(a) $f(x) < 0$

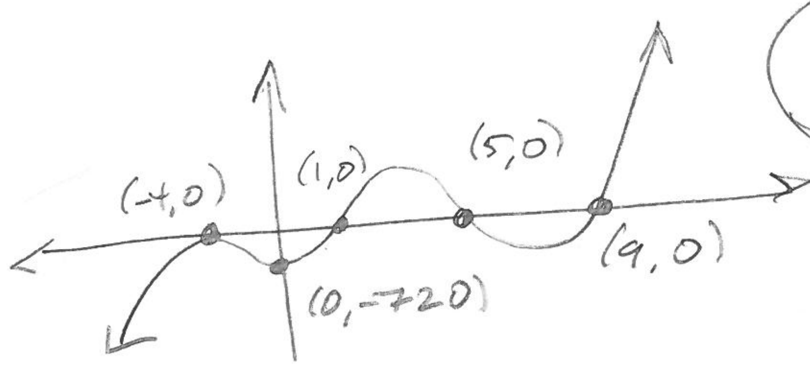


< 0

5pts

$\boxed{\Rightarrow x \in (-\infty, -4) \cup (-4, 1) \cup (5, 9)}$

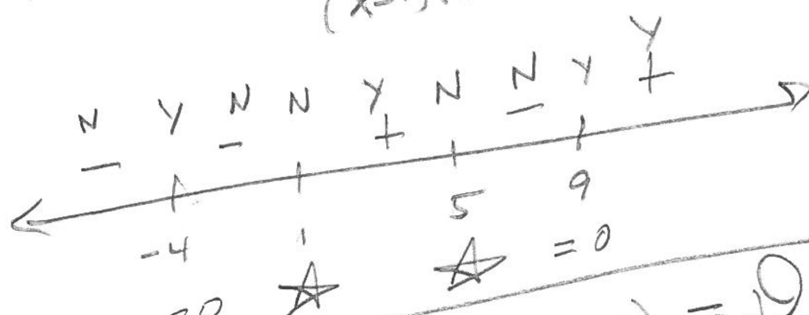
(b)



5pts

(c) Need $\dfrac{(x-9)(x+4)^2}{(x-1)(x-5)} \geq 0$

5pts



$\Rightarrow \boxed{\{4\} \cup (1,5) \cup [9, \infty) = \mathcal{D}(g)}$

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5 $P(x) = \underbrace{x^5 - 7x^4}_{1}\underbrace{}_{2} + 12x^3 + \underbrace{16x^2 - 37x}_{3} - 33$

a $\boxed{\text{3 or 1 positive zeros}}$ (At least one!)

$P(-x) = -x^5 - 7x^4 \underbrace{- 12x^3 + 16x^2}_{1} \underbrace{+ 37x - 33}_{2}$

5pts

$\boxed{\text{2 or 0 negative zeros}}$

$3 \,\underline{|\, 33}$
$\quad 11$

b $\frac{p}{q} : \frac{33}{1}$

$\boxed{\pm 1, \pm 3, \pm 11}$

c By

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6. $\pm 1, \pm 3, \pm 11$

Alread did $x=1$: "No"

$-1\rfloor$	1	-7	12	16	-37	-33
		-1	8	-20	4	33
$-1\rfloor$	1	-8	20	-4	-33	0
		-1	9	-29	33	
$3\rfloor$	1	-9	29	-33	0	
		3	-18	33		
	1	-6	11	0		

DONE W/ NEGATIVE GUESSES (FOUND 2!)

