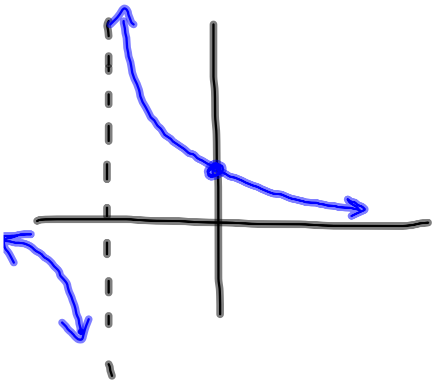
An easy online grapher to help you "see."

http://people.hofstra.edu/stefan_waner/realworld/functions/func.html

$\frac{1}{(x+11)^3}$, $\boxed{\frac{1}{x+11}}$, $\frac{1}{(x+11)^{371,522,201}}$

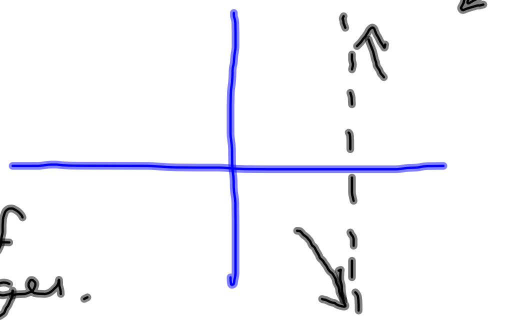


$x = -11$

$\frac{1}{(x+3)^2}$

Key for bigger problems is <u>behavior near the vertical asymptote</u>

$\frac{1}{x-7}$



For when $x-7$ is in the denominator of something bigger.

Take-Home Tests were handed out in class on Friday. They are due

WEDNESDAY, March 9th, *at the beginning of class*.

Sorry about the instructions about the "Testing Center" at the top of the page. That was a holdover from my ONLINE class.

H.A. Horizontal Asymptotes

$$\frac{x}{x^2-7} \begin{matrix} \leftarrow \text{deg } 1 \\ \leftarrow \text{deg } 2 \end{matrix}$$

x^2-7 eventually outstrips x
So $y=0$ is H.A.

$$\frac{x^3-2x+7}{x+5x^4-5} \qquad y=0$$

$5x^4$ dominates x^3

These are <u>PROPER</u>.

$$\frac{x^2-5x+4}{x^2+7x-11} \qquad y=1$$

Degree the same,
The big kids decide.

$$\frac{3x^3+11x^2+5}{-7x^3+5x-4} \qquad y=-\frac{3}{7}$$

$$\frac{21x+5x^4-3}{32x^4-3x^3} \qquad y=\frac{5}{32}$$

Mar 4-9:01 AM

Oblique Asymptotes (O.A.)

$$\frac{x^2-5x+11}{x-3}$$

Numerator is of higher degree than the denominator.

Asymptotes: V.A. $x=3$

O.A. Divide by $x-3$:

$$\begin{array}{r|rrr} 3 & 1 & -5 & 11 \\ & & 3 & -6 \\ \hline & 1 & -2 & 5 \\ & x' & c & r \end{array}$$

This says $\frac{x^2-5x+11}{x-3} = x-2+\frac{5}{x-3}$

(In 3.5, we said "$x^2-5x+11 = (x-3)(x-2)+5$")

What happens to $x-2+\frac{5}{x-3}$ as $x \to \pm\infty$?

$\frac{5}{x-3} \to 0$

So $\frac{x^2-5x+11}{x-3} \approx x-2$ as $x \to \pm\infty$

$y = x-2$ is Oblique Asymptote

$$\mathcal{D} = \{x \mid x \neq -1\}$$

$$R(x) = \frac{x^2-5x+6}{x+1} \qquad = (-\infty,-1)\cup(-1,\infty)$$

Fact: If $R(x)$ has an O.A., it won't have a H.A. (and vice-versa)

Simplify $R(x)$, if possible

$$R(x) = \frac{(x-2)(x-3)}{x+1}$$ No cancellations.

$R(x)$ is in Reduced form already.

