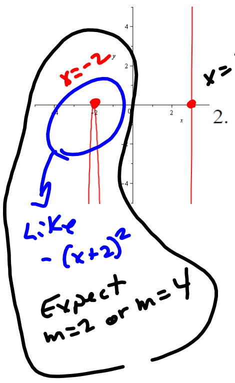
Use separate paper to do the work on this take-home test. Start a fresh sheet of paper to show work on #4. Use paper without lines. Use only one side of each sheet of paper. *I will not grade work written on the backs of pages.* Write clearly and make sure your pencil work is *dark*. It's a struggle for me to read faint print.

Do all your work on separate paper. Circle final answers on the separate paper. I do *not* want an answer key. I want to see work with the final answers all in the same place.

For the take-home, be *sure* to use a grapher of some sort to shorten the way. On the sit-down test, we cook things up to come out nice, with most of the answers in that *x* = +/- 1, 2, 3 range being about as far as you have to go to break things down.

$$f(x)=x^5-x^4-5x^3+9x^2-16x-60.$$

1. (3 pts) Describe the end behavior of the graph of f with a simple graphic.



x=3 ish

2. (3 pts) Use Descartes' Rule of Signs to determine the *possible* number of positive and negative zeroes of f.

$$f(x)=x^5-x^4-5x^3+9x^2-16x-60.$$

3 or 1

$$f(-x)=-x^5-x^4+5x^3+9x^2+16x-60$$

2 or 0

2, 3, 5, 7, 11, 13, 17, 19

3. (3 pts) Use the Rational Zeroes Theorem to determine the possible rational zeroes of f.

$$f(x)=x^5-x^4-5x^3+9x^2-16x-60.$$

$\boxed{\pm 1, \pm 2, \pm 3,}$ ± 5, ± 4,
± 6, ± 10, ± 15, ± 12, ± 30, ± 60

2 | 60
2 | 30
3 | 15
5

4. (3 pts) Informed by your work, above, *and a graphing utility of some sort*, use synthetic division to find the zeros. Each time you find a zero, it *should* reduce (depress) the question by one degree. Each time you find a zero, you should, thereafter, be working with a *depressed polynomial* that is of lesser degree.

Dividing by x-

3	1	-1	-5	9	-16	-60
		3	6	3	36	60
-2	1	2	1	12	20	0 sweet!
		-2	0	-2	-20	
-2	1	0	1	10	0 sweet!	
		-2	4	-10		
	1	-2	5	0		

$x^2-2x+5=0$

$x^2-2x+1^2=-5+1=-4$

$(x-1)^2=-4$

$x-1=\pm\sqrt{-4}=\pm 2i$

$x=1\pm 2i$

$f(x)=(x-3)(x+2)^2(x^2-2x+5)$

$f(x)=(x-3)(x+2)(x^3+x+10)$

Split off a factor of x-3

$f(x)=(x-3)(x^4+2x^3+x^2+12x+20)$

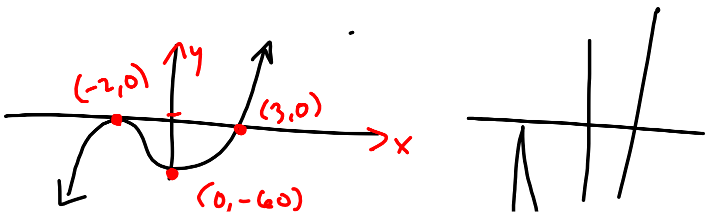
5. (3 pts) From you work, above, factor f over the real numbers. This will involve an irreducible quadratic factor.

$(x-3)(x+2)^2(x^2-2x+5)$

6. (3 pts) From your work above, factor f over the complex numbers. This should split f into linear factors.

$(x-3)(x+2)^2(x-(1+2i))(x-(1-2i))$

7. (3 pts) Give a rough sketch of f that shows all intercepts.



