Rock:

$y = x^2$ ✓

$6x - y = 5$

$6(1) - 1 = 5$ ✓

$6(5) - 25 = 5$ ✓

$\Longrightarrow$

$6x - (x^2) = 5$

$-x^2 + 6x - 5 = 0$

$x^2 - 6x + 5 = 0$

$(x-1)(x-5) = 0$

$x = 1$ or $x = 5$

$y = 1$ or $y = 25$

$\boxed{(1,1),\ (5,25)}$

Swap rows

Multiply row by #

Add multiples of rows together

Goal

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 5 & 6 \\ 0 & 0 & 1 & 7 \end{array}\right]$$

Echelon Form

This system is INDEPENDENT.

$\{(41, -29, 7)\}$

$z = 7$

$y + 5z = 6$

$y + 5(7) = 6$

$y + 35 = 6$

$y = -29$

$x + 2y + 3z = 4$

$x + 2(-29) + 3(7) = 4$

$x - 58 + 21 = 4$

$x - 37 = 4$

$x = 41$

$$3x = 4 + y \qquad 3x - y = 4$$
$$x + y = z - 1 \qquad x + y - z = -1$$
$$2z = 3 - x \qquad x + 2z = 3$$

$$\left[\begin{array}{ccc|c} 3 & -1 & 0 & 4 \\ 1 & 1 & -1 & -1 \\ 1 & 0 & 2 & 3 \end{array}\right] R1 \longleftrightarrow R3 \left[\begin{array}{ccc|c} 1 & 0 & 2 & 3 \\ 1 & 1 & -1 & -1 \\ 3 & -1 & 0 & 4 \end{array}\right]$$

$$\begin{array}{c} R1 \\ -R1+R2 \\ -3R1+R3 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 2 & 3 \\ 0 & 1 & -3 & -4 \\ 0 & -1 & -6 & -5 \end{array}\right] \begin{array}{c} R1 \\ R2 \\ R2+R3 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 2 & 3 \\ 0 & 1 & -3 & -4 \\ 0 & 0 & -9 & -9 \end{array}\right]$$

$$\begin{array}{rrrrr} -R1 & -1 & 0 & -2 & -3 \\ R2 & 1 & 1 & -1 & -1 \\ \hline & 0 & 1 & -3 & -4 \end{array} \qquad \begin{array}{rrrrr} -3R1 & -3 & 0 & -6 & -9 \\ R3 & 3 & -1 & 0 & 4 \\ \hline & 0 & -1 & -6 & -5 \end{array}$$

$$\begin{array}{c} R1 \\ R2 \\ -\frac{1}{9}R3 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 2 & 3 \\ 0 & 1 & -3 & -4 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$z = 1$

$y - 3z = -4$

$y - 3(1) = -4$

$y = -1$

$x + 2z = 3$

$x + 2(1) = 3$

$x = 1$

$$\boxed{\{(1, -1, 1)\}}$$

$$\begin{aligned} x - y + z &= 2 \\ 2x + y - z &= 1 \\ 2x - 2y + 2z &= 4 \end{aligned}$$

<u>Augmented Coefficient Matrix.</u>

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 2 & 1 & -1 & 1 \\ 2 & -2 & 2 & 4 \end{array}\right] \begin{array}{l} R1 \\ -2R1+R2 \\ -2R1+R3 \end{array} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 3 & -3 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{array}{c} \\ \frac{1}{3}R2 \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right] \qquad \begin{aligned} x - y + z &= 2 \\ y - z &= -1 \\ 0 &= 0 \end{aligned}$$

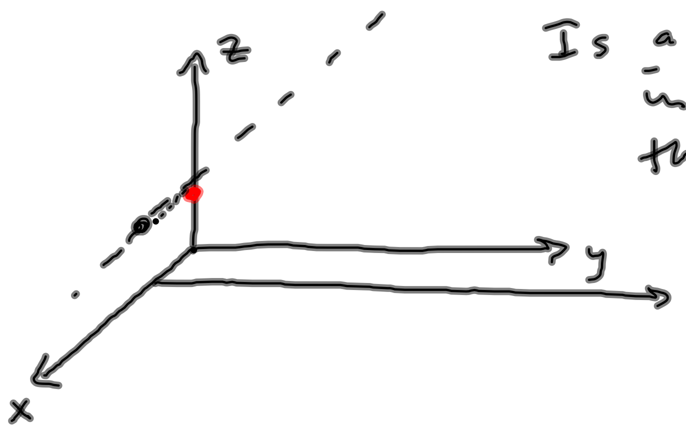
Get x & y in terms of z

$y = z - 1$

$$\begin{aligned} x - y + z &= 2 \\ x - (z-1) + z &= 2 \\ x - z + 1 + z &= 2 \\ x &= 1 \end{aligned}$$