$\boxed{\text{V.A.}: x=-1}$

$\boxed{x\text{-int}: (2,0), (3,0)}$

$\boxed{y\text{-int}: R(0) = \frac{6}{1} = 6 \rightsquigarrow (0,6)}$

$$R(x) = \frac{x^2-5x+6}{x+1}$$

$\boxed{\text{V.A.}: x=-1}$

$\boxed{x\text{-int}: (2,0), (3,0)}$

$\boxed{y\text{-int}: R(0) = \frac{6}{1} = 6 \rightsquigarrow (0,6)}$

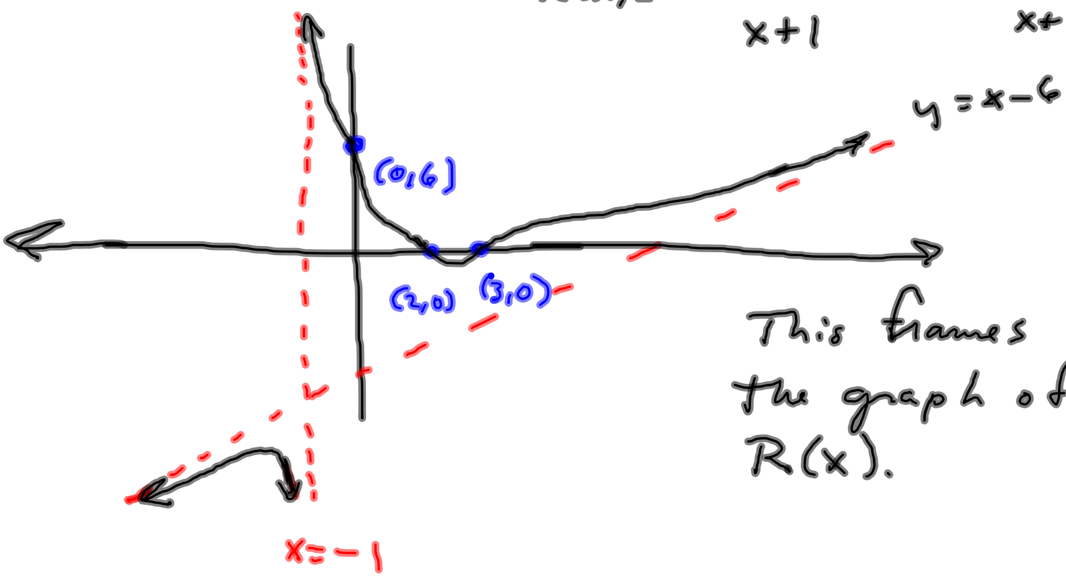
Now find O.A. by division:

$$\begin{array}{r|rrr} -1 & 1 & -5 & 6 \\ & & -1 & 6 \\ \hline & 1 & -6 & 12 \end{array}$$

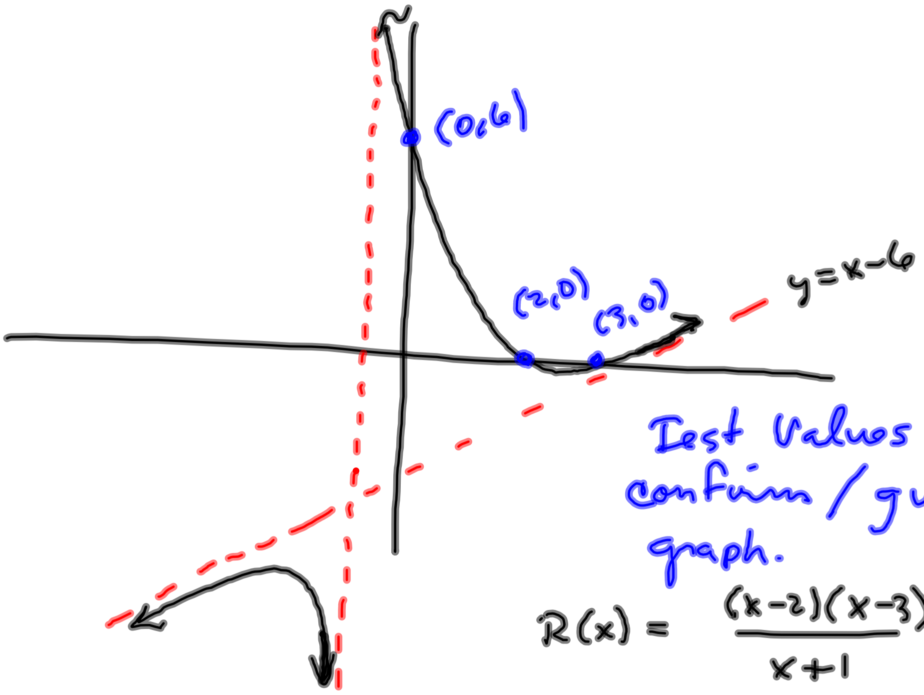
$\boxed{y = x-6 \text{ is O.A.}}$

cross or touch @ $x=2$?

