§5.1

$x + y = 1$
$2x - 3y = -8$

Is $(-1, 2)$ a sol'n?
(This is how we check)

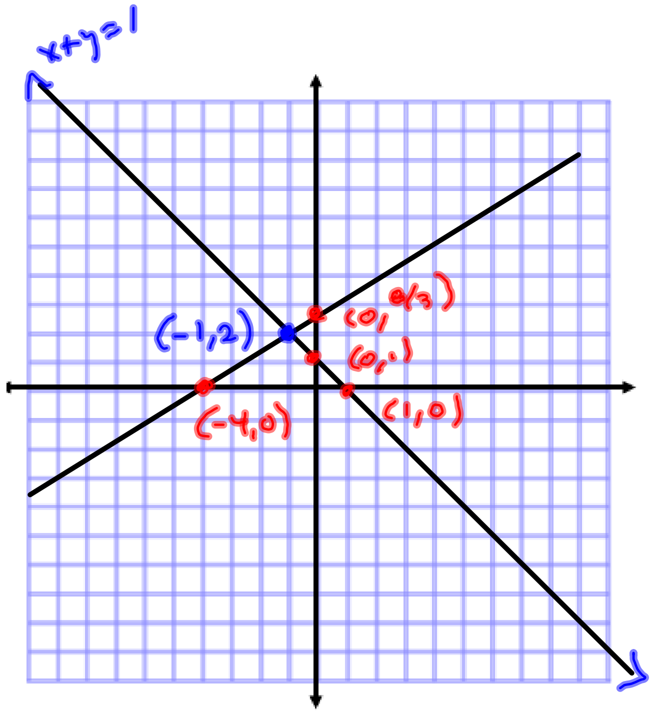
$-1 + 2 = 1$ ✓
$2(-1) - 3(2) = -2 - 6 = -8$ ✓

3 things can happen

Inconsistent No Sol'n
Dependent Infinitely many solutions
Independent Unique Sol'n.

what if you solution doesn't check?
It means you made a mistake, NOT that there's no solution.

Graphical Solutions – Not very precise, but very good for ballpark take.




$x+y=1$

| x | y |
|---|---|
| 0 | 1 |
| 1 | 0 |

$2x-3y=-8$

| x | y |
|---|---|
| 0 | $8/3 = 2\frac{2}{3}$ |
| -4 | 0 |
| 1 | |

Substitution Method

Solve for one variable. Stick it in the other equation(s).

$x + y = 1 \implies y = 1 - x$

$2x - 3y = -8 \implies 2x - 3(1-x) = -8$

Independent

$2x - 3 + 3x = -8$

$5x - 3 = -8$

$5x = -5$

$\boxed{x = -1}$

$\implies y = 1 - (-1)$

$\boxed{y = 2}$

$(x, y) = (-1, 2)$

Solution Set

$(x, y) \in \{(-1, 2)\}$

A set containing an ordered pair.

$y - 3x = 5 \Longrightarrow y = 3x + 5$

$3(x+1) = y - 2$ These are the same line!

$3x + 3 = y - 2$

$3x - y = -5 \Longrightarrow 3x - (3x+5) = -5$

$3x - 3x - 5 = -5$

$-5 = -5$

$0 = 0$

Infinite # of sol'ns.

Dependent System

$\{(x,y) \mid y = 3x + 5\}$

Standard reporting: Let y be free! Define x in terms of y. FOR 3x3's.

$y = 3x + 5$

$3x = 5 - y$

$x = \frac{5-y}{3}$

$\{(x,y) \mid x = \frac{5-y}{3},\ y = \text{any real \#}\}$

Addition Principle (Method)

$$A = B \implies$$

$$A + C = B + C$$

The Method:

1

$$x + y = 1$$

$$2x - 3y = -8$$

$$\begin{array}{lr} -2E1 & -2x - 2y = -2 \\ E2 & 2x - 3y = -8 \\ \hline & -5y = -10 \\ & y = 2 \end{array}$$

New System:

2

$$x + y = 1$$

$$\boxed{y = 2}$$

Back-Substitute:

$$x + 2 = 1$$

$$\boxed{x = -1}$$

$$\to (x, y) \in \{(-1, 2)\}$$

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## §5.2 3- and more-variable systems.

Here, there are more ways to have dependent and inconsistent systems.

