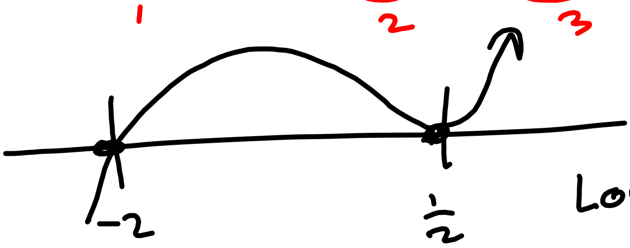
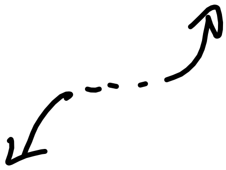
$$4x^5 - 8x^4 - 3x^3 + 39x^2 - 34x + 8$$

1 2 3 4



Looks like.

① $4x^5$



② sign changes

$f(x)$: 4

$\boxed{4, 2, \text{ or } 0 \text{ positive zeros}}$

$f(-x)$: 1

$$f(-x) = -4x^5 - 8x^4 + 3x^3 + 39x^2 + 34x + 8$$

1

$\boxed{1 \text{ negative zero}}$

③ $4x^5 \ldots\ldots 8$

q's p's

$\frac{p}{q}$: $\pm 1, \pm 2, \pm 4, \pm 8$

$\pm \frac{1}{2}, \cancel{\pm \frac{2}{2}}, \cancel{\pm \frac{4}{2}}, \cancel{\pm \frac{8}{2}}$

$\pm \frac{1}{4}, \cancel{\pm \frac{2}{4}}, \cancel{\pm \frac{4}{4}}, \cancel{\pm \frac{8}{4}}$

$$\boxed{\pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{2}, \pm \frac{1}{4}}$$

(4) Guesses! $x=-2$

$x+2$ $x=\frac{1}{2}$, twice

$-2)$	4	-8	-3	39	-34	8
		-8	32	-58	38	-8
$\frac{1}{2})$	4	-16	29	-19	4	0 sweet!
		2	-7	11	-4	
$\frac{1}{2})$	4	-14	22	-8	0 sweet	
		2	-6	8		
	4	-12	16	0 sweet!		

$4x^2-12x+16 = 4(x^2-3x+4)$

$a=1, b=-3, c=4$

$b^2-4ac = (-3)^2-4(1)(4) = 9-16 = -7$

No real sol'ns

(2 nonreal sol'ns.

So $\boxed{\begin{array}{l} x=-2, m=1 \\ x=\frac{1}{2}, m=2 \end{array}}$

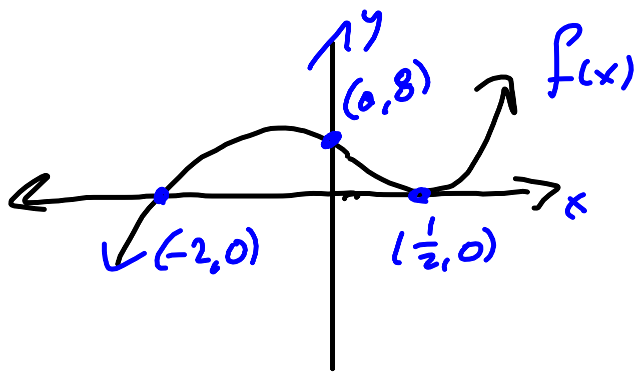
(5) $\boxed{4(x+2)(x-\frac{1}{2})^2(x^2-3x+4)}$ $=4x^5+$smaller

x x^2 x^2

$\boxed{(x+2)(2x-1)^2(x^2-3x+4)}$

$\boxed{(x+2)(x-\frac{1}{2})^2(4x^2-12x+16)}$

⑥



⑦

$4x^2 - 12x + 16$

$= 4(x^2 - 3x + 4) \overset{\text{SET}}{=} 0 \Longrightarrow$

$x^2 - 3x + 4 = 0$

$x^2 - 3x = -4$

$x^2 - 3x + \left(\frac{3}{2}\right)^2 = -4 + \frac{9}{4} = \frac{-16}{4} + \frac{9}{4} = \frac{-7}{4}$

$\left(x - \frac{3}{2}\right)^2 = -\frac{7}{4}$

$x - \frac{3}{2} = \pm\sqrt{-\frac{7}{4}} = \pm i\frac{\sqrt{7}}{2}$

$\Longrightarrow x = \frac{3 \pm i\sqrt{7}}{2}$

All OK

$4(x+2)\left(x - \frac{1}{2}\right)^2\left(x - \frac{3+i\sqrt{7}}{2}\right)\left(x - \frac{3-i\sqrt{7}}{2}\right)$