$a=1,\ b=-6,\ c=11$

$b^2-4ac = (-6)^2-4(1)(11)$

$= 36-44 = -8$

$x^2-6x+11$ is irreducible

Done with real roots

Zeros: $\boxed{x=-1,\ m=2\ ;\ x=3,\ m=1}$

$$\boxed{P(x) = (x+1)^2(x-3)(x^2-6x+11)}$$

Bonus

$\boxed{x=-1 \text{ is lower bound (Alternating Signs)}}$

$7\rfloor$	1	-7	12	16	-37	-33
		7	7	133	BIG	BIG
	1	0	19	BIG	BIG	BIG

All ≥ 0

All nonnegative ($x=3$ is smallest)

$\boxed{x=7 \text{ is upper bound}}$

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(7) $x^2-6x+11=0$

$x^2-6x+3^2=-11+9$

$(x-3)^2=-2$

$x-3=\pm\sqrt{2}\,i$

$x=3\pm\sqrt{2}\,i$

$$\boxed{P(x)=(x+1)^2(x-3)(x-(3+\sqrt{2}i))(x-(3-\sqrt{2}i))}$$

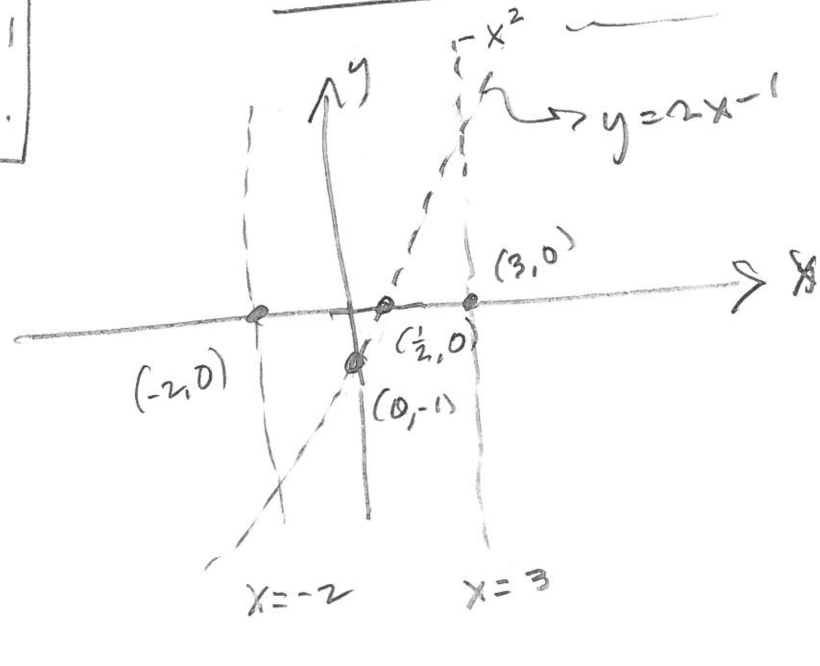
(8) $R(x)=\dfrac{2x^3-3x^2-3x+2}{x^2-x-6}=\dfrac{\text{STUFF}}{(x-3)(x+2)}$

V.A.: $x=-2$, $x=3$

O.A.:

$$\begin{array}{r}
2x-1\\
x^2-x-6\,\overline{\big)\,2x^3-3x^2-3x+2}\\
-(2x^3-2x^2-12x)\\
\hline
-x^2
\end{array}$$

$\boxed{y=2x-1 \text{ is O.A.}}$



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(9) $F(x) = \dfrac{3x^2-7x-26}{x^2+2x-15} = \dfrac{(3x-13)(x+2)}{(x+5)(x-3)}$