$$R(x) = \frac{x^2-5x+6}{x+1} = \frac{(x-2)(x-3)}{x+1}$$



This frames the graph of $R(x)$.



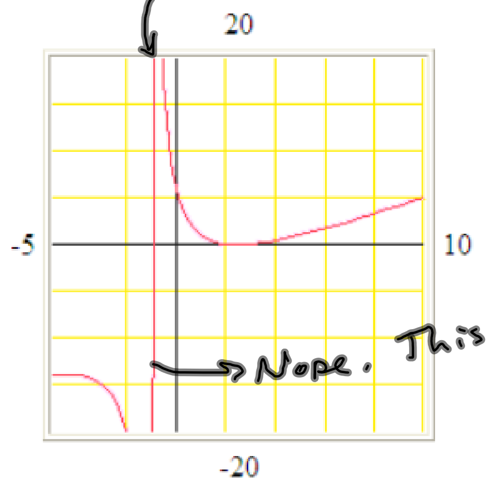
Test values can confirm/guide your graph.

$$R(x) = \frac{(x-2)(x-3)}{x+1}$$

$R(-2)$ for $(-\infty,-1)$
$R(0)$ for $(-1,2)$
$R(2.5)$ for $(2,3)$
$R(4)$ for $(3,\infty)$

To see if you're grokking this.

" Stranger in a Strange Land "



Nope. This is V.A.

$$R(x) = \frac{(x-2)(x-3)(x-4)}{(x+1)(x-4)}$$

$$R(x) = \frac{x^3 - 9x^2 + 26x - 24}{x^2-3x-4}$$ is typical starting point.

Your job is to graph it. If I didn't factor it for you, try Rational Zeros Theorem!

$\frac{p}{q} = \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$

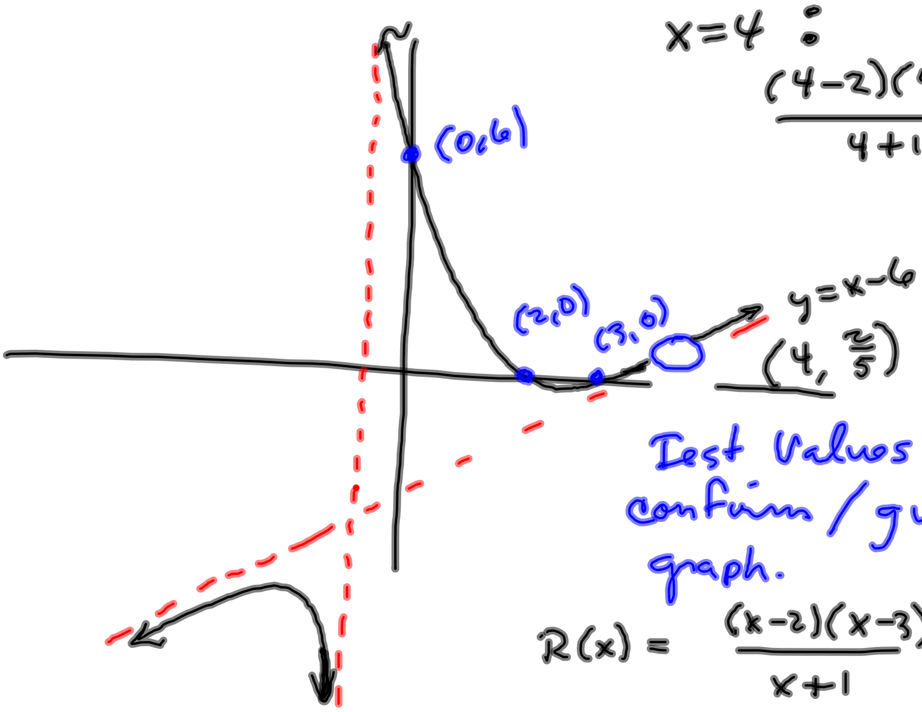
for rational zeros. To break down the numerator. To save time, use what we already know ...