$f(x) = 18x^5 + 21x^4 - 86x^3 - 35x^2 + 124x - 60$

$f(-x) = -18x^5 + 21x^4 + 86x^3 - 35x^2 - 124x - 60$

2 | 18
3 | 9
3

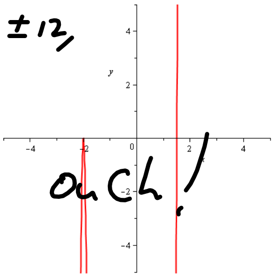
2 | 60
2 | 30
3 | 15
5

$\pm 2, \pm 3, \pm 4, \pm 5, \pm 6, \pm 10, \pm 12,$
$\pm 15, \pm 20, \pm 30, \pm 60,$

$\cancel{\pm \frac{2}{2}}, \pm \frac{3}{2}, \cancel{\pm \frac{4}{2}}, \pm \frac{5}{2}, \ldots$

$\pm \frac{2}{3}, \pm \frac{3}{3}, \pm \frac{4}{3}, \pm \frac{5}{3}, \ldots$

$\pm \frac{2}{6}, \pm \frac{2}{6}, \ldots$



ouch!

1. (3 pts) Describe the end behavior of the graph of f with a simple graphic.

2. (3 pts) Use Descartes' Rule of Signs to determine the *possible* number of positive and negative zeroes of f.

(3 pts) Use the Rational Zeroes Theorem to determine the possible rational zeroes of f.

(3 pts) Informed by your work, above, *and a graphing utility of some sort*, use synthetic division to find the zeros. Each time you find a zero, it *should* reduce (depress) the question by one degree. Each time you find a zero, you should, thereafter, be working with a *depressed polynomial* that is of lesser degree.

5. (3 pts) From you work, above, factor f over the real numbers. This will involve an irreducible quadratic factor.

6. (3 pts) From your work above, factor f over the complex numbers. This should split f into linear factors.

(3 pts) Give a rough sketch of f that shows all intercepts.

8. (3 pts) Sketch the graph of $R(x) = \dfrac{2x^2 - x - 3}{x^2 + 2x - 15}$. Show all asymptotes, intercepts and any holes.

Degree of the top = degree of the bottom -> Horizontal Asymptote

$$\frac{(2x-3)(x+1)}{(x+5)(x-3)}$$

$D = \mathbb{R} \setminus \{-5, 3\}$

V.A. = $x = -5$, $x = 3$

H.A = $y = 2$

$\boxed{\text{Deg Top} = \text{Deg Bottom}.}$ Check Big stuff

$x \to \text{Big}$

$$\frac{2x^2 - x - 3}{x^2 + 2x - 3} \to \frac{2x^2}{x^2} = 2 = y$$

$\boxed{\text{Deg Top} < \text{Deg Bottom}}$ $\to y = 0$

$$\frac{x^5 + \text{stuff}}{19x^7 + \text{stuff}}$$ Proper Rational Func.

$x \to \pm\infty$ $\to 0$

$\boxed{y = 0}$

$2x^2 - x - 3 = 0$

$a = 2, b = -1, c = -3$

$b^2 - 4ac = (-1)^2 - 4(2)(-3)$

$= 1 + 24$

$= 25$

$$x = \frac{-(-1) \pm \sqrt{25}}{2(2)}$$

$$= \frac{1 \pm 5}{4}$$ $\to \frac{6}{4} = \frac{3}{2}$, $\to -\frac{4}{4} = -1$

$\to 2(x+1)(x - \frac{3}{2})$

$= (x+1)(2)(x - \frac{3}{2})$

$= (x+1)(2x-3)$

8. (3 pts) Sketch the graph of $R(x)=\dfrac{2x^2-x-3}{x^2+2x-15}$. Show all asymptotes, intercepts and any holes.

