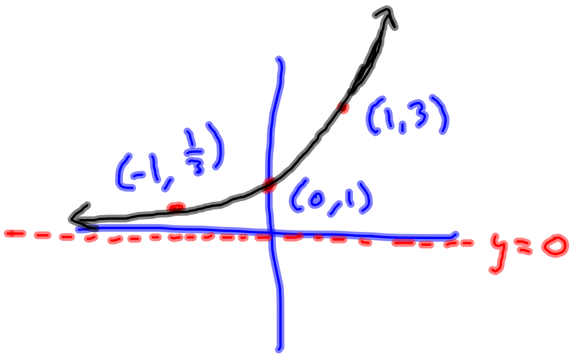
S 4.1 - Exponential Functions.

You need to know your properties of exponents and *all* your equation-solving skills, but especially *linear equations* and *quadratic equations*.

$f(x) = b^x$ b = base : $b > 0$, $b \neq 1$
x = eXponent

$f(x) = 3^x$

x	$f(x)$	
-1	$3^{-1} = \frac{1}{3}$	$\rightsquigarrow (-1, \frac{1}{3})$
0	$3^0 = 1$	$\rightsquigarrow (0, 1)$
1	$3^1 = 3$	$\rightsquigarrow (1, 3)$
-10	1.7E-5	



Smooth, concave up

1.7×10^{-5}

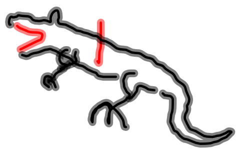
$0.00001.7 = .000017 > 0$ Always positive

$\mathcal{D} = (-\infty, \infty) = \mathbb{R}$

$\mathcal{R} = (0, \infty) = \{x \mid x > 0\}$

$= \{y \mid y > 0\}$ is more suggestive of range, since range relates to y-values or output.

$3 > 1$



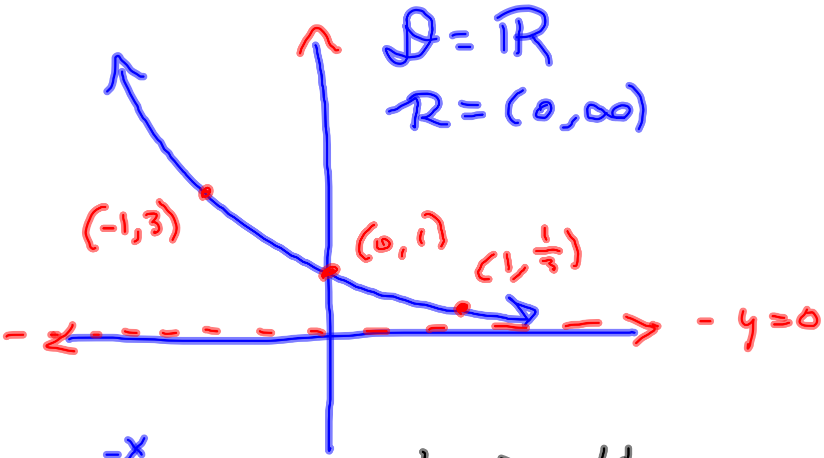
3^x is Exponential Growth.

$$f(x) = \left(\tfrac{1}{3}\right)^x$$

x	$f(x)$
-1	$\left(\frac{1}{3}\right)^{-1} = 3$
0	$\left(\frac{1}{3}\right)^{0} = 1$
1	$\left(\frac{1}{3}\right)^{1} = \frac{1}{3}$

$\mathcal{D} = \mathbb{R}$

$\mathcal{R} = (0, \infty)$



$$\left(\tfrac{1}{3}\right)^x = \left(3^{-1}\right)^x = 3^{-x}$$

$$\left(a^b\right)^c = a^{bc}$$

Its graph is the graph of $3^x = g(x)$ with x replaced by $-x$, i.e., $3^{-x} = g(-x)$

is another way to think of $f(x) = \left(\frac{1}{3}\right)^x = \left(3^{-1}\right)^x = 3^{-x}$

$A^x = A^y \Longrightarrow x = y$

$2^{3x+2} = 2^{5x} \Longrightarrow$

$3x+2 = 5x$

$-2x = -2$

$x = 1$

$8 = 2 \cdot 2 \cdot 2 = 2^3$

$\left(\frac{1}{2}\right)^x = 8 = 2^3$

$\left(2^{-1}\right)^x = 2^3$

$2^{-x} = 2^3$

$-x = 3$

$\boxed{x = -3}$

Either way:

$\left(\frac{1}{2}\right)^x = 2^3 = \left(\left(\frac{1}{2}\right)^{-1}\right)^3$

$\left(\frac{1}{2}\right)^x = \left(\frac{1}{2}\right)^{-3}$

$\boxed{x = -3}$

$$\left(\frac{4}{9}\right)^{x}\left(\frac{8}{27}\right)^{1-x} = \frac{2}{3}$$

$$\left(\frac{a}{b}\right)^{c} = \frac{a^c}{b^c}$$

$$\left(\frac{2^2}{3^2}\right)^{x}\left(\frac{2^3}{3^3}\right)^{1-x} = \frac{2}{3}$$

$$3(1-x) = 3-3x$$

$$\left(\left(\frac{2}{3}\right)^2\right)^{x}\left(\left(\frac{2}{3}\right)^3\right)^{1-x} = \frac{2}{3}$$

$$(a^b)(a^c) = a^{b+c}$$

$$2x + (3-3x)$$

$$= 2x+3-3x$$

$$= -x+3$$

$$\left(\frac{2}{3}\right)^{2x}\left(\frac{2}{3}\right)^{3-3x} = \frac{2}{3}$$

$$\left(\frac{2}{3}\right)^{-x+3} = \left(\frac{2}{3}\right)^{1}$$

$$-x+3 = 1$$

$$-x = -2$$

$$\boxed{x = 2}$$

Compound Interest.