$(x+2)(2x-1)^2(x \ldots\ldots)$

$(x+2)\left(x - \frac{1}{2}\right)^2\left(2x - (3+i\sqrt{7})\right)\left(2x - (3-i\sqrt{7})\right)$

8 $\dfrac{x^2-9}{x^2-6x+5} = \dfrac{(x-3)(x+3)}{(x-1)(x-5)} = R(x)$

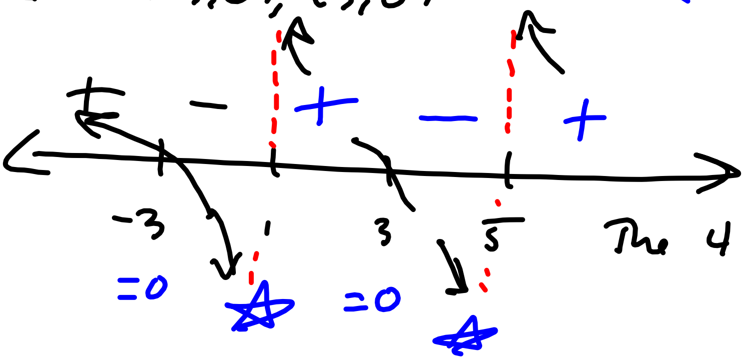
$\mathcal{D} = \mathbb{R} \setminus \{1, 5\}$

zeros: $x = \pm 3$ from $x^2 - 9 = 0$

V.A.: $x=1$, $x=5$

x-int: $(-3,0)$, $(3,0)$

$R(6) = \dfrac{(6-3)(6+3)}{(6-1)(6-5)} = +!$

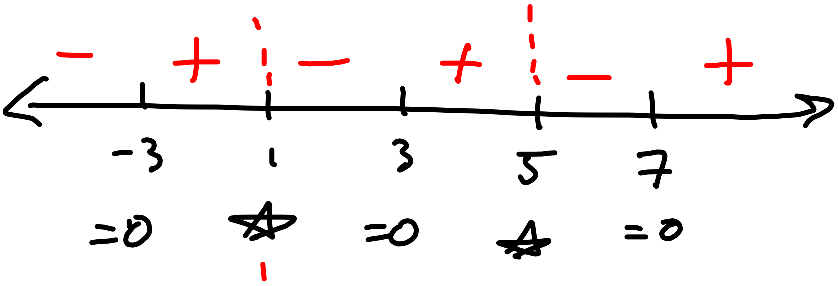


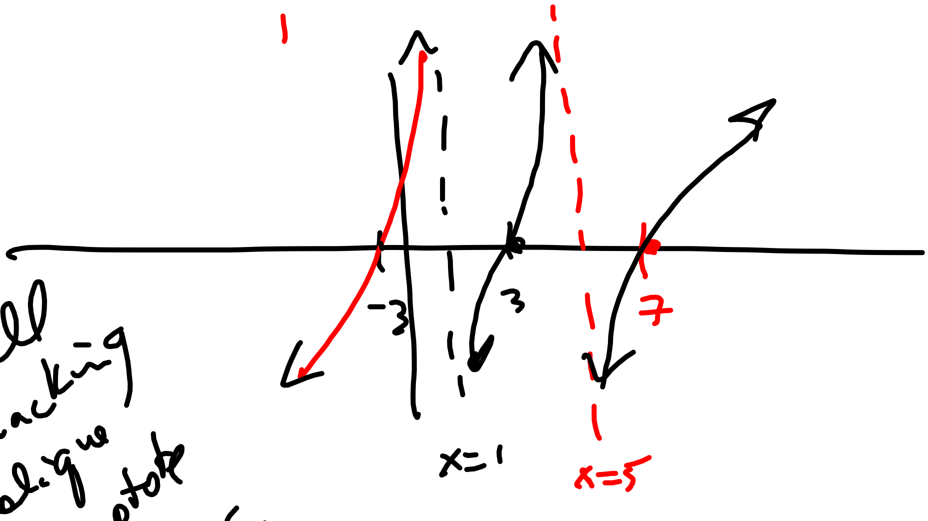
The 4 "critical points"