E1 $x + y - z = 0$

E2 $3x - y + 2z = 14$

E3 $2x - y + 3z = 18$

1x in top left is good. Use it to eliminate x in the other 2 equations.

$$
\begin{array}{lrrrl}
-3E1 & -3x & -3y & +3z & = 0 \\
E2 & 3x & -y & +2z & = 14 \\
\hline
 & & -4y & +5z & = 14
\end{array}
$$

$$
\begin{array}{lrrrl}
-2E1 & -2x & -2y & +2z & = 0 \\
E3 & 2x & -y & +3z & = 18 \\
\hline
 & & -3y & +5z & = 18
\end{array}
$$

New System:

$$
\begin{aligned}
x + y - z &= 0 \\
-4y + 5z &= 14 \\
-3y + 5z &= 18
\end{aligned}
$$

Tempting to just eliminate the z's, since their stars are aligned.

New System:

E1 $x + y - z = 0$

E2 $-4y + 5z = 14$

E3 $-3y + 5z = 18$

The mindless left-to-right method 1st

$-3E2 \quad 12y - 15z = -42$

$4E3 \quad -12y + 20z = 72$

$5z = 30$

$z = 6$

New System:

$x + y - z = 0$

$-4y + 5z = 14$

$\boxed{z = 6}$

Is triangular.

$-4y + 5(6) = 14$

$-4y + 30 = 14$

$-4y = -16$

$\boxed{y = 4}$

$x + 4 - 6 = 0$

$x - 2 = 0$

$\boxed{x = 2}$

$(x, y, z) \in \{(2, 4, 6)\}$

Not as obvious as eliminating the z's, but systematic:

Left to right

Top to bottom

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New System:

E1 $x + y - z = 0$

E2 $-4y + 5z = 14$

E3 $-3y + 5z = 18$

The 5z's are aligned.

$$\begin{array}{lr} -\text{E2} & 4y - 5z = -14 \\ \text{E3} & -3y + 5z = 18 \\ \hline & y = 4 \end{array}$$

New System

$$\begin{aligned} x + y - z &= 0 \\ -4y + 5z &= 14 \\ y &= 4 \end{aligned}$$

Back-substitution works, as before, to give

$(x,y,z) \in \{(2,4,6)\}$

Sometimes, it's tough to use this non-systematic way on big systems.

Apr 4-9:31 AM

# A Quick Glimpse of Matrix Methods.

$$\begin{array}{c|c} & \begin{array}{ccc} x & y & z \end{array} \\ \begin{array}{c} R1 \\ R2 \\ R3 \end{array} & \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 3 & -1 & 2 & 14 \\ 2 & -1 & 3 & 18 \end{array}\right] \end{array}$$

= Augmented coefficient matrix corresponding to the system.

$$\begin{array}{c} R1 \\ -3R1+R2 \\ -2R1+R3 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & -4 & 5 & 14 \\ 0 & -3 & 5 & 18 \end{array}\right]$$

New System:

E1 $x + y - z = 0$

E2 $-4y + 5z = 14$

E3 $-3y + 5z = 18$

FROM Before, just to compare.

| | | | | |
|---|---|---|---|---|
| $-3R2$ | 0 | 12 | −15 | −42 |
| $4R3$ | 0 | −12 | 20 | 72 |
| | 0 | 0 | 5 | 30 |
| Divide by 5 ~ | 0 | 0 | 1 | 6 |

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & -4 & 5 & 14 \\ 0 & 0 & 1 & 6 \end{array}\right]$$

TRIANGULAR

Back substitute to the solution.

A collection of quarters, dimes
is worth \$13.25.
80 coins in all.

How many of each coin?

Variables:

Let $x$ = the number of dimes
$y$ = " " " quarters.

They're worth \$13.25

$$\boxed{.1x + .25y = 13.25}$$

$$\left(\frac{\$.10}{\cancel{\text{dime}}}\right)\left(\cancel{\text{dimes}}\right) = \$ = \$$$

80 coins in all:

$$\boxed{x + y = 80}$$

coins = coins

Apr 4-9:47 AM

One day a store sold 20 sweatshirts. White ones cost $ 9.95 and yellow ones cost $11.50. In all, $223.80 worth of sweatshirts were sold. How many of each color were sold?

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How many white sweatshirts were sold?

Let x = the # of yellow sweatshirts.
y = " " " white " .

$11.50x + 9.95y = 223.80$

$\left(\frac{\$}{\cancel{\text{sweat}}}\right)\left(\frac{\#\ \text{of}}{\cancel{\text{sweat}}}\right)$

$x + y = 20$

Matrix:

$$\left[\begin{array}{cc|c} 11.5 & 9.95 & 223.80 \\ 1 & 1 & 20 \end{array}\right]$$

OR

$$\left[\begin{array}{cc|c} 1 & 1 & 20 \\ 11.5 & 9.95 & 223.80 \end{array}\right]$$