$f(x) = 2(x-1)^2(x+4)(x-5)^3$

$x = -4$, $m = 1$, cross

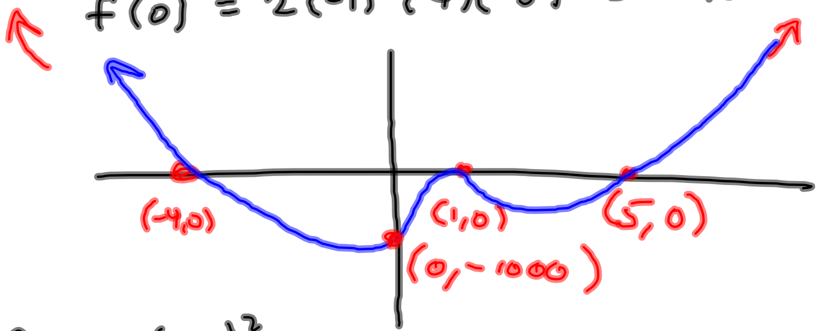
$x = 1$, $m = 2$, touches (Doesn't cross)

$x = 5$, $m = 3$, cross

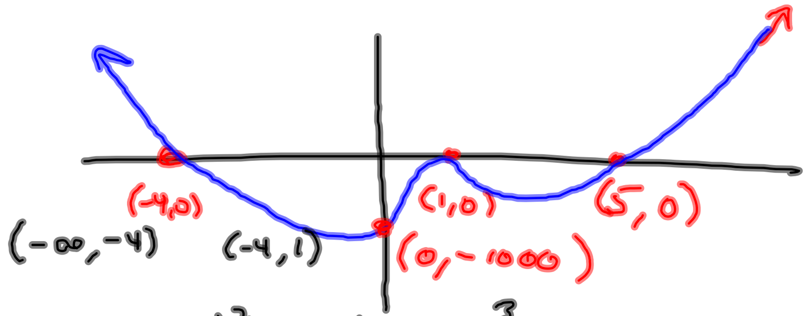
E.B. $2(x)^2(x)^1(x)^3 = 2x^6$

$f(0) = 2(-1)^2(4)(-5)^3 = -1000$

125
8



Solve $2(x-1)^2$



Solve $2(x-1)^2(x+4)(x-5)^3 > 0$

$+ \quad | \quad - \quad | \quad - \quad | \quad +$

$(-\infty,-4)$ -4 $(-4,1)$ 1 $(1,5)$ 5 $(5,\infty)$

$(-\infty,-4) \cup (5,\infty)$

$2(x-1)^2(x+4)(x-5)^3 < 0$

Same sign pattern, but now < 0

$(-4,1) \cup (1,5)$

$2(x-1)^2(x+4)(x-5)^3 \geq 0$

$(-\infty,-4] \cup \{1\} \cup [5,\infty)$

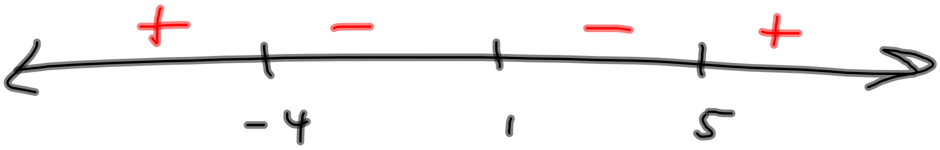
$2(x-1)^2(x+4)(x-5)^3 \leq 0$

$+ \quad | \quad - \quad | \quad - \quad | \quad +$

$(-\infty,-4)$ -4 $(-4,1)$ 1 $(1,5)$ 5 $(5,\infty)$

$[-4,5]$

$$\frac{(x-1)^2(x+4)}{(x-5)^3} \le 0 \qquad \text{E.B.} \quad \frac{(x)^2(x)}{x^3} = 1 > 0$$



$[-4, 5]$ but $x = 5 \notin \mathcal{D}$

So, $x \in [-4, 5)$

Find domain $\sqrt{\dfrac{(x-1)^2(x+4)}{(x-5)^3}}$

Need $\dfrac{(x-1)^2(x+4)}{(x-5)^3} \ge 0$

$(-\infty, -4] \cup (5, \infty)$

Mar 2-9:08 AM

Find the real zeros and factor over the Real Numbers.

$$f(x) = x^4 - 5x^3 + 15x^2 - 5x - 26$$

Descartes: 3 or 1 positive zeros

$$f(-x) = x^4 + 5x^3 + 15x^2 + 5x - 26$$

1 negative zero.

p: 26

q: 1

$$\frac{p}{q} = \pm 1, \pm 2, \pm 13, \pm 26$$

-1	1	5	15	5	-26
		-1	-4	-11	
	1	4	11	-6	No.