#10

$$\frac{x^3-7x^2-9x+63}{x^2-6x+5} = \frac{(x-3)(x+3)(x-7)}{(x-1)(x-5)}$$

y-int: $\left(0, \frac{63}{5}\right)$





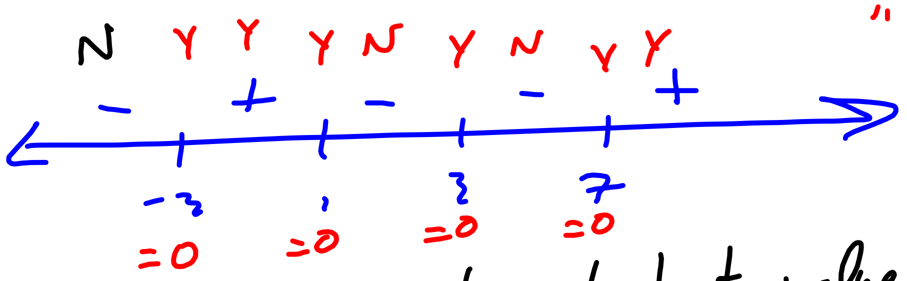
still lacking Oblique Asymptote features.

#4 $w(x) = \sqrt{(x+3)(x-1)(x-3)^2(x-7)}$

$\mathcal{D}(w)$: Need $(x+3)(x-1)(x-3)^2(x-7) \geq 0$

-3, 1, 3, 7

"≥ 0"

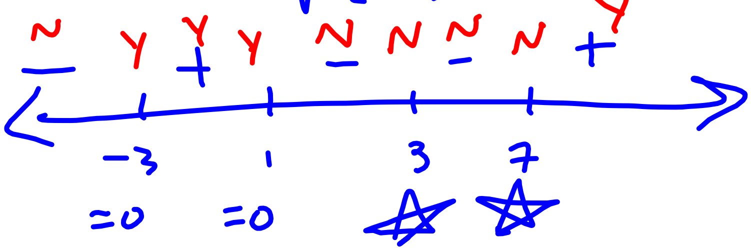


x=8's a decent test value.

$$\boxed{[-3,1] \cup \{3\} \cup [7,\infty)}$$

#12 $K(x) = \sqrt{\dfrac{(x+3)(x-1)}{(x-3)^2(x-7)}}$

"≥ 0"



Same as #11, but throw out 3 & 7!

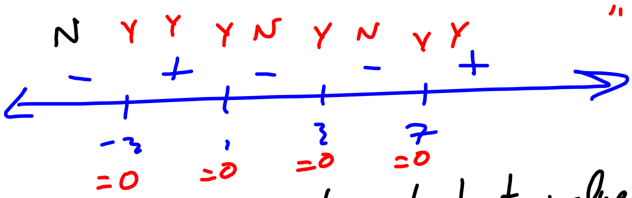
$$\boxed{\mathcal{D}(K) = [-3,1] \cup (7,\infty)}$$

#4 $w(x) = \sqrt{(x+3)(x-1)(x-3)^2(x-7)}$

$\mathcal{D}(w)$: Need $(x+3)(x-1)(x-3)^2(x-7) \geq 0$

"≥ 0"

$-3, 1, 3, 7$



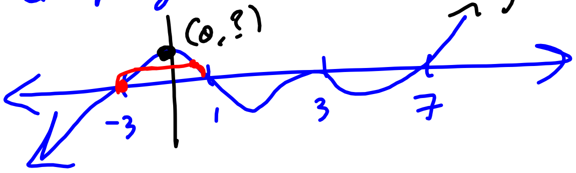
$x = 8$'s a decent test value.

$\boxed{[-3, 1] \cup \{3\} \cup [7, \infty)}$

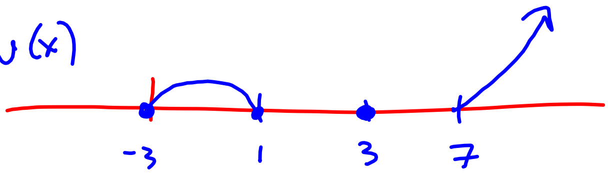
$W(x) = \sqrt{g(x)}$, where

$g(x) = (x+3)(x-1)(x-3)^2(x-7)$

Graph $g(x)$:

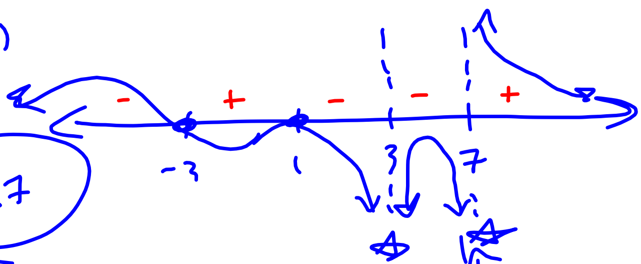


$w(x)$



$K(x) = \sqrt{z(x)}$, where $z(x) = \dfrac{(x+3)(x-1)}{(x-3)^2(x-7)}$

$z(x)$



§1.7

$K(x)$: