Quadratic Inequalities

$ax^2+bx+c \geq 0$

< 0

≤ 0

> 0

Method ① Solve the "=" version

② Analyze the sign pattern.

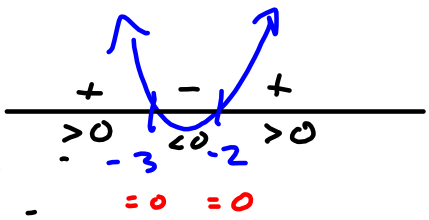
$x^2+5x+6 < 0$

$x^2+5x+6 = 0$

$(x+2)(x+3) = 0$

$\Rightarrow x=-2$ OR $x=-3$.

Picture:



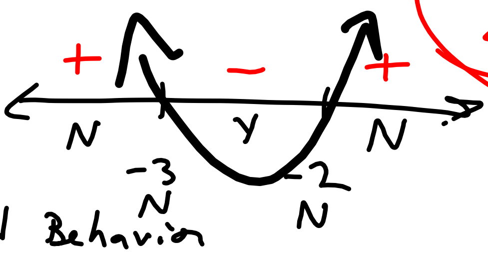
students WANT to do -

" $x < -2$ OR $x < -3$ "

but that's not how it works !!!!

$x \in (-3,-2)$

want < 0

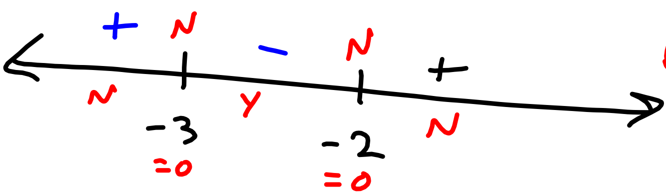


End Behavior

$+x^2$

Other Method Test Values

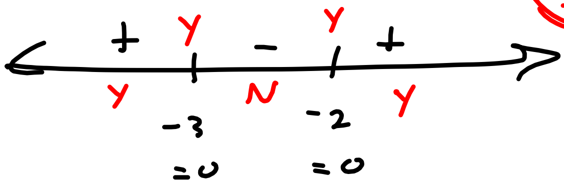
< 0



	Test	f(x)	
$(-\infty,-3)$	-4	$(-4)^2+5(-4)+6 = 2$	+
$(-3,-2)$	-2.5	$(-2.5)^2+5(-2.5)+6 = 6.25-12.5+6 = -.25$	−
$(-2,\infty)$	0	6	

$x^2+5x+6 \ge 0$

≥ 0



$(-\infty, -3] \cup [-2, \infty)$

I urge you to build these sign patterns off of general understanding of function shapes.

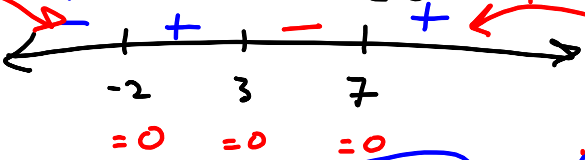
$x^3 - 8x^2 + x + 42 = (x+2)(x-7)(x-3)$

$x+2=0$
$-2=-2$
$x=-2$

$x^3 - 8x^2 + x + 42 \ge 0$

$(x+2)(x-7)(x-3) \ge 0$

$(x+2)(x-7)(x-3) \ge 0$



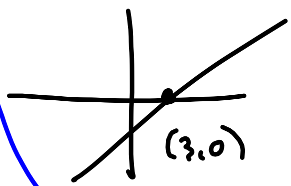
x^3

The bigger x gets, the more the x^3 controls.