-2	1	5	15	5	-26
		-2	-6	-18	26
1	1	3	9	-13	0
		1	4	13	
	1	4	13	0	

$(x+2)\boxed{(x^3+3x^2+9x-13)}$ Depressed Function.

$(x+2)(x-1)(x^2+4x+13)$

$x^2+4x+13=0$ Depressed Equation

$a=1,\ b=4,\ c=13$

$$\begin{aligned} b^2-4ac &= 4^2-4(1)(13) \\ &= 16-52 \\ &= -36 < 0 \end{aligned}$$

No real root.

So $f(x) = (x+2)(x-1)\boxed{(x^2+4x+13)} \rightarrow$ Irreducible quadratic factor.

$x = -2, 1$ are the real zeros.

Find All zeros of $f(x)$ and split it into linear factors.

$f(x) = x^4 - 5x^3 + 15x^2 - 5x - 26$

$x^2+4x+13=0$ Depressed Equation

$a=1, b=4, c=13$

$b^2-4ac = 4^2-4(1)(13)$

$= 16-52$

$= -36 < 0$ No real root.

So $f(x) = (x+2)(x-1)\boxed{(x^2+4x+13)} \rightarrow$ Irreducible quadratic factor.

$x=-2, 1$ are the real zeros.

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-4 \pm \sqrt{-36}}{2(1)}$$

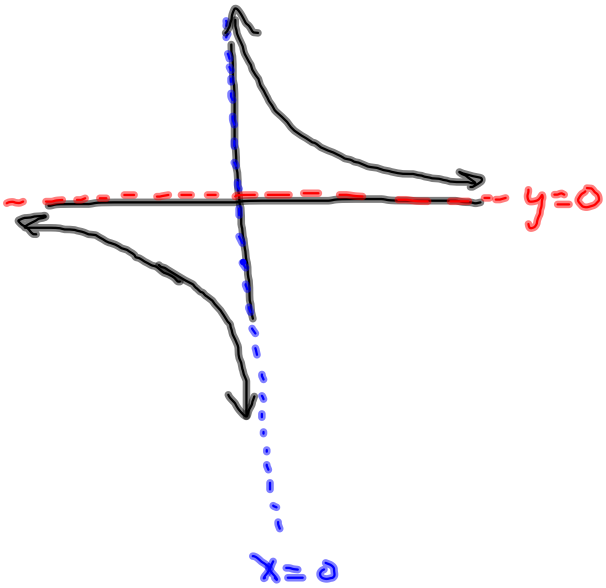
$$= \frac{-4 \pm 6i}{2} = \frac{\cancel{2}(-2 \pm 3i)}{\cancel{2}}$$

$$= -2 \pm 3i$$

Zeros: $-2, 1, -2 \pm 3i$

$f(x) = (x+2)(x-1)(x-(-2+3i))(x-(-2-3i))$

$f(x) = \frac{1}{x}$



x	$f(x)$
0	*
-2	$-\frac{1}{2}$
-1	-1
$-\frac{1}{2}$	-2
$-\frac{1}{10}$	-10
$-\frac{1}{100}$	-100
-100	$-\frac{1}{100}$

$f(-2) = \frac{1}{-2} = -\frac{1}{2}$

x-int:

$\frac{1}{x} = 0$

$x\left(\frac{1}{x}\right) = x \cdot 0$

$1 = 0$

Nevah!

$f(x) = \frac{1}{x}$

$f(-x) = \frac{1}{-x} = -\frac{1}{x}$

That's odd.