$$\frac{(2x-3)^1(x+1)^1}{(x+5)^1(x-3)^1}$$

$D = \mathbb{R} \setminus \{-5, 3\}$

V.A. $=$ $x=-5$, $x=3$

H.A $=$ $y=2$

x-int : $\left(\frac{3}{2}, 0\right)$ C $(-1,0)$ A

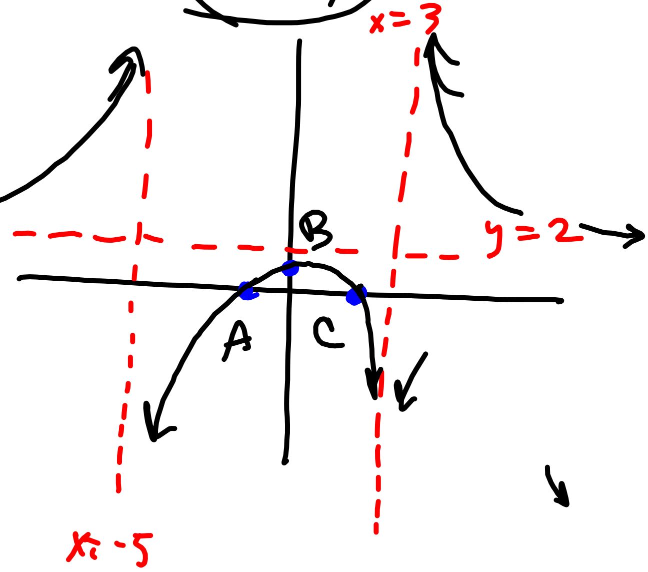
$2x-3=0$ $x+1=0$

$2x=3$ $x=-1$

$x=\frac{3}{2}$

y-int: $R(0)=\frac{-3}{-15}=\frac{1}{5}$

$\left(0, \frac{1}{5}\right)$ B



8. (3 pts) Sketch the graph of $R(x)=\frac{2x^2-x-3}{x^2+2x-15}$. Show all asymptotes, intercepts and any holes.

9. (3 pts) The graph of $g(x)=\frac{2x^3-9x^2+x+12}{x^3-2x^2-23x+60}$ differs from the graph of *f*, in #8, in only one small detail. Sketch the graph of *g*, showing all asymptotes, intercepts and holes.

$$g(x)=\frac{(2x-3)(x+1)\cancel{(x-4)}}{(x+5)(x-3)\cancel{(x-4)}}$$

Basically #8, all over again, only there's a hole!

$$\frac{2x^2-x-3}{x^2+2x-15} = \frac{(2x-3)(x+1)}{(x+5)(x-3)}$$

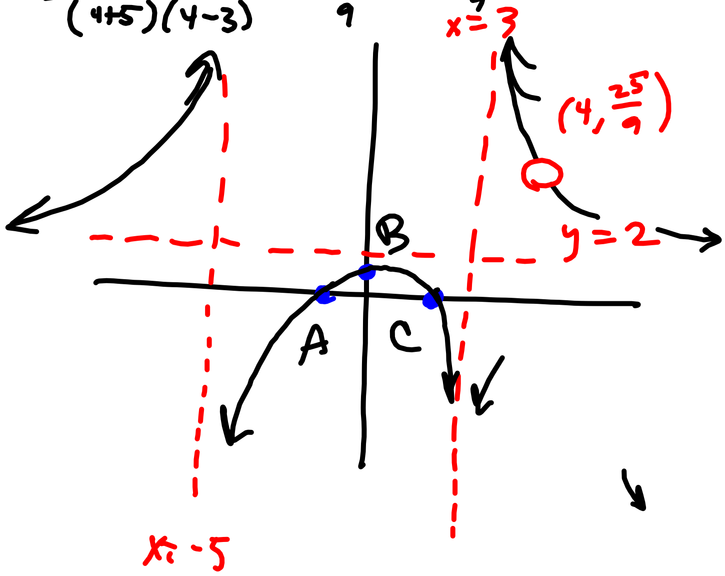
Hole @ $x=4$

$$\frac{(2(4)-3)(4+1)}{(4+5)(4-3)} = \frac{(5)(5)}{9} = \frac{25}{9}$$

$x^3-2x^2-23x+60$

$$\begin{array}{r|rrrr}
-5 & 1 & -2 & -23 & 60 \\
 & & -5 & 35 & -60 \\
\hline
3 & 1 & -7 & 12 & 0 \\
 & & 3 & -12 & \\
\hline
 & 1 & -4 & &
\end{array}$$

$\rightarrow x-4$



9. (3 pts) The graph of $g(x)=\dfrac{2x^3-9x^2+x+12}{x^3-2x^2-23x+60}$ differs from the graph of f, in #8, in only one small detail. Sketch the graph of g, showing all asymptotes, intercepts and holes.

Basically #8, all over again, only there's a hole!

10. (3 pts) Sketch the graph of $h(x) = -\dfrac{x^2 - 6x + 3}{x - 4}$, showing all asymptotes, intercepts and holes.