Chapter 4

Exponential and Logarithmic Functions.

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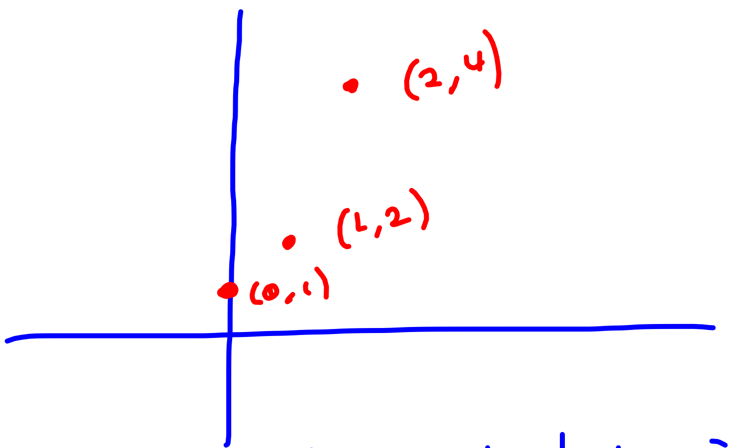
(3,8)

2^x, $x = 0, 1, 2, 3$



What if we started with 3 amoebas

$x=0$ (0,3)

$x=1$ (1,6)

$x=2$ (2,12)

$3 \cdot 2^x$, 3 = initial value, and

this is NOT 6^x

6^x: (0,1), (1,6), (2,36)

What's special about exponential growth?

The growth rate is proportional to the function.

$3x^2 \rightarrow 6x$

Growth rate of $y = 3x+2$

$m = 3$

Facts: Slope of $mx+b$ is

" " $3x^2$ is $6x$

" " $5x^3$ is $15x^2$

" " $2x^4$ is $8x^3$

Slope of 7^x is $K \cdot 7^x$

Variable in exponent is a big deal.

Slope of 7^x is $(\ln 7)\cdot 7^x$

The proportionality constant.

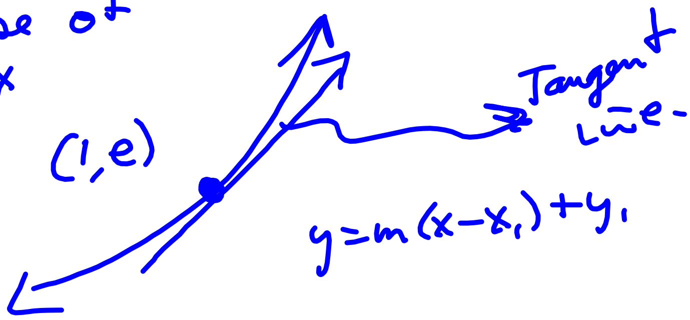
A very SPECIAL exponential function is the NATURAL exponential function

e^x $\qquad 2 < e < 3$

Euler

e^x is exactly as steep as it is tall.

The slope of e^x is e^x



$y = m(x - x_1) + y_1$

$y = e(x-1) + e$

$= ex - e + e$

$y = ex$

Logarithms are the inverses of exponentials.

Recall:

f^{-1} is the function that sends $f(x)$ back to x.

Example $y = 2x = f(x)$

$y = \frac{1}{2}x = f^{-1}(x)$, because

$f(f^{-1}(x)) = 2(f^{-1}(x)) = 2(\frac{1}{2}x) = x$

$\log_2(2^x) = x$

$2^{\log_2(x)} = x$

$$\boxed{\begin{array}{c}\log_2(x) = b \\ \text{means} \\ x = 2^b\end{array}}$$

$\log_2(16) = 4$

$= \log_2(2^4)$

base

We use logs to solve equations involving exponential functions

$3^x = 7$ clobber both sides with a log.

$\log_3(3^x) = x = \log_3(7)$

$$\log_7(x) = 13$$

$$7^{\log_7(x)} = 7^{13}$$

$$x = 7^{13}$$

write $3^x = 7$ in logarithmic form

$$\log_3(3^x) = \log_3(7)$$

$$x = \log_3(7)$$

Natural Exponential:

$e^{.7x}$

The .7 is the (relative) growth rate.

Applications:

Compound Interest. Exponential Growth

Pop. Growth

Radiometric Dating Exponential Decay

Feldspar

K-feldspar Orthoclase Feldspar.

Na-feldspar

Millsium