After College Career Network:

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1. (5 kindness points) I can really see how you made an effort to be clear, use one column per page, one side of writing per page, and plenty of room for my silly comments. Thanks for writing really dark, which helped me see your work so easily. You helped me serve your classmates, better, too, because your paper was quick and easy to grade. I thank you on their behalf.

2. Consider the relation $R = \{(1,3),(2,7),(3,4),(4,-2)\}$.

   a. (5 pts) Is R a function? If not, why not? Yes

   b. (5 pts) What is the domain of R ? $\{1, 2, 3, 4\}$

   c. (5 pts) What is the range of R ? $\{3, 7, 4, -2\}$

   d. (5 pts) If you answered "No" to part 'a.,' then the appropriate response, here, is "'1-to-1' only applies to functions, so this question is poorly posed." Assuming R is a function (You answered "Yes" to part 'a.,'), is R one-to-one? If not, explain why not.

   Yes. Each y-val. corresponds to exactly one x-value.

Sorted Pre-Test Scores, so you don't freak:

57
55
45
43
42
42
41
37
33
33
32
27
25
25
21
17
12

3. Let $f(x)=\sqrt{x+20}$ and $g(x)=x^2-13x+22$.

a. (5 pts) What is the domain of f?

b. (5 pts) What is the domain of g?

c. (5 pts) Write the function $\frac{f}{g}$. Do not simplify.

d. (5 pts) What is the domain of $\frac{f}{g}$?

e. (5 pts) Write the function $f \circ g$. Do not simplify.

f. (5 pts) What is the domain of $f \circ g$? (Highest level of synthesis.)

(a) Need $x+20 \geq 0 \Rightarrow$

$\mathcal{D} = \{x \mid x \geq -20\}$

(b) g is polynomial

$\Rightarrow \mathcal{D} = (-\infty, \infty)$

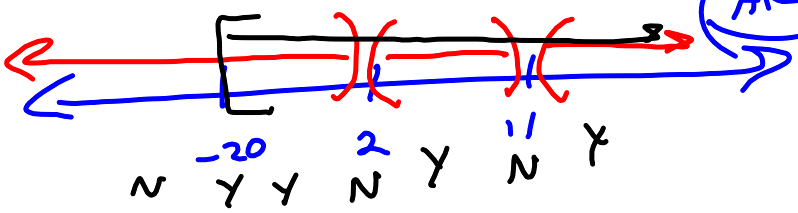
(c) $\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x+20}}{x^2-13x+22}$

(d) $\mathcal{D}\left(\frac{f}{g}\right) = \{x \mid x \in \mathcal{D}(g) \text{ and } x \in \mathcal{D}(f) \text{ and } g(x) \neq 0\}$

$= \{x \mid \text{No restrict. } \& \; x \geq -20 \text{ and } x \neq 2, 11\}$

$x \notin \{2, 11\}$

AND

$=$ 

Scratch

$g(x)=0$

$x^2-13x+22=0$

$=(x-2)(x-11)=0$

$\Rightarrow x \in \{2, 11\}$

$=[-20, 2) \cup (2, 11) \cup (11, \infty)$

(e) $(f \circ g)(x) = f(g(x))$

$= \sqrt{g(x)+20}$

$= \sqrt{x^2-13x+22+20}$

$= \sqrt{x^2-13x+42}$

Recall

$\mathcal{D}(f) = [-20, \infty)$

($x \ge -20$ is the condition)

(f) $\mathcal{D}(f \circ g) = \{x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f)\}$

$= \{x \mid \text{no restrict. and } g(x) \ge -20\}$

$g(x) \ge -20$

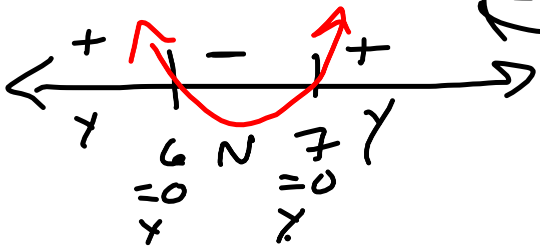
$x^2-13x+22 \ge -20$

$x^2-13x+42 \ge 0$

$(x-7)(x-6) \ge 0$

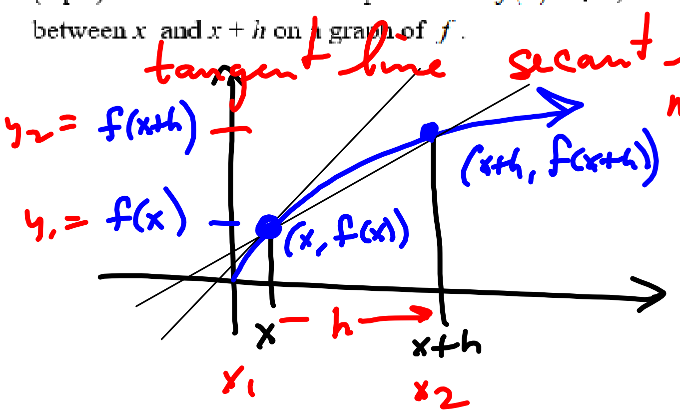
≥ 0

$= (-\infty, 6] \cup [7, \infty)$



$= (-\infty, 6] \cup [7, \infty)$

5. (5 pts) Write the difference quotient for $f(x)=\sqrt{x}$, and explain its connection to the slope of a secant line between x and $x+h$ on a graph of f.



$$m_{AVG} = \frac{f(x+h)-f(x)}{h} = \frac{y_2-y_1}{x_2-x_1}$$

Diff. Quotient is the slope of the line between 2 points on the curve.

