$$2^x = \left(\frac{1}{5}\right)^{2x-5} = 5^{5-2x}$$

$$\boxed{\ln(2)x = \ln(5)(5-2x)}$$

$$x\ln(2) = 5\ln(5) - 2x\ln(5)$$

$$x\ln(2) + 2x\ln(5) = 5\ln(5)$$

$$(\ln(2) + 2\ln(5))x = 5\ln(5)$$

$$x = \frac{5\ln(5)}{(\ln(2) + 2\ln(5))}$$

$$\left.\begin{aligned} \text{Let } a &= \ln(2) \\ b &= \ln(5) \end{aligned}\right\} \text{A timely substitution}$$

$$ax = b(5-2x)$$

$$ax = 5b - 2bx$$

$$ax + 2bx = 5b$$

$$(a+2b)x = 5b$$

$$x = \frac{5b}{a+2b}$$

$$= \frac{5\ln(5)}{\ln(2) + 2\ln(5)}$$

Last time, we solved the equation, below, in class (Nice job, Graham.). I wanted to share how I built the equation, but by doing so, perhaps some of you were distracted by the graph of the function on the right hand side, which I created while I thought you all would be working on the equation. The graph, itself is worthy of study, and we'll give it a look.

$$2^x = \left(\frac{1}{5}\right)^{2x-5} = 5^{5-2x}$$

7

Mar 28-7:40 AM

Vince asks us this question: $A(t) = Ae^{-kt}$

If 70% of a radioactive substance remains after 400 million years, then what part remains active after 500 million years? What is the half-life?

$A(400) = .7A = Ae^{-400k}$

To me, it's all about the radioactive decay model, and interpreting the sentences in the word problem in terms of the model. When we worked in-class on a similar problem, Melissa was the first one to put this part into words:

1. 70% of a radioactive substance remains after 400 million years. This sentence will give us an equation that can be solved for the decay rate, *k*. That will complete the model for us.

2. What part remains after 500 million years? This question can only be answered after we find the decay rate, from the above. We're just evaluating the function at $t = 500$ million yrs.

3. What is the half-life? THIS is the equation that I think Melissa saw the other day, but it's the same *idea* as the first part.

① $Ae^{-400k} = .7A$

$e^{-400k} = .7$

$\ln(e^{-400k}) = \ln(.7)$

$-400k = \ln(.7)$

$k = \frac{\ln(.7)}{-400} \approx .000891687$

② $A(500) = Ae^{-500k} = xA$

Solve for x

$\boxed{e^{-500k} = x}$

$x = e^{-500(\ln(.7)/-400)}$

$\approx .6402838535$

```
e^(-500*ln(.7)*-
400)
               0
e^(-500*ln(.7)/-
400)
     .6402838535
```

```
400)
               0
e^(-500*ln(.7)/-
400)
     .6402838535
ln(.7)/-400
  8.916873598E-4
```

$.0008.916873598$

③ $Ae^{-kt} = \frac{1}{2}A$

$\frac{1}{2}$-life setup

K is known.

etc.

log property test questions

Write as a single log.

$3\log(x) + \frac{1}{4}\log(y) - 3\log(z)$

$= \log(x^3) + \log(y^{\frac{1}{4}}) - \log(z^3)$

$= \log\left(\frac{x^3 y^{1/4}}{z^3}\right)$

$\frac{a^b}{a^c} = a^{b-c}$

$\log\left(\frac{A}{B}\right) =$
$\log A - \log B$

Mar 28-9:08 AM

Write as the sum/difference of logs.

$$\log\left(\frac{\sqrt[3]{x}\;y^2}{z^5}\right) = \log\left(\frac{x^{\frac{1}{3}}y^2}{z^5}\right)$$

$$= \log(x^{\frac{1}{3}}) + \log(y^2) - \log(z^5)$$

$$= \frac{1}{3}\log(x) + 2\log(y) - 5\log(z)$$

Mar 28-9:13 AM

$$f(ax+b) = \boxed{f\left(a\left(x+\frac{b}{a}\right)\right)}$$

a controls horizontal:
stretches $x \to \frac{1}{a}x$
flips (if a is negative)

