

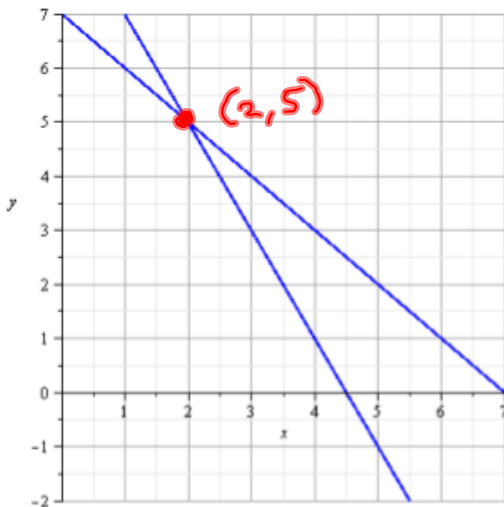
Equivalent Systems:

A modest justification of the Addition Method (Otherwise known as *Gaussian Elimination*).

Multiplying an equation by a constant does not change the solution set of a system.

Adding two equations together does not change the solution set of a system.

We illustrate this with an example, with a graph of each of the equivalent systems.



With the addition method, we're trying to *get* to

$$x = 2$$

$$y = 5$$

as the final system. But we typically start with a system like this one, where the answer isn't so obvious (and we can't always graph one very quickly).

$$2x + 2y = 14$$

$$2x + y = 9$$

$$\begin{array}{r} -E1 \\ -2x - 2y = -14 \end{array}$$

$$\begin{array}{r} E2 \\ 2x + y = 9 \end{array}$$

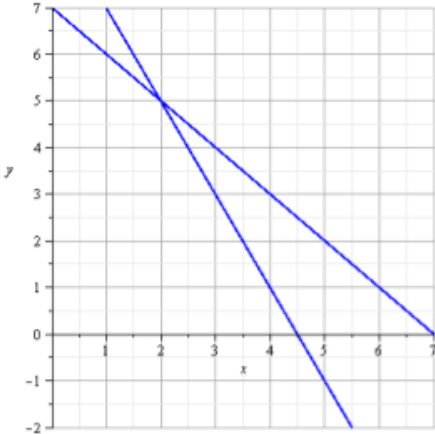
$$\begin{array}{r} -E1+E2 \\ -y = -5 \end{array}$$

$$y = 5$$

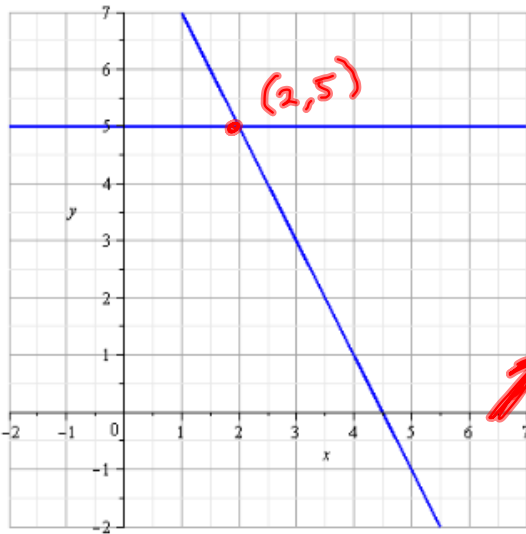
New System

$$2x + 2y = 14$$

$$y = 5$$



$$2x + y = 9$$
$$2x + 2y = 14$$



$$\begin{aligned} 2x + y &= 9 \\ y &= 5 \end{aligned}$$

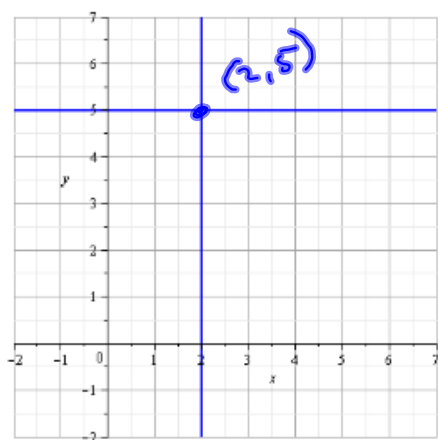
New System

$$\begin{aligned} 2x + 2y &= 14 \\ y &= 5 \end{aligned}$$

$$\begin{array}{r} E1 \quad 2x + 2y = 14 \\ -2E2 \quad \quad -2y = -10 \\ \hline E1 - 2E2 \quad 2x = 4 \\ \quad \quad \quad x = 2 \end{array}$$

New system:

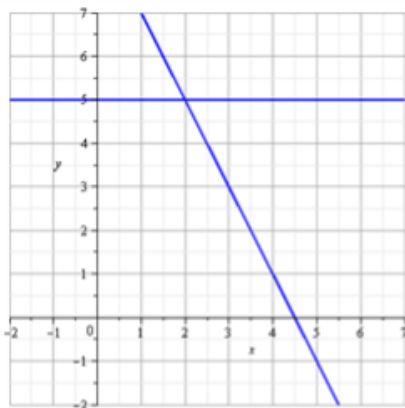
$$\begin{aligned} x &= 2 \\ y &= 5 \end{aligned}$$



$$x = 2$$
$$y = 5$$

New system:

$$x = 2$$
$$y = 5$$



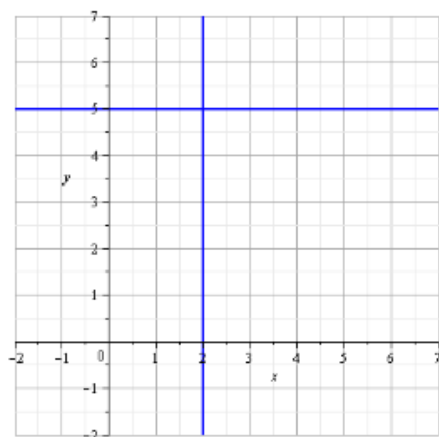
$$\begin{aligned} 2x + y &= 9 \\ y &= 5 \end{aligned}$$

Back-tracking a step, to the next-to-last system, I want to point out that you can CONTINUE to eliminate (and the eliminations are easier)

With the $y = 5$ in hand, we quickly found the $x = 2$ by back-substituting $y = 5$ into the equation $2x + y = 9$ and solving for x .

But there's another way, that completes the reversal of the process we followed before. We can subtract (add -1 times) the 2nd equation from the 1st equation to get rid of the y in the first equation.

This is called Gauss-Jordan reduction. Students wishing to use this method are free to do so. It's the procedure you'll do a million times if you ever take a Linear Algebra course.



$$\begin{aligned} x &= 2 \\ y &= 5 \end{aligned}$$

Summary:

Two "addition methods" at our disposal:

1. Gaussian Elimination - Back-SUBSTITUTE to find the remaining variables, after you've arrived at a TRIANGULAR system.
2. Gauss-Jordan reduction - Elimination worked so good, you back-ELIMINATE your way to a DIAGONAL system, from which you can read the solutions directly.

Word to the wise: Tortoise wins the race. The less of a hurry you're in and the more you SHOW what you did, the fewer mistakes you will make and the more partial credit I can award (because I can follow what you did!).

The more systematic you are about these (and the fewer shortcuts you pursue), the less likely it is that you will go in endless circles when we jack this up to 3 variables.