121 TEST 2 SPR '16

① $f = \{(-2,3), (1,1), (2,2), (3,3)$

ⓐ f is a function

ⓑ $\mathcal{D} = \{-2, 1, 2, 3\}$

ⓒ $\mathcal{R} = \{3, 1, 2\}$

ⓓ f is not 1-to-1.

$f(-2) = f(3) = 3$

② $f(x) = \dfrac{x+11}{x-2}$, $g(x) = \sqrt{x+7}$

ⓐ $\dfrac{f}{g} = \dfrac{\frac{x+11}{x-2}}{\sqrt{x+7}}$

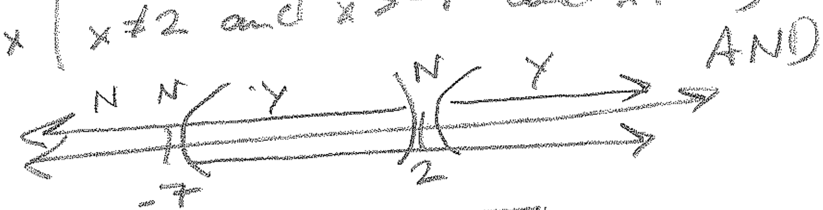
ⓑ $\mathcal{D}\left(\frac{f}{g}\right) = \{x \mid x \in \mathcal{D}(f) \text{ and } x \in \mathcal{D}(g) \text{ and } g(x) \neq 0\}$

$= \{x \mid x \neq 2 \text{ and } x \geq -7 \text{ and } x \neq -7\}$

$\mathcal{D}(f) = \{x \mid x \neq 2\}$

$\mathcal{D}(g) = \{x \mid x \geq -7$

$g(x) \neq 0 \Rightarrow x \neq -7$

$=$ 

$= \boxed{(-7, 2) \cup (2, \infty)}$

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(2) $(f\circ g)(x) = \boxed{f(g(x)) = \dfrac{\sqrt{x+7}+11}{\sqrt{x+7}-2}}$

$$D(f\circ g) = \{x \mid x\in D(g) \text{ and } g(x)\in D(f)\}$$

$$= \{x \mid x \ge -7 \text{ and } \sqrt{x+7} \ne 2\}$$

$$= \{x \mid x\ge -7 \text{ and } x \ne -3\}$$

AND

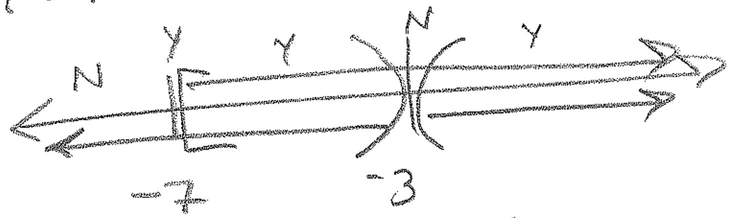
Scratch:

$\sqrt{x+7}\ne 2$

$x+7\ne 4$

$x\ne -3$

=



$$= \boxed{[-7,-3)\cup(-3,\infty)}$$

(3) $f(x) = 3x^2-2x \longrightarrow \dfrac{f(x+h)-f(x)}{h} = \dfrac{3(x+h)^2-2(x+h)-(3x^2-2x)}{h}$