$\frac{b}{a}$ controls horizontal shift $x \to x - \frac{b}{a}$

Graph $g(x) = 2^{3x-6} = 2^{3(x-2)} = f(\underline{3(x-2)})$

$f(\underline{x}) = 2^x$

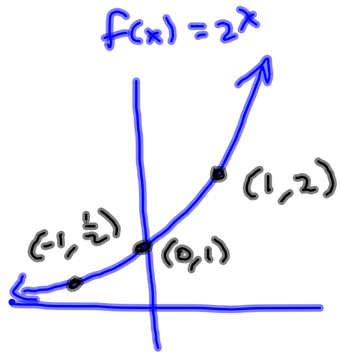
$x \longrightarrow 3x \longrightarrow 3(x-2)$

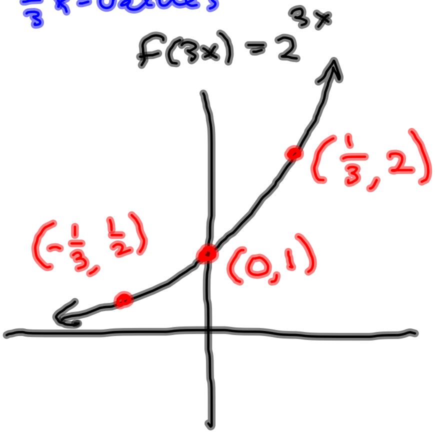
$2^x \longrightarrow 2^{3x} \longrightarrow 2^{3(x-2)}$

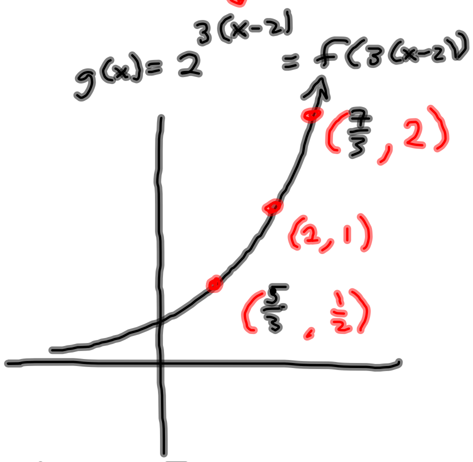
$f(x) \longrightarrow f(3x) \longrightarrow f(3(x-2))$

$\frac{1}{3}$ x-values

Add 2 to x-values (Right 2)







$-\frac{1}{3}+2 = \frac{5}{3}$

$\frac{1}{3}+2 = \frac{7}{3}$

Mar 28-9:16 AM

$g(x) = 2^{-3x+12} = 2^{-3(x-4)} = f(-3(x-4))$

$-\frac{1}{3}$ x-values

Right 4
x-values + 4

①	②	
$(-1, \frac{1}{2}) \rightarrow$	$(\frac{1}{3}, \frac{1}{2})$	$\rightarrow (\frac{1}{3}+4, \frac{1}{2}) = (\frac{13}{3}, \frac{1}{2})$
$(0, 1) \rightarrow$	$(0, 1)$	$\rightarrow (0+4, 1) = (4, 0)$
$(1, 2) \rightarrow$	$(-\frac{1}{3}, 2)$	$\rightarrow (-\frac{1}{3}+4, 2) = (\frac{11}{3}, 2)$
2^x	2^{-3x}	$2^{-3(x-2)}$

Horizontal Reflection.

$-5 \cdot 3^{4-7x} + 4$

① Basic Function: $f(x) = 3^x$

② Handle all vertical/horizontal stretches/shrinks and reflections.

③ Handle vertical/horizontal shifts.

