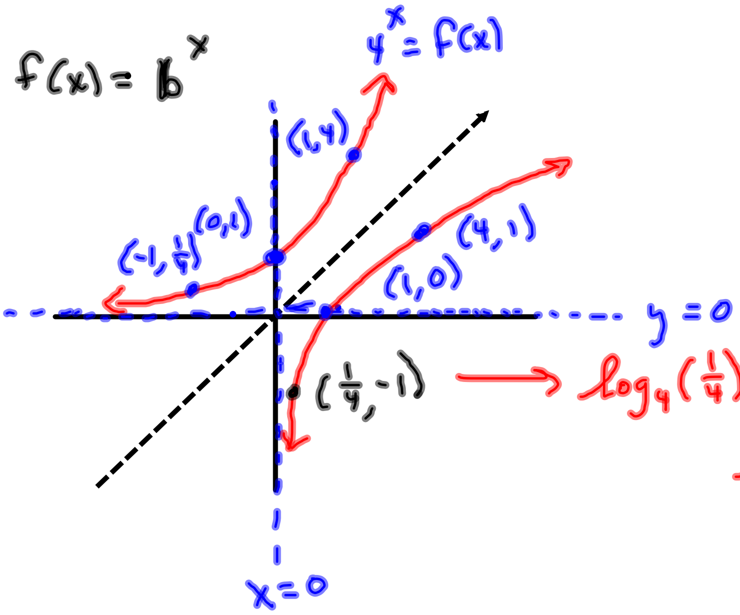
§ 4.2 $\log_b x$ is the inverse of

$f(x) = b^x$



So what's its graph look like?

$g(x) = \log_4 x$

$\longrightarrow$ $\log_4\left(\frac{1}{4}\right) = -1$, because

$\frac{1}{4} = 4^{-1}$

$\log_4(1) = 0$ b/c

$1 = 4^0$

$\lim_{x \to 0^+} \left(\log_4(x)\right) = -\infty$

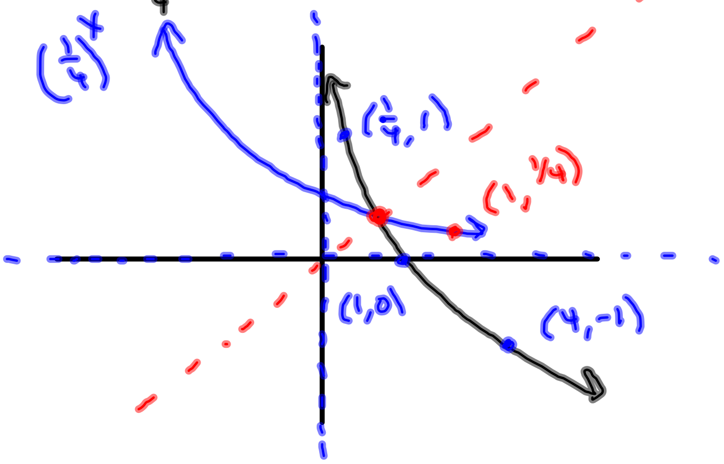
Increasing on its domain.

Concave Down

$\mathcal{D} = (0, \infty) =$ Range of $f(x) = 4^x$

$\mathcal{R} = (-\infty, \infty) =$ Domain of $f(x) = 4^x$

$\log_{\frac{1}{4}}(x) = g(x)$



x	y
$\frac{1}{4}$	1
1	0
4	-1

$\left(\frac{1}{4}\right)^x = \log_{\frac{1}{4}} x$

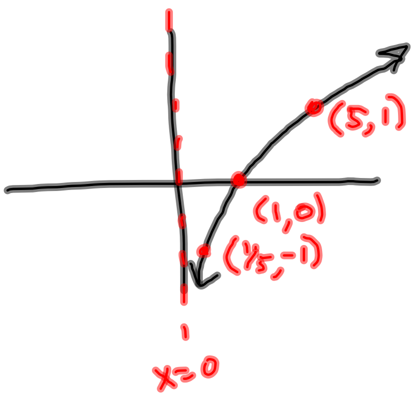
The solution is where they meet along the line $y = x$