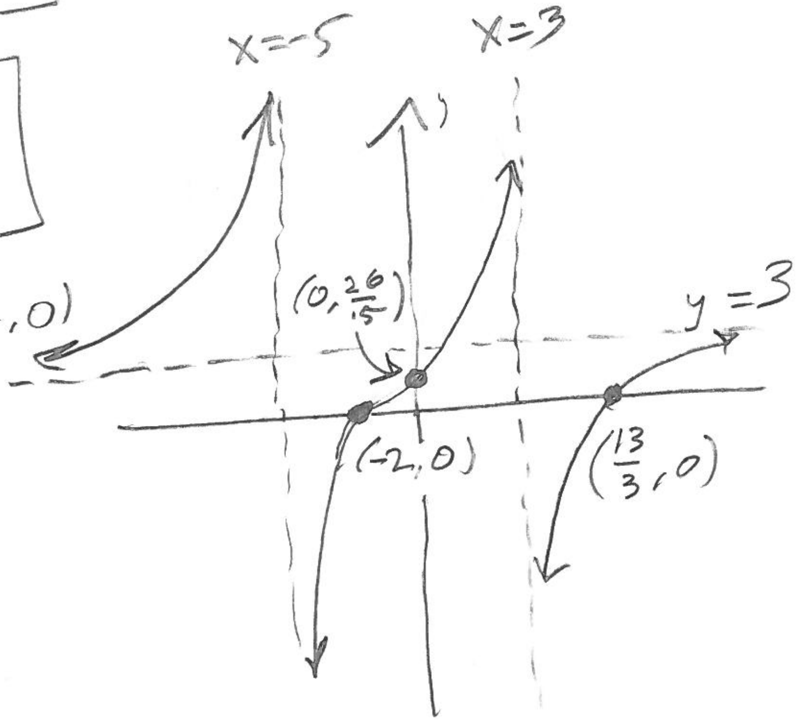
$D = \mathbb{R} \setminus \{-5, 3\}$

$\boxed{\text{V.A.: } x=-5,\ x=3}$

$\boxed{\text{H.A.: } \dfrac{3x^2}{x^2} = 3 = y}$

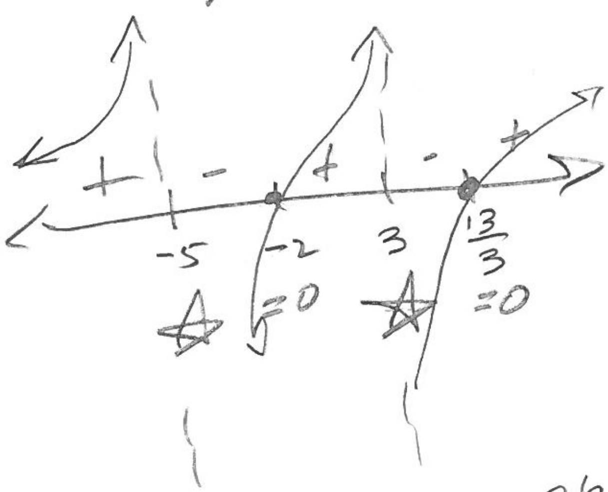
x-int: $\left(\frac{13}{3}, 0\right), (-2, 0)$

$(4.\bar{3}, 0)$



$-2,\ 4.\bar{3},\ -5,\ 3$

$-5,\ -2,\ 3,\ 4.\bar{3}$



y-int: $\left(0, \frac{26}{15}\right)$

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B1 (a) $|2x-5|-2 \ge 8$

$|2x-5| \ge 10$

$2x-5 \ge 10$ OR $2x-5 \le -10$

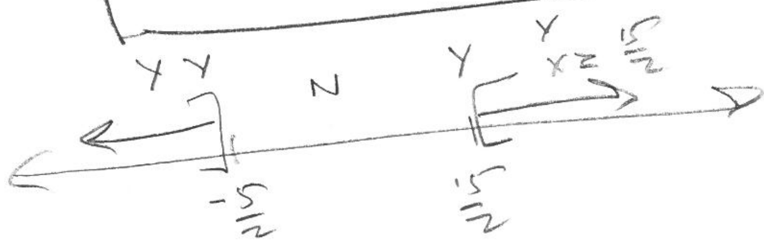
$2x \ge 15$ $\quad$ $2x \le -5$

$$\boxed{\left\{x \mid x \ge \frac{15}{2} \text{ OR } x \le -\frac{5}{2}\right\}}$$

$$= \boxed{\left(-\infty, -\frac{5}{2}\right] \cup \left[\frac{15}{2}, \infty\right)}$$