$$R(x) = \frac{(x-2)(x-3)\cancel{(x-4)}}{(x+1)\cancel{(x-4)}} = \frac{(x-2)(x-3)}{x+1}$$ = the one we just did,

EXCEPT FOR $x \neq 4$. Because the $x-4$ cancels with an $x-4$ upstairs, this graph will look exactly like the previous, EXCEPT there will be a HOLE @

$x=4$:

$$\frac{(4-2)(4-3)}{4+1} = \frac{(2)(1)}{5}$$



$(4, \frac{2}{5})$ HOLE.

Test Values can confirm/guide your graph.

$$R(x) = \frac{(x-2)(x-3)}{x+1}$$

$$R(x) = \frac{2x^3-3x^2-2x+3}{x^3-4x^2+x+6} = \frac{(x-1)(2x-3)\cancel{(x+1)}}{\cancel{(x+1)}(x-2)(x-3)}$$

$D = \{x \mid x \neq -1, 2, 3\}$

$\boxed{\text{V.A.: } x=2,\ x=3}$

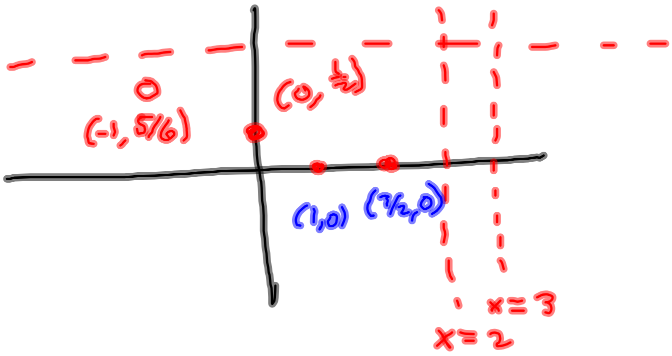
$\boxed{\text{HOLE: } x=-1}$ $\rightsquigarrow$ $\dfrac{(-1-1)(2(-1)-3)}{(-1-2)(-1-3)} = \dfrac{(-2)(-5)}{(-3)(-4)} = \dfrac{5}{6}$

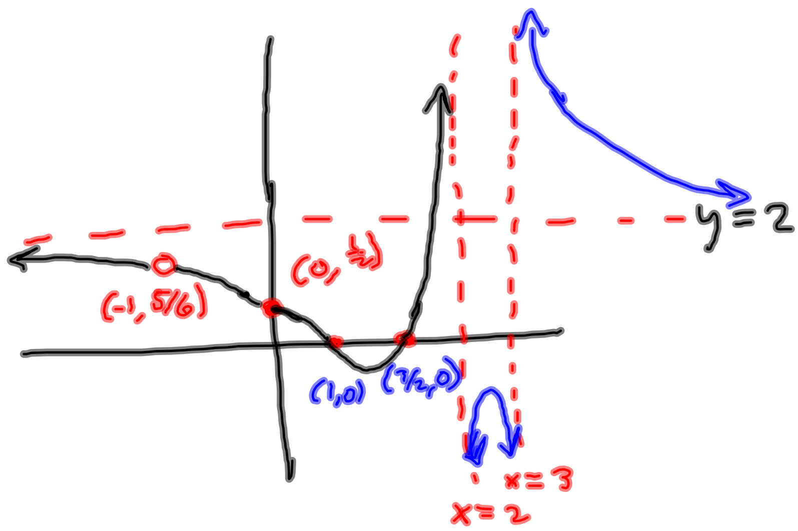
$\boxed{\left(-1, \frac{5}{6}\right)}$

$\boxed{\text{x-int: } (1,0),\ \left(\tfrac{3}{2}, 0\right)}$

$\boxed{\text{H.A.: } \dfrac{2x^3}{1x^2} = 2 = y}$

$\boxed{\text{y-int: } \left(0, \tfrac{1}{2}\right)}$





$$R(x) = \frac{(x-1)(2x-3)}{(x-2)(x-3)} \quad (x \neq -1)$$

Mar 4-10:04 AM