Thx, Sarah.

$$\{(x, y, z) \mid (1, z-1, z)\}$$



Is a line in 3-space that your teacher can't draw w/o thinking.

$$\begin{aligned} x - y + z &= 2 \\ 2x + y - z &= 1 \\ 2x - 2y + 2z &= 5 \end{aligned} \qquad \left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 2 & 1 & -1 & 1 \\ 2 & -2 & 2 & 5 \end{array}\right]$$

$$\begin{array}{l} R1 \\ -2R1+R2 \\ -2R1+R3 \end{array} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 3 & -3 & -3 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

No solution.

$0 = 1$!?

Matrix addition ✓

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} -3 & 5 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 7 \\ 5 & 5 \end{bmatrix}$$

Scalar Multiplication ✓

$$-3\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} -3 & -6 \\ -9 & -12 \end{bmatrix}$$

Matrix Multiplication

$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ is 2×3 Rows x Columns

Let $A = \begin{bmatrix} 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

1×2 $\quad$ 2×1

AB is defined $(2=2)$

The product is 1×1

$$AB = \begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} (1)(3) + (2)(4) \end{bmatrix} = \underset{1\times 1}{\begin{bmatrix} 11 \end{bmatrix}}$$

Row times column gives a real number.

$A = \begin{bmatrix} 1 & 2 \\ 5 & 6 \end{bmatrix}$ $\qquad \overset{A}{2 \times 2}$ ✓ $\overset{B}{2 \times 1}$

2×1 result.

$$AB = \begin{bmatrix} 1 & 2 \\ 5 & 6 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} (1)(3) + (2)(4) \\ (5)(3) + (6)(4) \end{bmatrix} = \begin{bmatrix} 11 \\ 39 \end{bmatrix} = C$$

$$= \begin{bmatrix} (1,1)\text{-entry} \\ (2,1)\text{-entry} \end{bmatrix} = \begin{bmatrix} c_{11} \\ c_{21} \end{bmatrix} = \begin{bmatrix} \text{From Row 1 times column 1} \\ \text{From Row 2 times column 1} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 5 & -6 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$$

2×3 $\quad$ 2×2

Nope.

$$\begin{bmatrix} 1 & 5 \\ 2 & -6 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \\ c_{31} & c_{32} \end{bmatrix}$$

3×2 ✓ 2×2 $\quad$ 3×2

$$= \begin{bmatrix} (1)(1) + (5)(2) & (1)(-1) + (5)(4) \\ (2)(1) + (-6)(2) & (2)(-1) + (-6)(4) \\ (3)(1) + (2)(2) & (3)(-1) + (2)(4) \end{bmatrix} = \begin{bmatrix} 11 & 19 \\ -10 & -26 \\ 7 & 5 \end{bmatrix}$$

3×2

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \end{bmatrix}$$ is the same as

2×2 $\quad$ 2×1

$$\begin{bmatrix} 1x + 2y \\ 3x + 4y \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \end{bmatrix},$$ which means $$\begin{aligned} x + 2y &= 7 \\ 3x + 4y &= 8 \end{aligned}$$

This is the connection between systems and matrix multiplication.

Apr 11-9:38 AM

$$\boxed{\{(1,-1,1)\}}$$

$$\begin{aligned} 3x - y \quad &= 4 \\ x + y - z &= -1 \\ x \quad + 2z &= 3 \end{aligned}$$

means $\begin{bmatrix} 3 & -1 & 0 \\ 1 & 1 & -1 \\ 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix}$

Matrix multiplication to check our answer.

$\{(1,-1,1)\}$ means $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

To see if it works, multiply:

$$\begin{bmatrix} 3 & -1 & 0 \\ 1 & 1 & -1 \\ 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} (3)(1) + (-1)(-1) + (0)(1) \\ (1)(1) + (1)(-1) + (-1)(1) \\ (1)(1) + (0)(-1) + (2)(1) \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix} \checkmark$$