OR

Scratch



(b) $|2x-5|-2 < 8$

$|2x-5| < 10$

$2x-5 < 10$ and $2x-5 > -10$

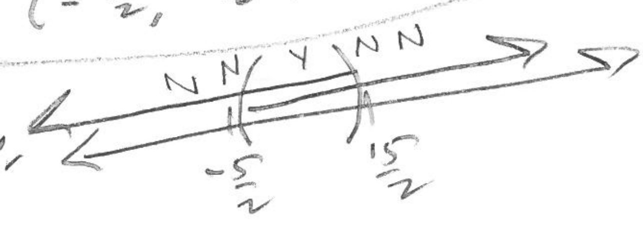
$2x < 15$ $\quad$ $2x > -5$

$$\boxed{\left\{x \mid x < \frac{15}{2} \text{ and } x > -\frac{5}{2}\right\}}$$

$$= \boxed{\left(-\frac{5}{2}, \frac{15}{2}\right)}$$

AND

Scratch!



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B2) $R(x) = \dfrac{2x^3 - 3x^2 - 3x + 2}{x^2 - x - 6}$. HINT: $x = 2$ is zero

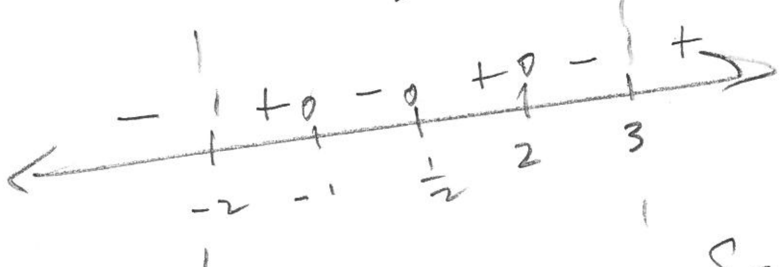
2	2	-3	-3	2
		4	2	-2
	2	1	-1	0

$(x-2)(2x^2 + x - 1)$

$= (x-2)(2x-1)(x+1)$

So $R(x) = \dfrac{(x-2)(2x-1)(x+1)}{(x-3)(x+2)}$

Critical #s: $x = -2, -1, \frac{1}{2}, 2, 3$



O.A.:

$$\begin{array}{r|l} & 2x - 1 \\ x^2 - x - 6 & 2x^3 - 3x^2 - 3x + 2 \\ & -(2x^3 - 2x^2 - 12x) \\ & -x^2 \end{array}$$

So, $y = 2x - 1$ is O.A.

