§ 8.1 Sequences & Series

$n! = n$ factorial $= n(n-1)(n-2)\cdots(3)(2)(1)$

$0! = 1$

$1! = 1$

$2! = 2\cdot 1 = 2$

$3! = 3\cdot 2\cdot 1 = 6$

$4! = 4\cdot 3\cdot 2\cdot 1 = 24$

$5! = 5\cdot 4! = 120$

$\vdots$

Fun with 'em

$$\frac{10!}{8!} = \frac{10\cdot 9\cdot \cancel{8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2}}{\cancel{8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2}}$$

$$= 10\cdot 9 = 90$$

$\frac{10!}{8!} = 10\cdot 9 = 90$, since you "see" 8! chowin' up the rest of the 10!!

$a_n = \frac{(-1)^n}{n!}$ 1^{st} 5 terms are:

$a_1 = \frac{(-1)^1}{1!} = \frac{-1}{1} = -1$

$a_2 = \frac{(-1)^2}{2!} = \frac{1}{2}$

$a_3 = \frac{(-1)^3}{3!} = \frac{-1}{3\cdot 2} = -\frac{1}{6}$

$a_4 = \frac{1}{24}$

$a_5 = -\frac{1}{120}$

See how $(-1)^n$ gives us alternating signs?

ODD-index terms negative $\overset{1}{-1}, \overset{2}{\frac{1}{2}}, \overset{3}{-\frac{1}{6}}, \overset{4}{\frac{1}{24}}, \overset{5}{-\frac{1}{120}}$

How would you define b_n, if the first 5 terms were $1, -\frac{1}{2}, \frac{1}{6}, -\frac{1}{24}, \frac{1}{120}$?

even-index terms negative.

Vince says the 2^{nd} sequence is -1 times the first sequence:

$$b_n = -a_n = -\left(\frac{(-1)^n}{n!}\right) = (-1)\left(\frac{(-1)^n}{n!}\right)$$

$$= \frac{(-1)^1(-1)^n}{n!} = \frac{(-1)^{n+1}}{n!}$$

$(-1)^{n-1}$ would have the same effect as $(-1)^{n+1}$

even-index terms negative.

$$\begin{array}{cccccc} k=1 & k=2 & k=3 & k=4 & & \\ 6, & 8, & 10, & 12, & \ldots, & a_n = ? \end{array}$$

Each term is 2 more than the one before.

2K is part of it

$2k + ? = 6$

$2(1) + ? = 6$

$? = 4$

Try $2k + 4 = a_k$

$a_1 = 2(1) + 4 = 6$ ✓

$a_2 = 2(2) + 4 = 8$ ✓

$\vdots$

$\boxed{a_n = 2n + 4}$ $n = 1, 2, 3, \ldots$

Could you do a version that started with $K = 4$?

$$\begin{array}{cccccc} k=4 & k=5 & k=6 & k=7 & & \\ 6, & 8, & 10, & 12, & \ldots, & a_n = ? \end{array}$$

$2K + ?$

$2(4) + ? = 6$

$8 + ? = 6$

$? = -2$

$\boxed{a_n = 2n - 2, \ n = 4, 5, 6, \ldots}$

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$$\overset{1}{1}, -\overset{2}{\frac{1}{8}}, \overset{3}{\frac{1}{27}}, -\overset{4}{\frac{1}{64}}, \dots, \quad a_n = ?$$

Start with $k=1$

Pattern of cubes with alternating sign.

Negative exponent

$$(-1)^{n-1}(\ ?\ )$$

$k=1$:

$$1 = (-1)^{1-1}\left(\frac{1}{1^3}\right) = (-1)^{1-1}(1)^{-3} = a_1$$

$k=2$:

$$-\frac{1}{8} = (-1)^{2-1}\left(\frac{1}{2^3}\right) = (-1)^{2-1}(2)^{-3} = a_2$$

$k=3$:

$$\frac{1}{27} = (-1)^{3-1}\left(\frac{1}{3^3}\right) = (-1)^{3-1}(3)^{-3} = a_3$$

$$a_n = (-1)^{n-1}\left(\frac{1}{n^3}\right) = (-1)^{n-1}(n)^{-3} = a_n$$

$$= \frac{(-1)^{n-1}}{n^3} = (-1)^{n-1}\cdot n^{-3} = (-1)^{n-1}\left(\frac{1}{n}\right)^3$$

Recursion

a_n is built from a_{n-1}

OR

a_{n+1} is built from a_n — probably more convenient.

$$a_{n+1} = 1 - \frac{1}{a_n} \quad , \quad a_1 = 2$$

First 4 terms are?

$a_1 = 2$, $a_2 = 1 - \frac{1}{a_1} = 1 - \frac{1}{2} = \frac{1}{2}$

$a_3 = 1 - \frac{1}{a_2} = 1 - \left(\frac{1}{\left(\frac{1}{2}\right)}\right) = 1 - 2 = -1$

$a_4 = 1 - \frac{1}{-1} = 1 + 1 = 2 = a_1$

$a_5 = a_2$ we just went full circle

$a_6 = a_3$

$a_7 = a_4$

$\vdots$

$a_n = a_{n-3}$ as it turns out.

$\boxed{\text{Series}}$ – Goal is Present Value of an Annuity.

→ Exponential Growth

what lump sum today, if put in the bank, would yield the same future value as a stream (sequence) of payments placed in the bank?

→ Geometric Growth.

Series is the sum of a sequence.

$\{a_1, a_2, a_3, a_4\}$ is a finite sequence.

$a_1 + a_2 + a_3 + a_4$ is a finite series.

$= \sum_{k=1}^{4} a_k$ is Sigma Notation.

$$\sum_{k=1}^{5}(2k+1) = 2(1)+1 + 2(2)+1 + 2(3)+1 + 2(4)+1 + 2(5)+1$$

$$= 3+5+7+9+11 = 35$$

Gauss: Boy Mutant

$$\sum_{k=1}^{100} k = \frac{100(101)}{2} = 50(101) = 5050$$

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{100} 1 = 100$$

$$\sum_{k=1}^{100} 2 = 200$$

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$$\frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \frac{1}{13}$$

1 2 3 4 5

write in Σ-notation, starting with $k=1$

$$\sum_{k=1}^{5} (-1)^{k-1}\left(\frac{1}{2k+3}\right)$$

$k=1$ $\frac{1}{5} = \frac{1}{2k+?} = \frac{1}{2k+3}$

$k=2$ $\frac{1}{7} = \frac{1}{2(2)+3}$? Yes

Next time:
Geometric
Series
& J.G. Wentworth.

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