$$= \frac{3(x^2+2xh+h^2)-2x-2h-3x^2+2x}{h}$$

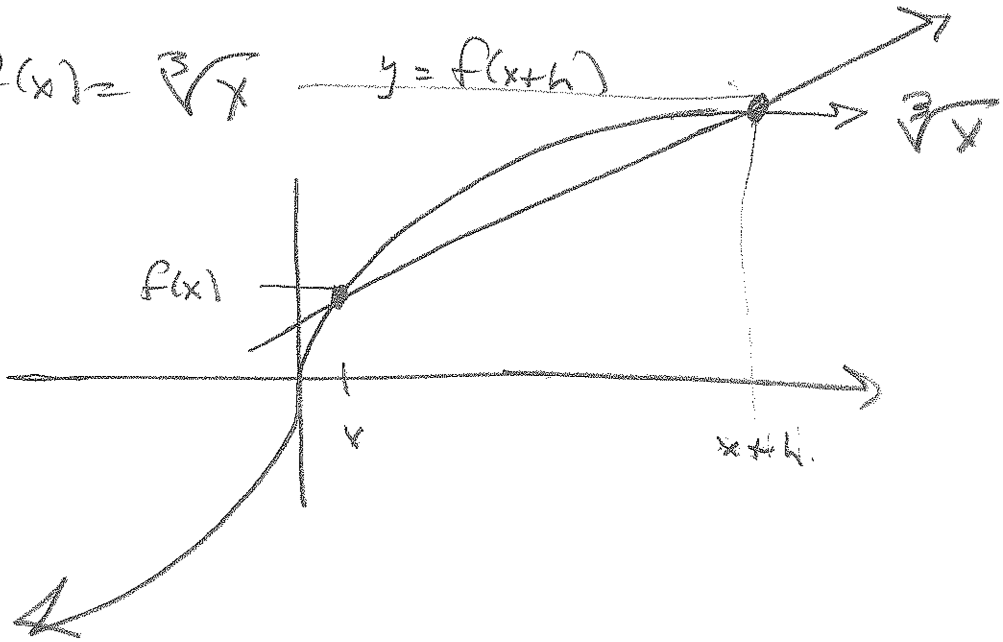
$$= \frac{3x^2+6xh+3h^2-2h-3x^2}{h} = \frac{6xh+3h^2-2h}{h}$$

$$= \frac{\cancel{h}(6x+3h-2)}{\cancel{h}} = \boxed{6x+3h-2}$$

$\xrightarrow{h\to 0} 6x-2 \longrightarrow$ Calculus!

121 TEST 2

(4) $f(x) = \sqrt[3]{x}$



$$\frac{f(x+h) - f(x)}{h} = \frac{\sqrt[3]{x+h} - \sqrt[3]{x}}{h}$$ is the

average slope of f(x), given by the slope of the line between $(x, f(x)) = (x_1, y_1)$ and $(x+h, f(x+h)) = (x_2, y_2)$

& $$m_{AVG} = \frac{y_2 - y_1}{x_2 - x_1}$$

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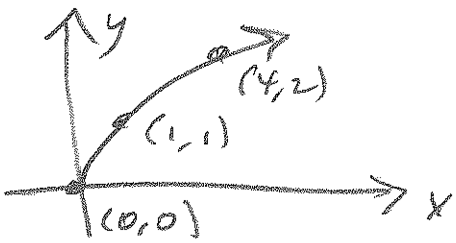
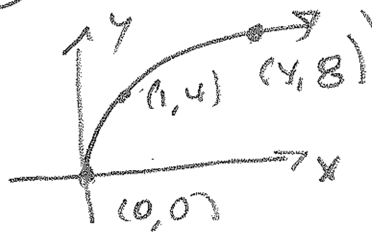
(5) $g(x) = 4\sqrt{-5x-10} - 7$