$4-7x = -7x+4 = -7(x-\frac{4}{7})$

$-5 \cdot 3^{4-7x} + 4$

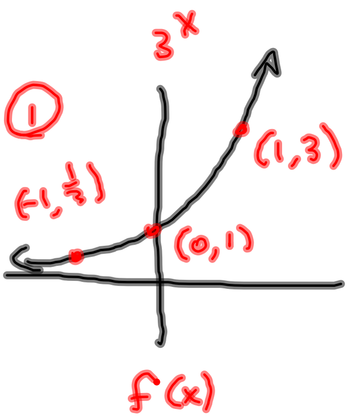
$x \longrightarrow -7x \longrightarrow -7(x-4/7)$

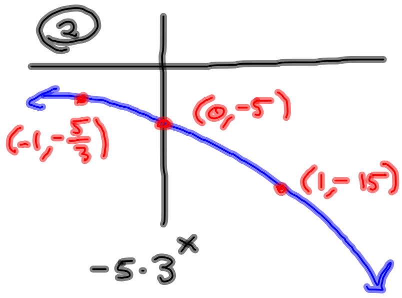
$f(x) = 3^x$

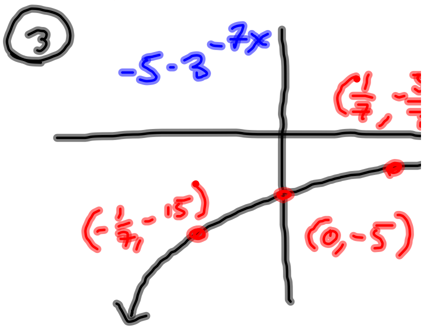
Plan: ② $3^x \longrightarrow -5 \cdot 3^x \longrightarrow -5 \cdot 3^{-7x}$

① $f(x) \longrightarrow -5f(x) \longrightarrow -5f(-7x)$

② $-5 *$ y-values $\longrightarrow -\frac{1}{7} *$ x-values.







$-\frac{1}{7} *$ x-values is a shrink & flip

the '7' The '−

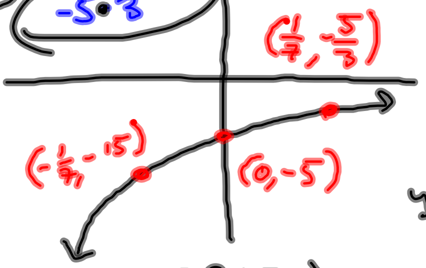
The finish line:
Handle the horizontal & vertical shifts.

We have $-5 \cdot 3^{-7x}$ done.

(4) $-5 \cdot 3^{-7x} + 4$ up 4

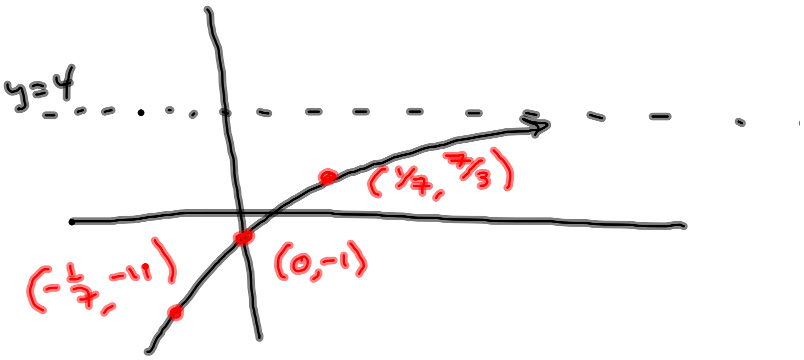
$-5 \cdot 3^{-7(x - 4/7)}$ Right $\frac{4}{7}$

(3) $-5 \cdot 3^{-7x}$



$-5f(-7x)$

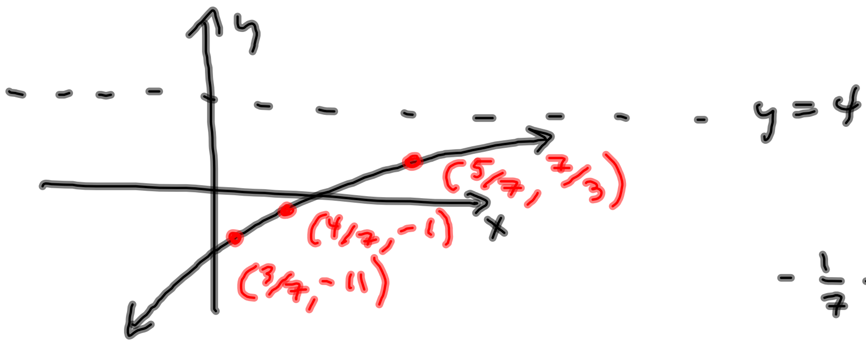
(4) $-5 \cdot 3^{-7x} + 4$
$= -5f(-7x) + 4$



