

30 pts

Due Monday, November 3<sup>rd</sup>, at the beginning of class. I will have *zero* patience for stapling during class time.

Use separate paper to do the work on this take-home test. Start a fresh sheet of paper to show work on #4. Use paper without lines. Use only one side of each sheet of paper. *I will not grade work written on the backs of pages.* Write clearly and make sure your pencil work is *dark*. It's a struggle for me to read faint print.

Let  $f(x) = x^5 - x^4 - 5x^3 + 9x^2 - 16x - 60$ . We'll say everything we can about this polynomial that's worth saying.

1. (3 pts) Describe the end behavior of the graph of  $f$  with a simple graphic.
2. (3 pts) Use Descartes' Rule of Signs to determine the *possible* number of positive and negative zeros of  $f$ .
3. (3 pts) Use the Rational Zeroes Theorem to determine the possible rational zeroes of  $f$ .
4. (3 pts) Informed by your work, above, *and maybe a graphing utility of some sort*, use synthetic division to find the zeros. Each time you find a zero, it *should* reduce (depress) the question by one degree. Each time you find a zero, you should thereafter be working with a *depressed polynomial* that is of lesser degree.
5. (3 pts) From your work, above, factor  $f$  over the real numbers. This will involve an irreducible quadratic factor.
6. (3 pts) From your work above, factor  $f$  over the complex numbers. This should split  $f$  into linear factors.
7. (3 pts) Give a rough sketch of  $f$  that shows all intercepts.
8. (3 pts) Sketch the graph of  $R(x) = \frac{x^2 - 5x - 6}{x^2 - 5x + 6}$ . Show all asymptotes, intercepts and any holes.
9. (3 pts) The graph of  $g(x) = \frac{x^3 - 9x^2 + 14x + 24}{x^3 - 9x^2 + 26x - 24}$  differs from the graph of  $f$ , in #8, in only one small detail.

Sketch the graph of  $g$ , showing all asymptotes, intercepts and holes.

10. (3 pts) Sketch the graph of  $R(x) = \frac{x^3 - 4x^2 - 7x + 10}{x^2 - x - 6}$ , showing all asymptotes, intercepts and holes.