Same picture, roughly, for

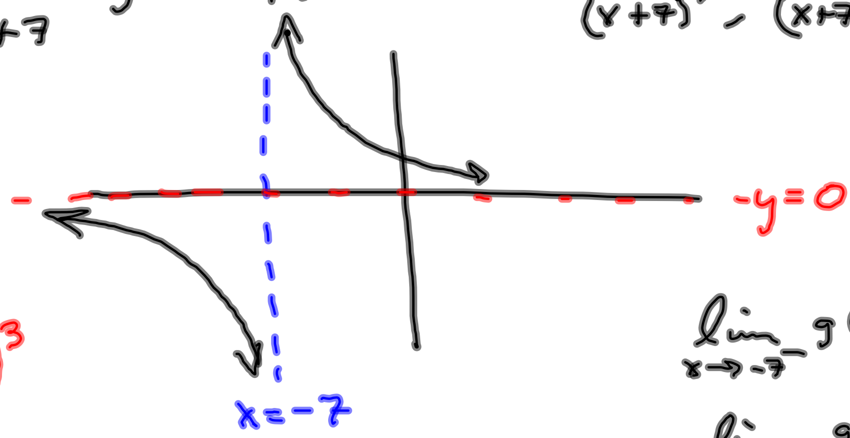
$f(x) = \frac{1}{x^3}, \frac{1}{x^5}, \frac{1}{x^7},$

$\frac{1}{x^{2n+1}}$

Mar 2-9:30 AM

$\frac{1}{x+7} = g(x) = f(x+7)$

$\frac{1}{(x+7)^3}, \frac{1}{(x+7)^5}, \frac{1}{(x+7)^{111}}$



$(\quad)^3$

$\lim_{x \to -7^-} g(x) = -\infty$

$\lim_{x \to -7^+} g(x) = +\infty$

$x=-7$ is vertical Asymptote

$y=0$ is $\underbrace{\text{horizontal Asymptote.}}$

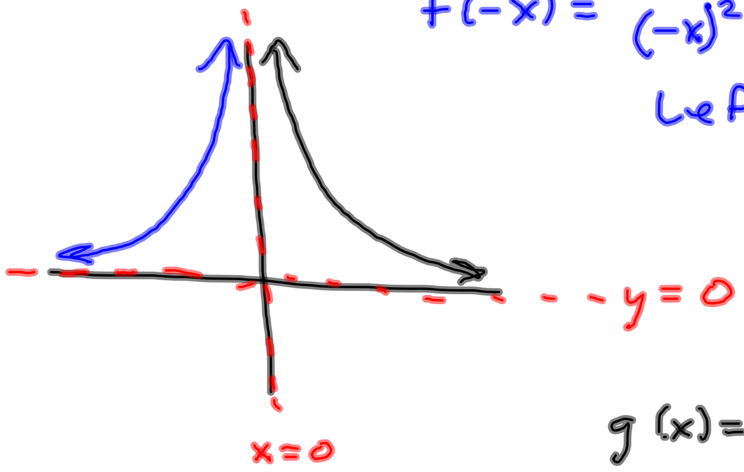
$\lim_{x \to \infty} g(x) = 0$

$\lim_{x \to -\infty} g(x) = 0$

$$\frac{1}{x^2} = f(x)$$

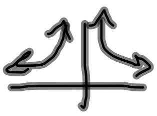
$$f(-x) = \frac{1}{(-x)^2} = \frac{1}{x^2} = f(x)$$

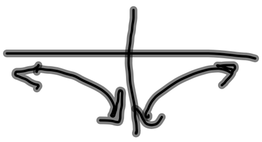
Left $\frac{1}{2}$ for free!

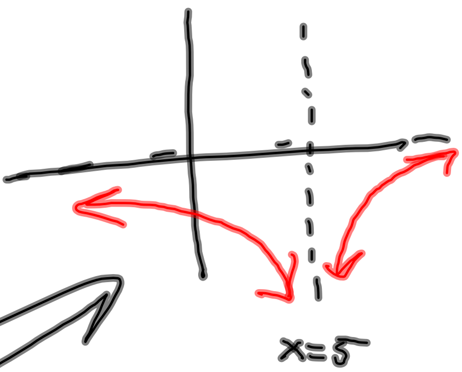


$$g(x) = -\frac{3}{(x-5)^2}$$

$$f(x) = \frac{1}{x^2} \longrightarrow \frac{-3}{x^2} \longrightarrow \frac{-3}{(x-5)^2}$$







$$\frac{-257}{(x-5)^{22}}$$

Graph

Graph $g(x) = \frac{2x+3}{x+1} = 2 + \frac{1}{x+1}$

One approach:

Divide:

$$\begin{array}{r|rr} -1 & 2 & 3 \\ & & -2 \\ \hline & 2 & 1 \\ & c & r \end{array}$$

So, $2x+3 = (x+1)(2) + 1$

OR $\frac{2x+3}{x+1} = 2 + \frac{1}{x+1}$

$f(x) = \frac{1}{x} \longrightarrow \frac{1}{x+1} \longrightarrow \frac{1}{x+1} + 2$