$(\frac{1}{7}, -\frac{5}{3})$ becomes
$(\frac{1}{7}, -\frac{5}{3} + 4) = (\frac{1}{7}, \frac{7}{3})$

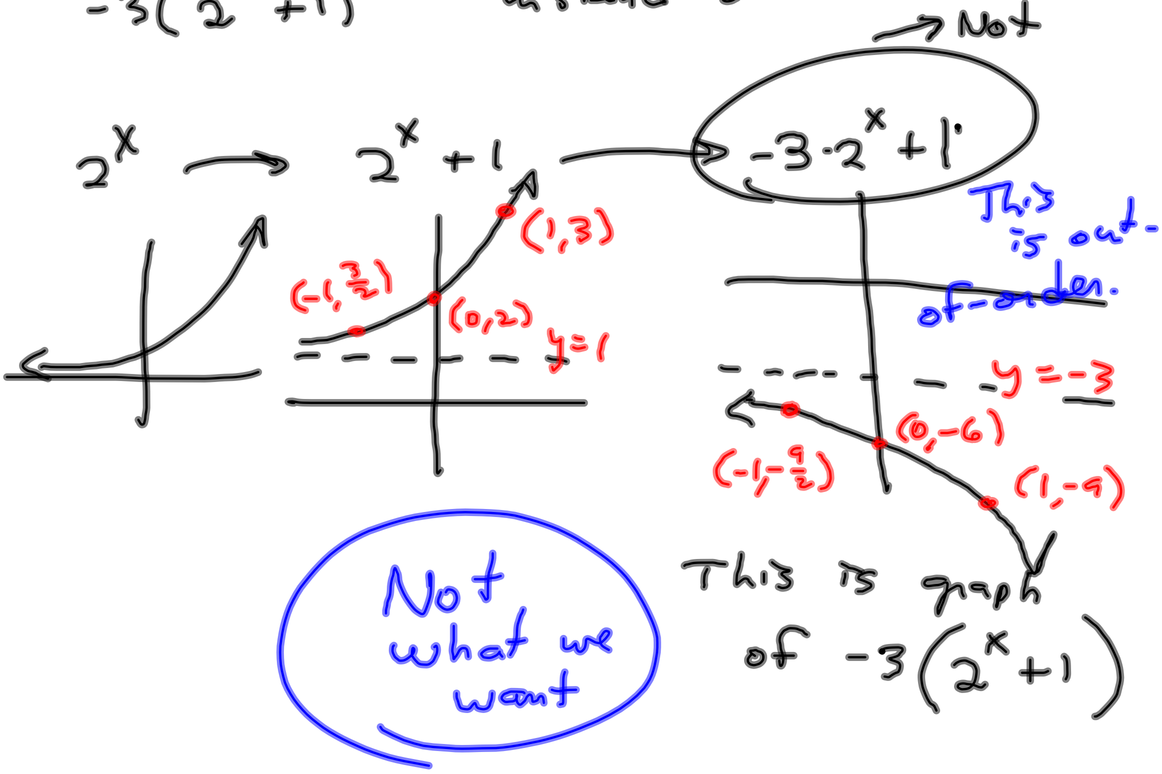
Finally,

$g(x) = -5 \cdot 3^{-7(x-4/7)} + 4 = -5f(-7(x-4/7)) + 4$



$-\frac{1}{7} + \frac{4}{7} = \frac{3}{7}$

Do Stretches/shrinks/Flips
B4 Rigid shifts.
If not, you'll get
$-3(2^x+1)$ instead of $-3\cdot 2^x+1$



This is graph of $-3(2^x+1)$