4. (5 pts) Simplify the difference quotient for $f(x)=5x^2+2x+1$.

$$\frac{f(x+h)-f(x)}{h}$$

$$= \frac{5(x+h)^2+2(x+h)+1-(5x^2+2x+1)}{h}$$

$$= \frac{5(x^2+2xh+h^2)+2x+2h+1-5x^2-2x-1}{h}$$

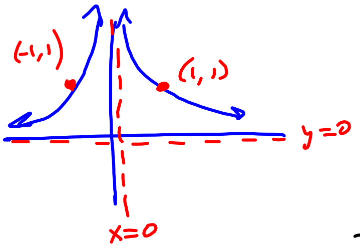
$$= \frac{5x^2+10xh+5h^2+2h-5x^2}{h}$$

$$= \frac{10xh+5h^2+2h}{h} = \frac{h(10x+5h+2)}{h}$$

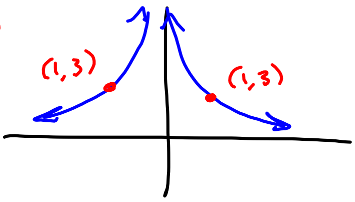
$$= \boxed{10x+5h+2}$$

6. Let $g(x)=\dfrac{3}{(5x-20)^2}-7$. $\qquad f(x)=\dfrac{1}{x^2}$

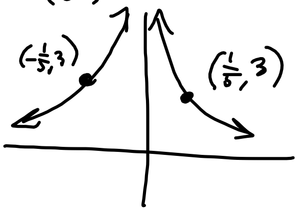
a. (10 pts) Sketch the graph of $g(x)$, by transforming the basic function $f(x)=\dfrac{1}{x^2}$. I want to see 2 points labeled in the graph of f preferably $(1,1)$ and $(\ 1,1)$ and track where those points are moved to after every step, as demonstrated by, uh, me. This will take 5 graphs, counting the first graph of $f(x)=\dfrac{1}{x^2}$ as the first. (I've been numbering them 0 thru 4, for some reason, this semester.)



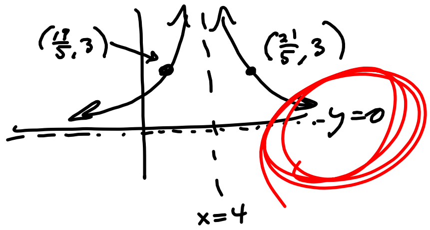
$$\frac{3}{x^2}=3\left(\frac{1}{x^2}\right)=3f(x)$$



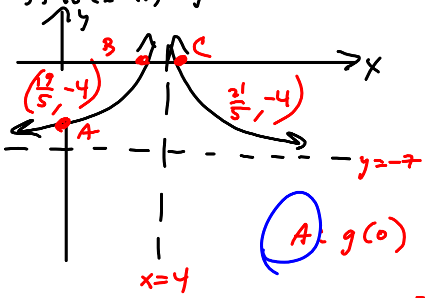
$$\frac{3}{(5x)^2}=3f(5x)$$



$$3f(5x-20)=$$ delay by 4
$$=3f(5(x-4))$$



$$3f(5(x-4))-7$$



$\frac{1}{5}+4$

$-\frac{1}{5}+4=$

A: $g(0)=\dfrac{3}{(5(0)-20)^2}-7$

$=\dfrac{3}{400}-7$

$=\dfrac{3-2800}{400}=-\dfrac{2797}{400}$

$$\boxed{A=\left(0,-\frac{2797}{400}\right)}$$

B: $g(x)=0$

$\dfrac{3}{(5x-20)^2}-7=0$

$\dfrac{3}{(5x-20)^2}=7$

$3=7(5x-20)^2$

$7(5x-20)^2=3$

$(5x-20)^2=\dfrac{3}{7}$

$5x-20=\pm\sqrt{\dfrac{3}{7}}=\pm\dfrac{\sqrt{21}}{7}$

$5x=20\pm\dfrac{\sqrt{21}}{7}$

$x=4\pm\dfrac{\sqrt{21}}{35}$

$$\boxed{\begin{aligned}B&=\left(4-\frac{\sqrt{21}}{35},0\right)\\ C&=\left(4+\frac{\sqrt{21}}{35},0\right)\end{aligned}}$$

b. (5 pts) State the domain and range of $g(x)$, based on your final graph.

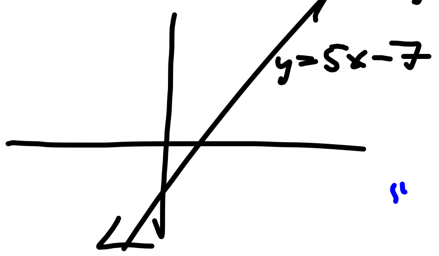
c. (5 pts) Find the x- and y-intercepts of $g(x)$, and label them, clearly, on the graph.