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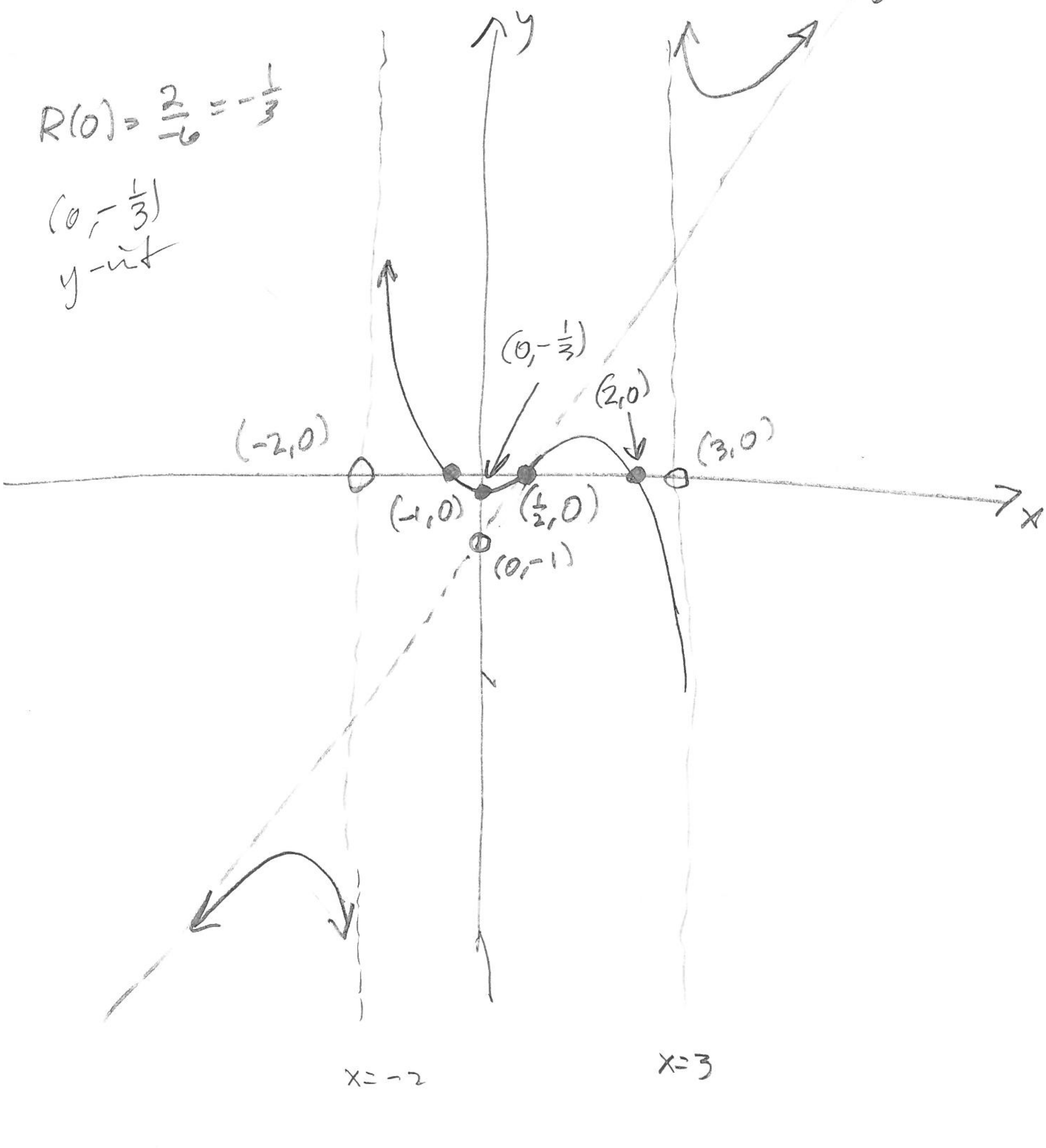
B2 cnt'd

$R(0) = \frac{2}{-6} = -\frac{1}{3}$

$(0, -\frac{1}{3})$

y-int



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B3 $x^3 - 5x^2 - 29x + 105 = (x-3)(x+5)(\ ?\ )$

$$\begin{array}{r|rrrr} 3 & 1 & -5 & -29 & 105 \\ & & 3 & -6 & -105 \\ \hline -5 & 1 & -2 & -35 & 0 \\ & & -5 & 35 & \\ \hline & 1 & -7 & & \end{array}$$

$(x-7)$ is the added factor

so $x = 7$ is where the hole is and

$$G(x) = \frac{3x^3 - 28x^2 + 23x + 182}{x^3 - 5x^2 - 29x + 105}$$

$$= \frac{(3x-13)(x+2)(x-7)}{(x-3)(x+5)(x-7)} = G^*(x) = \frac{(3x-13)(x+2)}{(x-3)(x+5)},$$

when $x \neq 7$, where there's a hole:

$$G^*(7) = \frac{(21-13)(7+2)}{(7-3)(7+5)}$$

$$= \frac{\cancel{(8)}^{2}\cancel{(9)}^{3}}{\cancel{(4)}_{1}\cancel{(12)}_{\cancel{4}\,2}} = \frac{3}{2}$$