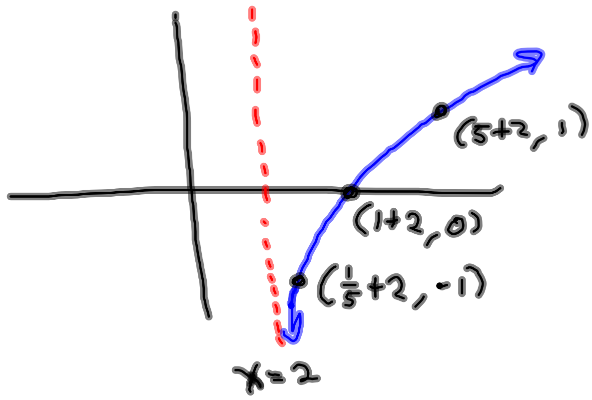
Graph $\log_5(x-2)$

$\boxed{f(x) = \log_5(x)}$

Then $\log_5(x-2) = f(x-2)$

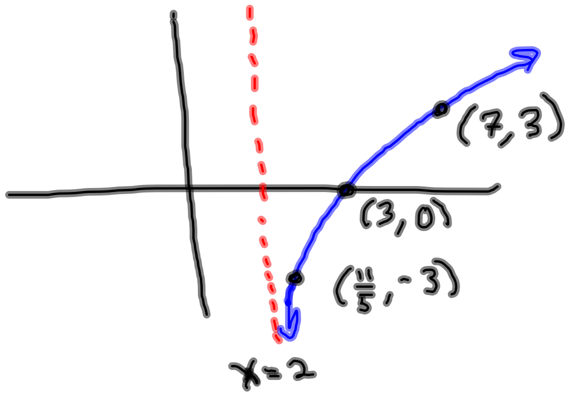
Right 2





$g(x) = 3\log_5(x-2)$ $\quad f(x) = \log_5 x$

$= 3f(x-2)$



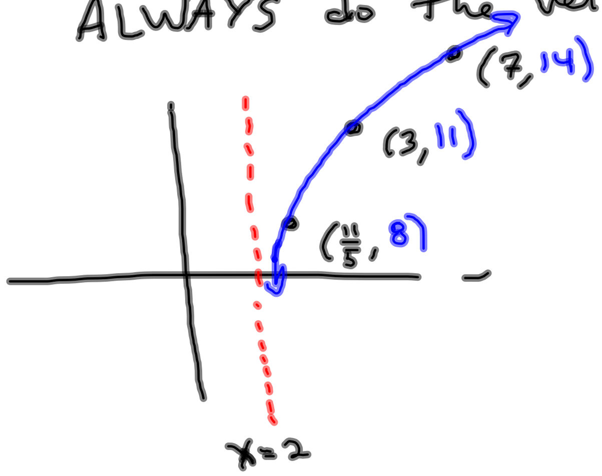
3 times y-values.

$\cancel{3\left(\log_5(x-2) + \frac{11}{3}\right)}$

$3\log_5(x-2) + 11$

Add 11 to previous y-values.

ALWAYS do the vertical shifts LAST.



3 times y-values.

$\log_4 16 = 2$ b/c $16 = 4^2$

Logarithmic Form

Exponential Form

I always remember that $\log_b x$ & b^x are inverse functions. This means

$4^{\log_4 x} = x$

$\log_4(4^x) = x$

$\log_4 x = 3$

$x = 4^3$

$4^{\log_4 x} = 4^3$

$x = 4^3$

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$e^{x-1} = 9$

$\log_e x = \ln x$

$\ln x$ & e^x are inverses.

$\ln(e^{x-1}) = \ln(9)$

$x-1 = \ln(9)$

$\boxed{x = \ln(9) + 1}$

$2^{x-1} = 9$

9 is an unfriendly base.

$\log_2(2^{x-1}) = \log_2(9)$

$x-1 = \log_2(9)$

$x = \log_2(9) + 1$

Logarithmic Form

$\ln(x-3) = \ln(2x-9)$

Exponential Form

$x-3 = 2x-9$

$e^{\ln(x-3)} = e^{\ln(2x-9)}$

Take e to the power of both sides.

$x-3 = 2x-9$

$x^3 x^6 = x^{3+6}$

$\underline{x^M x^N} = \underline{x^{M+N}}$

$\log((1000)(100)) = \log(10^5) = 5$

$= \log(1000) + \log(100)$

$= 3 + 2 = 5$

$\log_3(81) = \log_3(3^4) = 4$

$\log_3(81) = \log_3(3^4) = 4\underline{\log_3(3)} = 4$

$(a^b)^c = a^{bc} \longleftrightarrow \log_a(b^c) = c\log_a(b)$

$\frac{x^M}{x^N} = x^{M-N}$

$\log_a\left(\frac{M}{N}\right) =$

$\log_a(M) - \log_a(N)$

$\log_2(16) = 4$

$\log_2\left(\frac{64}{4}\right) = \log_2(64) - \log_2(4)$

$= 6 - 2 = 4$

Cool. It works.

$b^x = M$

$\log_c(b^x) = \log_c(M)$

$x \log_c(b) = \log_c(M)$

$$\boxed{x = \frac{\log_c(M)}{\log_c(b)} = \log_b(M)}$$

Change of base formula, when $\log_b(M)$ won't work in your calculator.

$b^x = M$

$\log_b(b^x) = \log_b(M)$

$x = \log_b(M)$

$$\log_3(7) = \frac{\log(7)}{\log(3)} = \frac{\ln(7)}{\ln(3)}$$

$$= \frac{\log_{500}(7)}{\log_{500}(3)}$$

1.945910
1.771244

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Solve

$$\cancel{300}e^{-.5x} = \frac{1}{2}\cdot\cancel{300}$$

$$e^{-.5x} = \frac{1}{2}$$

$$\ln(e^{-.5x}) = \ln\left(\frac{1}{2}\right)$$

$$-.5x = \ln\left(\frac{1}{2}\right)$$

$$x = \frac{\ln\left(\frac{1}{2}\right)}{-.5}$$

Found the $\frac{1}{2}$-life of a substance decaying at 50% annual rate.

1.386294

t = time in years.

$$\cancel{5000}\left(1+\frac{.04}{12}\right)^{12t} = 2\cdot\cancel{5000}$$

$$\left(1+\frac{.04}{12}\right)^{12t} = 2$$

$$\ln\left(\left(1+\frac{.04}{12}\right)^{12t}\right) = \ln(2)$$

$$12t\ln\left(1+\frac{.04}{12}\right) = \ln(2)$$

$$t = \frac{\ln(2)}{12\ln\left(1+\frac{.04}{12}\right)} \approx 17.36 \text{ years}$$

```
^(365*9)
     7166.505715
5000e^(.04*9)
     7166.647073
ln(2)/(12*ln((1+
.04/12)))
     17.35754463
```

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