Compounded Annually

$4\left(\frac{1}{100}\right)$ Principle

t = time (in years)

r = APR → Annual Percentage Rate $= 4\% = .04$

m = # of periods per year $= 1$

P = Principal = Initial Deposit. = \$5000

A = ~~Future Value~~ ← Save this term for Annuities.

= Accumulated Amount

After one year, we have

$$A = P + Pr = P(1+r)$$
$$= 5000 + 5000(.04) = 5000(1+.04) = 5200$$

After 2 years, we have

$$A = P(1+r) + P(1+r)r$$
$$= P(1+r)[1+r] = P(1+r)^2$$
$$= 5200 + 5200(.04) = 5000(1+.04)^2$$

```
              -406/15
5000*1.04
                 5200
5200+5200*.04
                 5408
5000(1.04)^2
                 5408
```

Rock asked me this one:

A deposit of \$5,000 earns 4% annual interest. Find the amount in the account at the end of 9 years and the amount of interest earned during the 9 years if the interest is compounded

(a) annually (b) quarterly (c) monthly, and (d) daily.

(a) At the end of 9 years, we have

$$A = P(1+r)^9$$
$$= 5000(1.04)^9 \approx \$7116.56$$

(b) The periods are different. There are 4 per year. What's the interest rate per quarter? $\frac{.04}{4} = \frac{r}{4} = \frac{r}{m} = .01 = i$

After 9 years, that amounts to

$4 \cdot 9 = mt = 36$

$$A = P\left(1+\frac{r}{m}\right)^{mt} = 5000\left(1+\frac{.04}{4}\right)^{4 \cdot 9}$$

The setup.

$\approx \$7153.84$

```
5000(1.04)^2
                5408
5000(1.04)^9
         7116.559062
5000*(1+.04/4)^(
4*9)
         7153.843918
```

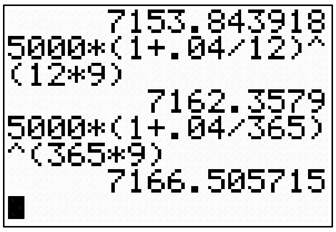
Rock asked me this one:

A deposit of \$5,000 earns 4% annual interest. Find the amount in the account at the end of 9 years and the amount of interest earned during the 9 years if the interest is compounded

(a) annually (b) quarterly (c) monthly, and (d) daily.

(c) $m=12, \; t=9, \; r=.04$

$A = P\left(1+\frac{r}{m}\right)^{mt} = 5000\left(1+\frac{.04}{12}\right)^{12\cdot 9} \approx \$7162.36$



(d) $5000\left(1+\frac{.04}{365}\right)^{365\cdot 9}$

$\approx \$7166.51$

Continuous Compounding $\quad m = \frac{r}{m} \cdot r$

$P(1+\frac{r}{\infty})^{\infty t}$ is the idea

$$\lim_{m\to\infty}\left(1+\frac{r}{m}\right)^{mt} = \lim_{m\to\infty}\left(1+\frac{r}{m}\right)^{\frac{r}{m}\cdot rt}$$

Euler

$$= \lim_{m\to\infty}\left(\left(1+\frac{r}{m}\right)^{\frac{r}{m}}\right)^{rt} = e^{rt}$$

"e" for Euler.

"e" is the NATURAL base.

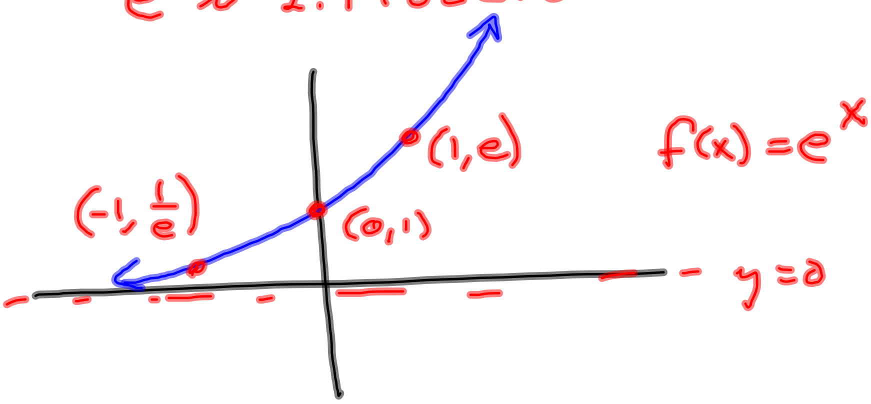
$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n} = e$ & "e" stands for Euler.

→ The formal definition of "e".

Continuous Compounding:

$A = Pe^{rt}$ is the bottom line

$e \approx 2.71828\underline{1}8$



(e) Compounded Continuously

$A = Pe^{rt} = 5000e^{(.04)(9)}$ ≈ \$7166.65

is close to "daily"

```
      7153.843918
5000*(1+.04/12)^
(12*9)
        7162.3579
5000*(1+.04/365)
^(365*9)
      7166.505715
█
```

```
(12*9)
        7162.3579
5000*(1+.04/365)
^(365*9)
      7166.505715
5000e^(.04*9)
      7166.647073
```