$\boxed{\text{HOLE: } \left(7, \tfrac{3}{2}\right)}$

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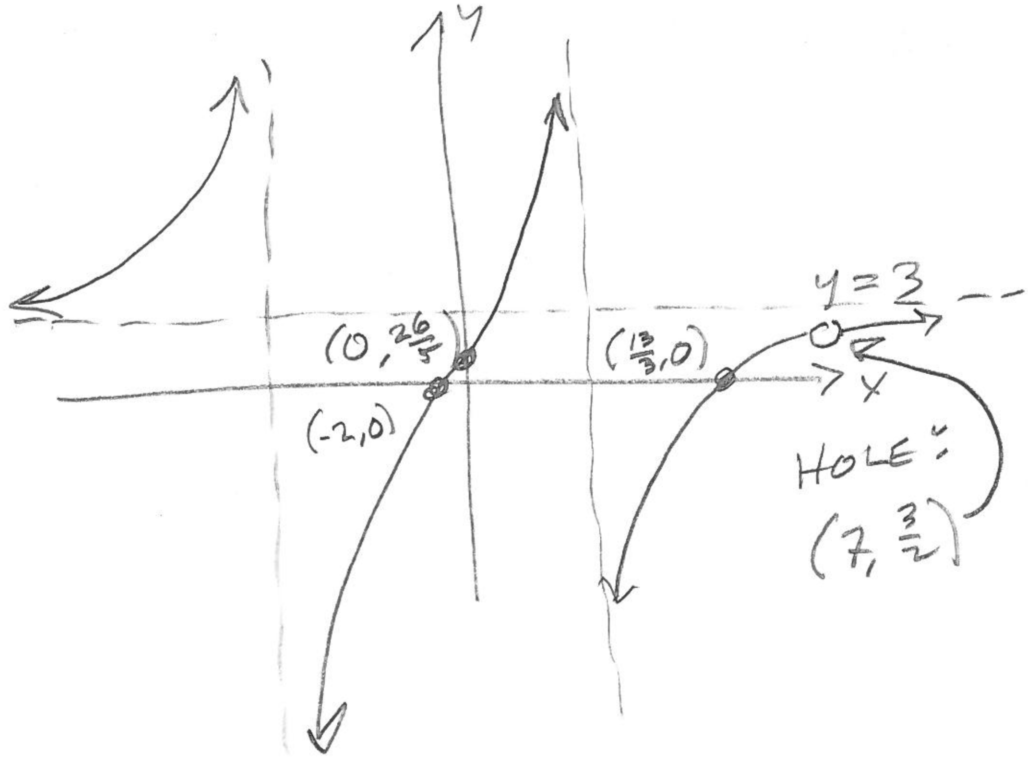
B3 Cont'd



$x = -5$ $\qquad$ $x = 3$

$$\begin{array}{r} 14 \\ \underline{\times 14} \\ 56 \\ \underline{140} \end{array}$$

B4 $7x^2 - 3x - 10$

$$= 7\left(x^2 - \frac{3}{7}x + \left(\frac{3}{14}\right)^2\right) - 10 - 7\left(\frac{9}{\cancel{196}_{28}}\right)$$

$$= \boxed{7\left(x - \frac{3}{14}\right)^2 - \frac{289}{28}}$$

$$-10 - \frac{9}{28} = \frac{-280 - 9}{28} = \frac{-289}{28}$$

$$\boxed{(h, k) = \left(\frac{3}{14}, -\frac{289}{28}\right)}$$

$$= (.2\overline{142857}, 10.32\overline{142857})$$

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B5 $f(x) = \begin{cases} (x+1)^2 & \text{if } x < 1 \\ 2x-1 & \text{if } x \geq 1 \end{cases}$