121 WP #1

#1-3 Find all real/non-real soln's w/ quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

(1) $x^2 - x - 6 = 0$

$a = 1, b = -1, c = -6$

$\Rightarrow b^2 - 4ac = (-1)^2 - 4(1)(-6)$

$= 1 + 24 = 25$

$= \frac{1 \pm \sqrt{25}}{2(1)} = \frac{1 \pm 5}{2}$

$\frac{1+5}{2} = 3$ $\quad \frac{1-5}{2} = -2$

$\Rightarrow x \in \{3, -2\}$

Went old-school on #2, on the prime factorizations of composite integers.

(2) $6.2x^2 - 14.63x + 7.26 = 0$ Times 100 $\Rightarrow$

$\Rightarrow 620x^2 - 1463x + 726 = 0$

$a = 620, b = 1463, c = 726$

$b^2 - 4ac = (1463)^2 - 4(620)(726)$

$= 339,889$ & so

$$x = \frac{-(-1463) \pm \sqrt{339,889}}{2(620)} \approx \begin{cases} -.029838709677 \\ -.970161290323 \end{cases}$$

$= \frac{1463 \pm 583}{1240}$

$\frac{2046}{1240} = \frac{1023}{620} = \frac{33 \cdot 31}{20 \cdot 31} = \frac{33}{20}$

$\frac{880}{1240} = \frac{88}{124} = \frac{44}{62} = \frac{22}{31}$

$\Rightarrow x \in \{\frac{33}{20}, \frac{22}{31}\}$

$\approx \{1.6500, .7097\}$

11	339,889	1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53,
11	30,899	
53	2809	
	53	

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(3) $4x^2 + 4x + 6 = 0 \Rightarrow$

$a = 4,\ b = -4,\ c = 6 \Rightarrow$

$b^2 - 4ac = (-4)^2 - 4(4)(6)$

$= 16 - 96 = -80$

$2 \mid 80$
$2 \mid 40 \Rightarrow \sqrt{-80} =$
$2 \mid 20$
$2 \mid 10 \quad \sqrt{-2^4 \cdot 5} =$
$5 \quad 2^2 i\sqrt{5} = 4i\sqrt{5}$

$x = \dfrac{-(-4) \pm 4i\sqrt{5}}{2(4)} = \dfrac{4 \pm 4i\sqrt{5}}{2(4)} = \dfrac{4(1 \pm i\sqrt{5})}{2(4)}$

$= \boxed{\dfrac{1 \pm i\sqrt{5}}{2}}$

$x \in \boxed{\left\{ \dfrac{1 \pm \sqrt{5}\, i}{2} \right\}}$

(4) $2\pi r^2 + 2\pi h r - S' = 0$

$a = 2\pi,\ b = 2\pi h,\ c = -S'$

$\Rightarrow b^2 - 4ac = (2\pi h)^2 - 4(2\pi)(-S')$

$= 4\pi^2 h^2 + 8\pi S'$

$r = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-2\pi h \pm \sqrt{4\pi^2 h^2 + 8\pi S'}}{2(2\pi)}$

$= \dfrac{-2\pi h \pm 2\sqrt{\pi^2 h^2 + 2\pi S'}}{4\pi}$

$= \boxed{\dfrac{-\pi h \pm \sqrt{\pi^2 h^2 + 2\pi S'}}{2\pi}}$

Simplified: $\dfrac{2(-\pi h \pm \sqrt{\pi^2 h^2 + 2\pi S'})}{4\pi}$

$\sqrt{4\pi^2 h^2 + 8\pi S'}$
$= \sqrt{4(\pi^2 h^2 + 2\pi S')}$
$= \sqrt{4}\sqrt{\pi^2 h^2 + 2\pi S'}$

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(5) $x^2 - 5x - 6 = 0$

Sledgehammer

$a = 1, b = -5, c = -6$

$b^2 - 4ac = (-5)^2 - 4(1)(-6)$

$= 25 + 24 = 49$

$x = \frac{5 \pm \sqrt{49}}{2(1)} = \frac{5 \pm 7}{2}$

$\frac{5+7}{2} = 6 = x$

$\frac{5-7}{2} = -1 = x$

$\Rightarrow (x-6)(x-(-1)) = 0$

$\Rightarrow (x-6)(x+1) = 0$

$\Rightarrow x \in \{-1, 6\}$

$2c = -6 = -(2)(3)$ Meh

$= -(6)(1)$ Ahh

$-6 + 1 = -5$

$x^2 - 6x + 1x - 6 = 0$

$x(x-6) + 1(x-6) = 0$

$(x-6)(x+1) = 0$

$x \in \{-1, 6\}$

$2 \mid 182$

$7 \mid 91$

13

(6) $10x^2 - 37x - 182 = 0$

$(10)(-182) = -(2)(5)(2)(7)(13) = -1820$

$-37 = -57 + 20 \qquad -1140$ Higher!

$= -67 + 30 \qquad -2010$ Lower!

$= -60 + 23 \qquad -1380$ Higher!

$= -65 + 28 \qquad -1820$ Sweet!

