

$$\textcircled{1} \quad x + y + z = 6$$

$$x - y - z = -4$$

$$2x + y - z = 3$$

$$\textcircled{2} \quad x + y + z = 6$$

$$2y + bz = c$$

$$dy + ez = f$$

$$\textcircled{3} \quad \begin{array}{l} x + y + z = 6 \\ 2y + bz = c \\ gz = h \end{array} \quad \begin{array}{l} \text{Get } x \\ \text{Get } y \\ \text{Gives } z \end{array} \quad \left. \vphantom{\begin{array}{l} x + y + z = 6 \\ 2y + bz = c \\ gz = h \end{array}} \right\} \text{Back-substitute.}$$

$$-E1 \quad -x - y - z = -6$$

$$E2 \quad x - y - z = -4$$

$$\hline -E1 + E2 \quad -2y - 2z = -10$$

$$\Rightarrow y + z = 5$$

(Divide by -2)

$$-2E1 \quad -2x - 2y - 2z = -12$$

$$E3 \quad 2x + y - z = 3$$

$$\hline -y - 3z = -9$$

$$\text{Times } -1: \quad y + 3z = 9$$

$$-E2 \quad -y - z = -5$$

$$E3 \quad y + 3z = 9$$

$$\hline -E2 + E3 \quad 2z = 4$$

$$\text{Divide by 2: } z = 2$$

$$\boxed{(x, y, z) = (1, 3, 2)}$$

$$E1 \quad x + y + z = 6$$

$$-E1 + E2 \quad y + z = 5$$

$$-2E1 + E3 \quad y + 3z = 9$$

$$E1 \quad x + y + z = 6$$

$$E2 \quad y + z = 5$$

$$-E2 + E3 \quad z = 2$$

Sweet!

Triangular!

Back-Substitute

$$\textcircled{z=2}:$$

$$E2: \quad y + z = 5$$

$$y + 2 = 5$$

$$\textcircled{y=3}$$

$$E1: \quad x + y + z = 6$$

$$x + 3 + 2 = 6$$

$$x + 5 = 6$$

$$\textcircled{x=1}$$

$$\begin{aligned} x+y+z &= 6 \\ x-y-z &= -4 \\ 2x+y-z &= 3 \end{aligned} \quad \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & -1 & -1 & -4 \\ 2 & 1 & -1 & 3 \end{array} \right]$$

$$\begin{array}{l} R1 \\ -R1+R2 \\ -2R1+R3 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -2 & -2 & -10 \\ 0 & -1 & -3 & -9 \end{array} \right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 1 & 5 \\ 0 & 1 & 3 & 9 \end{array} \right]$$

$$\begin{array}{l} R1 \\ R2 \\ -R2+R3 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 1 & 5 \\ 0 & 0 & 2 & 4 \end{array} \right]$$

$$\begin{aligned} x+y+z &= 6 \\ y+z &= 5 \\ 2z &= 4 \end{aligned}$$

$$z = 2$$

$$\begin{aligned} x+y+z &= 6 \Rightarrow z = -x-y+6 \\ x-y-z &= -4 \Rightarrow z = -x+y-4 \\ 2x+y-z &= 3 \Rightarrow z = 2x+y-3 \end{aligned}$$

See previous page for x & y

Any toughies on homework?
 Substitution:

$$x + y + z = 6 \Rightarrow x = 6 - y - z$$

Use this for x in other 2 equations

$$x - y - z = -4 \Rightarrow (6 - y - z) - y - z = -4 \Rightarrow -2y - 2z = -10$$

$$2x + y - z = 3 \Rightarrow 2(6 - y - z) + y - z = 3 \Rightarrow -y - 3z = -9$$

New System: 2x2:

$$-2y - 2z = -10 \Rightarrow y + z = 5$$

$$-y - 3z = -9 \Rightarrow y + 3z = 9$$

Use this for y in the 3rd equation

$$y = 5 - z$$

$$(5 - z) + 3z = 9$$

$$5 + 2z = 9$$

$$\text{So, } (x, y, z) = (1, 3, 2)$$

$$2z = 4$$

$$z = 2$$

$$y = 5 - z = 5 - 2 = 3 = y$$

$$x = 6 - y - z$$

$$= 6 - 3 - 2 = 1 = x$$