(b) $\mathcal{D} = \mathbb{R} \setminus \{4\} = (-\infty, 4) \cup (4, \infty)$

$\mathcal{R} = (-7, \infty)$

7. (5 pts) Prove that $\rho(x) = 5x - 7$ is one-to-one.

(1) picture & say "passes the horiz. line test"



"Roh"

(2) $\rho(x)$ is 1-to-1 iff $\rho(x_1) = \rho(x_2)$

$\Rightarrow 5x_1 - 7 = 5x_2 - 7$

$\Rightarrow 5x_1 = 5x_2$

$\Rightarrow x_1 = x_2$

$\Rightarrow$ 1-to-1.

8. (5 pts) The force of gravity, F, varies jointly with the masses, m_1 and m_2 of the two planets, and inversely with the square of the distance, r, between them. (To be precise, r is the distance between their respective centers of mass.). Using the proportionality constant, G, which stands for Newton's *gravitational constant*, write an equation relating force to the masses of the two planets and the distance between them.

$$F = G \frac{m_1 m_2}{r^2}$$

9. (5 pts) Explain why $x = |2y - 4|$ does *not* define y as a function of x.

Solve for y:

$|2y-4| = x$

$2y-4 = \pm x$

$2y = 4 \pm x$

$y = 2 \pm \frac{1}{2}x$

$x = 2$:

$y = 2 \pm \frac{1}{2}(2)$

$= 2 \pm 1$ → 3, 1

This says $(2,1)$ & $(2,3)$ are in the relation.

Means more than one y-val. for one x-val.

B1. Write down your answer to #4, again, and pass to the limit as h approaches zero, and show me some calculus.

$$10x + 5h + 2 \xrightarrow{h \to 0} \boxed{10x + 2} = f'(x)$$

B2. Simplify the difference quotient for the function $f(x) = \sqrt{x}$ (See #5.). Then pass to the limit, as h approaches zero, and demonstrate an early aptitude for Calculus.

$$\underbrace{\left(\frac{\sqrt{x+h} - \sqrt{x}}{h}\right)}_{(a-b)}\underbrace{\left(\frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}\right)}_{(a+b)} = \frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \frac{h}{h(\sqrt{x+h} + \sqrt{x})} = \frac{1}{\sqrt{x+h} + \sqrt{x}} \xrightarrow{h \to 0} \frac{1}{\sqrt{x} + \sqrt{x}}$$

$$\boxed{= \frac{1}{2\sqrt{x}}}$$

B3. Add the line to your picture in #5, that represents the tangent to f at the point $(x, f(x))$

B4. Complete the square to re-write the function $h(x) = 3x^2 - 5x - 4$ in the form $a(x-h)^2 + k$. What is the vertex?

$$h(x) = 3x^2 - 5x - 4$$

$$= 3\left(x^2 - \frac{5}{3}x \qquad\right) - 4$$

$$= 3\left(x^2 - \frac{5}{3}x + \left(\frac{5}{6}\right)^2\right) - 4 - 3\left(\frac{5}{6}\right)^2$$

Added

$$= 3\left(x - \frac{5}{6}\right)^2 - \frac{73}{12}$$

$$-4 - 3\left(\frac{25}{36}\right)$$

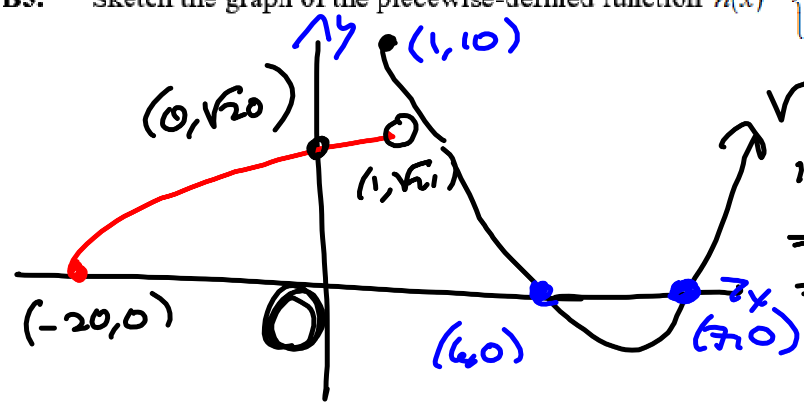
$$= -4 - \frac{25}{12}$$

$$= \frac{-48 - 25}{12} = \frac{-73}{12}$$

$$(h, k) = \left(\frac{5}{6}, -\frac{73}{12}\right)$$

B5. Sketch the graph of the piecewise-defined function $h(x) = \begin{cases} \sqrt{x+20} & \text{if } x < 1 \\ x^2 - 13x + 22 & \text{if } x \geq 1 \end{cases}$.

$\sqrt{21}$



$$\sqrt{1+20} = \sqrt{21}$$

$$1^2 - 13(1) + 22$$

$$= -12 + 22$$

$$= 10$$