(1) $f(x) = \sqrt{x}$

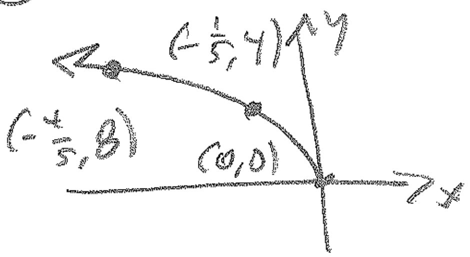


(2) $4f(x) = 4\sqrt{x}$



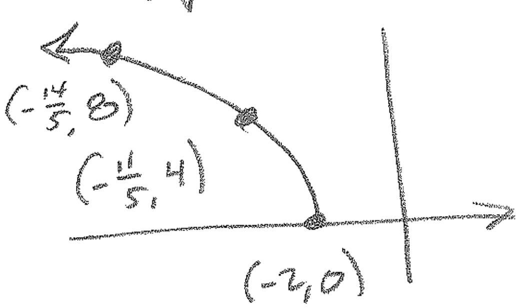
M1

(3) $4\sqrt{-5x} = 4f(-5x)$



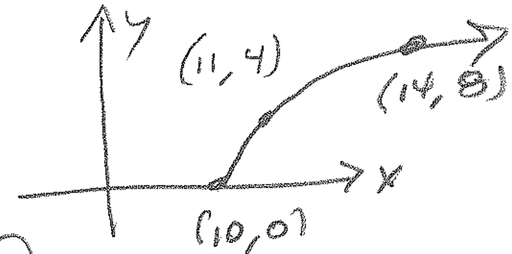
(4) $4f(-5(x+2))$

$= 4\sqrt{-5(x+2)}$

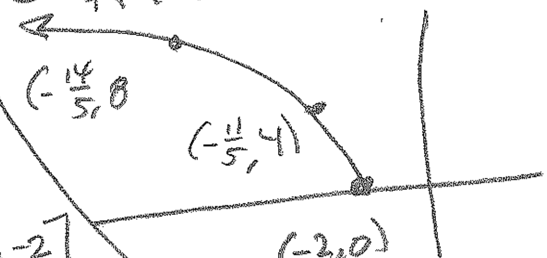


M2

(3) $4f(x-10)$



(4) $4f(-5x-10)$



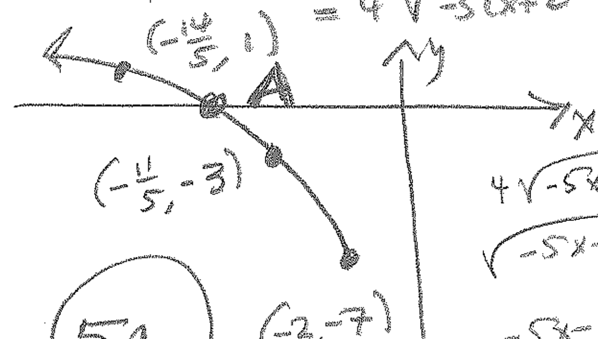
5b

$D = (-\infty, -2]$

$R = [-7, \infty)$

(5) $g(x) = 4\sqrt{-5x-10} - 7$

$= 4\sqrt{-5(x+2)} - 7$



$4\sqrt{-5x-10} = 7$

$\sqrt{-5x-10} = \frac{7}{4}$

$-5x-10 = \frac{49}{16}$

$-5x = \frac{209}{16}$

$x = \frac{209}{-80} = -2.6\overline{12}$

5c

$\boxed{A = (-\frac{209}{80}, 0) = (-2.6\overline{12}, 0)}$

$-\frac{4}{5} - 2 = -\frac{14}{5}$

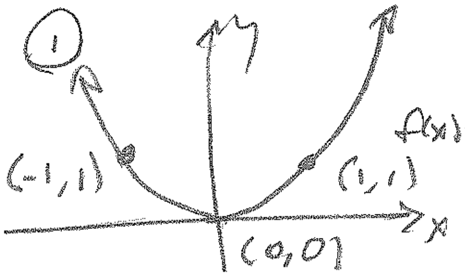
$-\frac{1}{5} - 2 = -\frac{11}{5}$

$$
\begin{array}{r}
2.6\overline{12} \\
8\,\overline{)\,20.9000} \\
16.0000 \\
\hline
4.9000 \\
4.8000 \\
\hline
.1000 \\
.0800 \\
\hline
.0200
\end{array}
$$

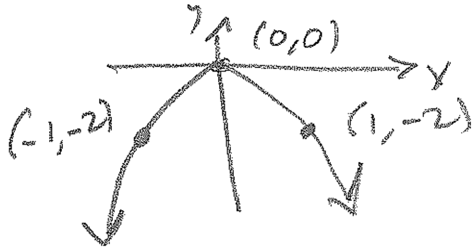
121 TEST 2

6) $r(x) = -2(x+5)^2 + 3$

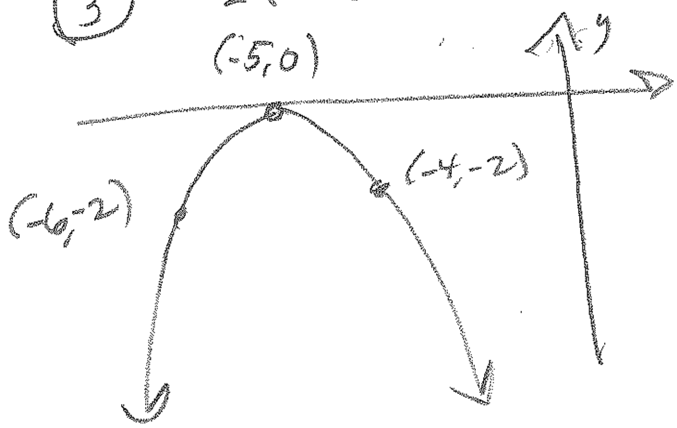
1)