END Behavior

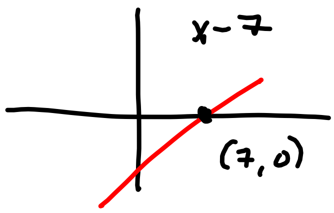
Changes sign @ x=3

$x-3$



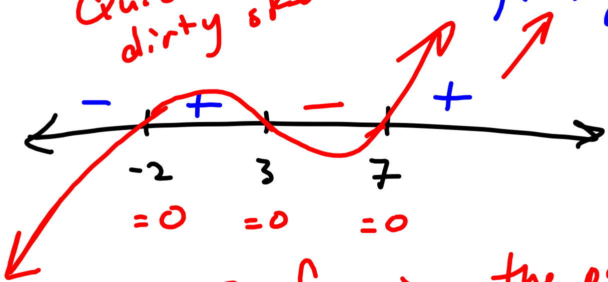
Changes sign at x=7

$x-7$



So they MAKE the big function change sign.

Quick dirty sketch



Informing the picture with the sign pattern.

$$\frac{(3)(-2)(-11)}{(5)(7)} \quad \text{is } +$$

$$\frac{(-3)(-2)(-11)}{(5)(7)} \quad -$$

So, one factor changes sign, the whole thing changes sign!

Long Division

$$\begin{array}{r} 93 \text{ r } 2 \\ 3 \overline{)281} \\ -\ 270 \\ \hline 11 \\ -\ 9 \\ \hline 2 \end{array}$$

This says

$$\frac{281}{3} = 93 + \frac{2}{3}$$

$$\left(\frac{281}{3} = 93 + \frac{2}{3}\right)(3)$$

$$\boxed{281 = (93)(3) + 2}$$

$$\frac{4x^4}{2x^2} = 2x^2 \qquad \frac{8x^3}{2x^2} = 4x = \frac{8}{2}\cdot x^{3-2} \qquad \frac{2x^2}{2x^2} = 1$$

Dividend = Quotient • Divisor + Remainder

$$\begin{array}{r} 2x^2 + 4x + 1 \text{ r } -7x-1 \\ 2x^2+2 \overline{)4x^4 + 8x^3 + 6x^2 + x + 1} \\ -\left(4x^4 \qquad\quad 4x^2\right) \\ \hline 8x^3 + 2x^2 + x + 1 \\ -\left(8x^3 \qquad\quad +8x\right) \\ \hline 2x^2 - 7x + 1 \\ -\left(2x^2 \qquad\quad 2\right) \\ \hline -7x - 1 \end{array}$$

This says

$$\frac{4x^4+8x^3+6x^2+x+1}{2x^2+2} = 2x^2+4x+1 + \frac{-7x-1}{2x^2+2}$$

OR

$$4x^4+8x^3+6x^2+x+1 = (2x^2+4x+1)(2x^2+2) - 7x - 1$$

Dividend = Quotient • Divisor + Remainder

Called "Division Algorithm."

Special: when divisor is of the form $x-c$, use synthetic Division

$$
\begin{array}{r l}
 & x^2+7x+8 \quad r\,18 \\
x-2 & \overline{)\,x^3+5x^2-6x+2} \\
 & -(x^3-2x^2) \\
\hline
 & 7x^2-6x+2 \\
 & -(7x^2-14x) \\
\hline
 & +8x+2 \\
 & -(8x-16) \\
\hline
 & 18
\end{array}
$$

$$
\begin{array}{r|rrrr}
2 & 1 & 5 & -6 & 2 \\
 & & 2 & 14 & 16 \\
\hline
 & 1 & 7 & 8 & 18 \\
 & x^2 & x^1 & c & r
\end{array}
$$

This says

$f(x) = x^3+5x^2-6x+2 = (x^2+7x+8)(x-2) + 18$

what's $f(2)$?

Remainder Theorem:

$f(c)$ = Remainder when dividing $f(x)$ by $x-c$.

$f(x) = x^3 - 8x^2 + x + 42$

Find $f(8)$

$$
\begin{array}{r|rrrr}
8 & 1 & -8 & 1 & 42 \\
  &   & 8 & 0 & 8 \\
\hline
  & 1 & 0 & 1 & 50
\end{array}
$$

$50 = f(8)$