$x \in \left\{-\frac{14}{5}, \frac{13}{2}\right\}$

So, $10x^2 - 65x + 28x - 182 = 0$

$5x(2x-13) + 14(2x-13) = 0$

$(2x-13)(5x+14) = 0 \Rightarrow x = \frac{13}{2}$ OR $x = -\frac{14}{5}$

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(6) Sledgehammer

$a = 10,\ b = -37,\ c = -182$

$b^2 - 4ac = (-37)^2 - 4(10)(-182)$

$= 1369 + 7280 = 8649$

$\pm\sqrt{8649} = 93 \longrightarrow$

$$x = \frac{-(-37) \pm 93}{2(10)}$$

$$\frac{37+93}{20} = \frac{130}{20} = \frac{13}{2}$$

$$\frac{37-93}{20} = -\frac{56}{20} = -\frac{28}{10} = -\frac{14}{5}$$

$\longrightarrow x \in \left\{-\frac{14}{5}, \frac{13}{2}\right\}$.

Now, to write it factored, to show you see the connection & can "simulate" the factoring skill:

$10\left(x - \frac{13}{2}\right)\left(x - \left(-\frac{14}{5}\right)\right)$

$= 10\left(x - \frac{13}{2}\right)\left(x + \frac{14}{5}\right)$

$= 5 \cdot 2\left(x - \frac{13}{2}\right)\left(x + \frac{14}{5}\right)$

$= 2\left(x - \frac{13}{2}\right)(5)\left(x + \frac{14}{5}\right)$

simulate solving by factoring:

$= (2x - 13)(5x + 14) = 0$

= Looks like factored form!

$2x - 13 = 0$ OR $5x + 14 = 0$

$2x = 13$ OR $5x = -14$

$x = \frac{13}{2}$ OR $x = -\frac{14}{5}$

$x \in \left\{-\frac{14}{5}, \frac{13}{2}\right\}$

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#s 7-10 Solve by completing the square

(7) $x^2+4x-12=0$

$x^2+4x = 12$

$x^2+4x+2^2 = 12+4$

$(x+2)^2 = 16$

$x+2 = \pm\sqrt{16} = \pm 4$

$x = -2 \pm 4$ → $-2+4=2$, $-2-4=-6$ → $x \in \{-6, 2\}$

(8) $x^2-5x-11=0$

$x^2-5x = 11$

$x^2-5x+\left(\frac{5}{2}\right)^2 = 11+\frac{25}{4} = \frac{44+25}{4} = \frac{69}{4}$

$3\overline{)69}$
23
No help

$\left(x-\frac{5}{2}\right)^2 = \frac{69}{4}$

$x-\frac{5}{2} = \pm\sqrt{\frac{69}{4}} = \pm\frac{\sqrt{69}}{\sqrt{4}} = \pm\frac{\sqrt{69}}{2}$

$x = \frac{5\pm\sqrt{69}}{2}$ ⟹ $x \in \left\{\frac{5\pm\sqrt{69}}{2}\right\}$

$x \approx 6.653311932, -1.653311932$

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(9) $3x^2 - 4x + 3 = 0$

$x^2 - \frac{4}{3}x + 1 = 0$

$x^2 - \frac{4}{3}x = -1$

$x^2 - \frac{4}{3}x$

$\frac{\frac{4}{3}}{2} = \left(\frac{4}{3}\right)\left(\frac{1}{2}\right) = \frac{4}{6} = \frac{2}{3}$

$x^2 - \frac{4}{3}x + \left(\frac{2}{3}\right)^2 = -1 + \frac{4}{9} = \frac{-9+4}{9} = -\frac{5}{9}$

$\left(x - \frac{2}{3}\right)^2 = -\frac{5}{9}$

$x - \frac{2}{3} = \pm\sqrt{-\frac{5}{9}} = \pm i\sqrt{\frac{5}{9}} = \pm i\frac{\sqrt{5}}{\sqrt{9}} = \pm i\frac{\sqrt{5}}{3}$

$x = \frac{-2 \pm i\sqrt{5}}{3} \Rightarrow$ $x \in \frac{2 \pm \sqrt{5}\, i}{3}$

121 WP #1

(40) $4x^2 - 20x + 23 = 0$

$x^2 - 5x + \frac{23}{4} = 0$

$x^2 - 5x = -\frac{23}{4}$

$x^2 - 5x + \left(\frac{5}{2}\right)^2 = -\frac{23}{4} + \frac{25}{4} = \frac{2}{4}$

$\left(x - \frac{5}{2}\right)^2 = \frac{2}{4}$ ALT: $\frac{1}{2}$

$x - \frac{5}{2} = \pm\sqrt{-\frac{2}{4}} = \pm i\sqrt{\frac{2}{4}} = \pm i\frac{\sqrt{2}}{\sqrt{4}} = \pm i\frac{\sqrt{2}}{2}$

ALT: $x - \frac{5}{2} = \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{1}}{\sqrt{2}} = \pm\frac{1}{\sqrt{2}}$

$= \pm\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$

Rationalized

$x = \frac{5 \pm \sqrt{2}}{2}$ $\Longrightarrow$ $x \in \frac{5 \pm \sqrt{2}}{2}$

$x \approx 3,207106781,\ 1,792893219$