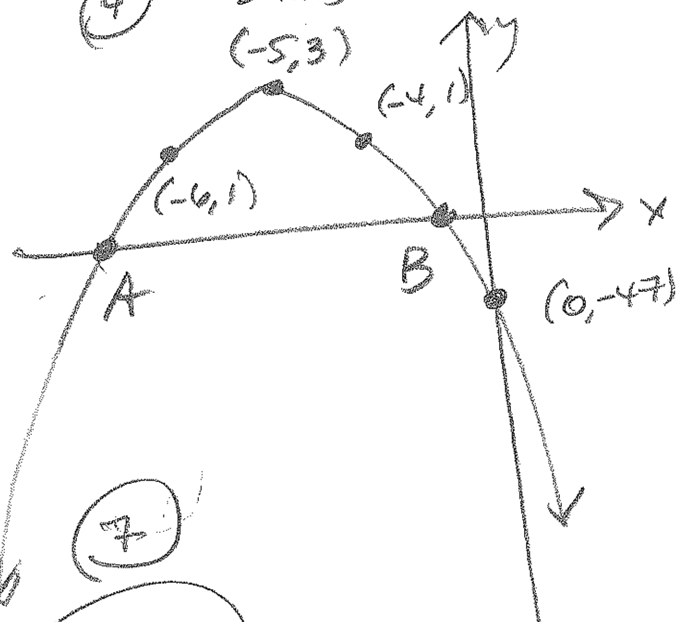
2) $-2f(x) = -2x^2$



3) $-2(x+5)^2 = -2f(x+5)$



4) $-2(x+5)^2 + 3$



7)

y-int:

$r(0) = -2(25)+3$

$= -50 + 3$

$= -47$

x-int $-2(x+5)^2 = -3$

$(x+5)^2 = \frac{3}{2}$

$x+5 = \pm\sqrt{\frac{3}{2}} = \pm\frac{\sqrt{6}}{2}$

$x = -5 \pm \frac{\sqrt{6}}{2}$

$$\boxed{\begin{aligned} A &= \left(-5-\tfrac{\sqrt{6}}{2}, 0\right) \\ B &= \left(-5+\tfrac{\sqrt{6}}{2}, 0\right) \end{aligned}}$$

121 TEST 2

8 $f(x) = \sqrt[3]{2x+8}$

$\sqrt[3]{2y+8} = x$

$2y+8 = x^3$

$2y = x^3 - 8$

$y = \boxed{\frac{x^3-8}{2} = f^{-1}(x)}$

9 $I = k\left(\frac{1}{r^2}\right)$

I = intensity

r = distance from source

B1 $f(x) = \frac{1}{\sqrt{x}} \Rightarrow \frac{f(x+h)-f(x)}{h} = \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h}$

$= \frac{1}{h}\left[\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}\right] = \frac{1}{h}\left[\frac{\sqrt{x} - \sqrt{x+h}}{\sqrt{x+h}\sqrt{x}}\right]$

$x-(x+h)$
$= x - h - x$

$= \frac{1}{h}\left[\left(\frac{\sqrt{x}-\sqrt{x+h}}{\sqrt{x(x+h)}}\right)\left(\frac{\sqrt{x}+\sqrt{x+h}}{\sqrt{x}+\sqrt{x+h}}\right)\right] = \frac{1}{h}\left[\frac{x-(x+h)}{\sqrt{x(x+h)}(\sqrt{x}+\sqrt{x+h})}\right]$

$= \frac{1}{h}\left[\frac{-h}{\sqrt{x(x+h)}(\sqrt{x}+\sqrt{x+h})}\right] = -\frac{1}{\sqrt{x(x+h)}(\sqrt{x}+\sqrt{x+h})}$

$\xrightarrow{h \to 0} \frac{-1}{\sqrt{x^2}(\sqrt{x}+\sqrt{x})} = \frac{-1}{2\sqrt{x^2}\sqrt{x}} = \boxed{-\frac{1}{2\sqrt{x^3}}}$

121 TEST 2

B2 $h(x) = 7x^2 - 11x + 3$

$= 7\left(x^2 - \frac{11}{7}x + \left(\frac{11}{14}\right)^2\right) - 7\left(\frac{121}{196}\right) + 3$

$= \boxed{7\left(x + \frac{11}{14}\right)^2 - \frac{37}{28}}$

$\begin{array}{r} 14 \\ \times 14 \\ \hline 56 \\ 140 \\ \hline \end{array}$

$\frac{7(121)}{196} = \frac{121}{28}$

$\begin{array}{r} 28 \\ \times 3 \\ \hline 84 \end{array}$

$\frac{-121}{28} + \frac{84}{28} = \frac{-37}{28}$

B3 $r(x) = \dfrac{x^{77} - 5x^{12} + 17}{x^2 + 5x + 7}$

Need

$x^2 + 5x + 7 \neq 0$

$a = 1, b = 5, c = 7 \Rightarrow$

$b^2 - 4ac = 5^2 - 4(1)(7)$

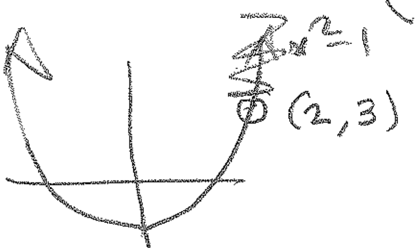
$= 25 - 28 = -3$

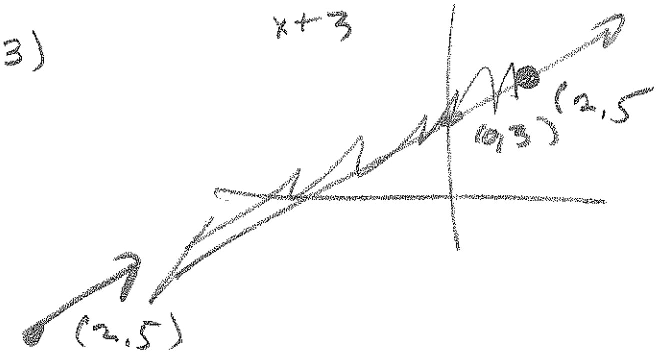
No real zeros $\Longrightarrow$ $\boxed{D = (-\infty, \infty)}$

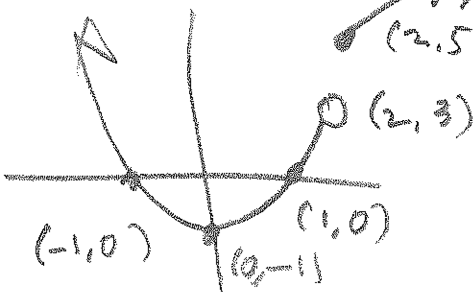
121 TEST 2

B4

$$f(x) = \begin{cases} x^2-1 & \text{if } x<2 \\ x+3 & \text{if } x\ge 2 \end{cases}$$







B5

$$g\circ f = g(f(x)) = \sqrt{\frac{x+11}{x-2}+7}$$

$$\mathcal{D} = \{x \mid x \in \mathcal{D}(f) \text{ and } f(x) \in \mathcal{D}(g)\}$$

$$= \left\{x \;\middle|\; x \ne 2 \text{ and } \frac{x+11}{x-2}+7 \ge 0\right\}$$

$$= \{x \mid x \ne 2 \text{ and } (x \ge 2 \text{ OR } x \le \tfrac{3}{8})\}$$

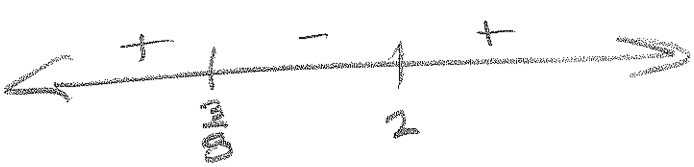
$=$ 

$$= \boxed{(-\infty, \tfrac{3}{8}] \cup (2, \infty)}$$

$$\frac{x+11}{x-2} + 7 \ge 0$$

$$\frac{x+11+7x-14}{x-2} \ge 0$$

$$\frac{8x-3}{x-2} \ge 0$$



121 TEST 2

B6

John: $\frac{1}{6}x = \left(\frac{1}{6}\frac{\text{Job}}{\cancel{hr}}\right)(x\ \cancel{hrs}) = \frac{1}{6}x$ is in "job" units.

Bob: $\frac{1}{10}y$

Let x = # of hours John works

y = " " " Bob works

John starts 2 hrs early $\Longrightarrow$ $x = y + 2$ OR $y = x - 2$, whichever.

$\frac{1}{6}x + \frac{1}{10}y = 1$ job done.

$6 = 2 \cdot 3$ LCD $= 2 \cdot 3 \cdot 5$

$10 = 2 \cdot 5$ $= 30$

$30\left[\frac{1}{6}x + \frac{1}{10}(x-2) = 1\right]$

$5x + 3x - 6 = 30$

$8x = 36$

$x = \frac{36}{8} = \frac{9}{2} = \boxed{4.5 = x} \Longrightarrow y = 4.5 - 2$

$\boxed{y = 2.5}$