15 pts on Test 1?

No way! You guys must be lying to me!!!

E1 E2 E2P2 E2P3 P2+P3 WS

$$(\text{WS}\ \%)\left(\tfrac{1}{2}\right)(\underline{\text{Missed}}) + E2 = E2C$$

Steve needs to revisit Test 1 scores.

Steve found the following in his notes, from the day the Test 1's were handed back:

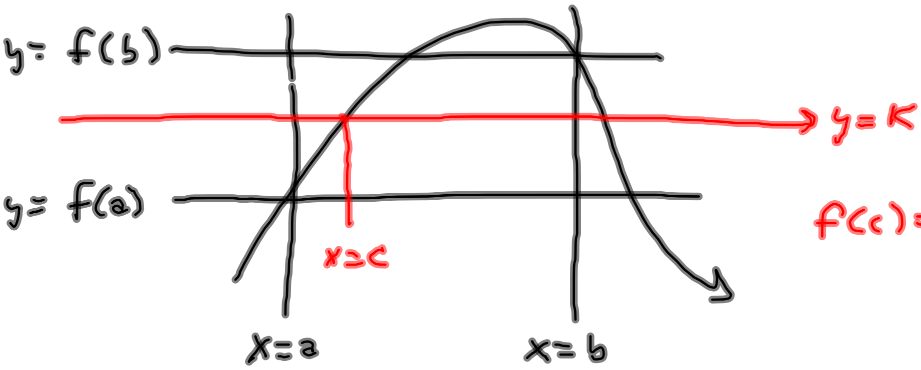
Test 1 Add 15 pts

I told you you guys were lying. And this proves... uh.... OK. Add 15 points to Test 1 scores. I'm making the change in the gradebook, now.

Continuity

If f is a polynomial, $f(a) \neq f(b)$ and $f(a) < K < f(b)$, then there is a $c \in (a, b)$ such that $f(c) = K$. Intermediate Value Theorem. IVT.

Continuous Function.



$f(c) = K$

$1 + 1.414 = 2.414$

2, 3

Show that $f(x) = x^2 - 2x - 1$ has a root in $(2,3)$, without solving $f(x) = 0$.

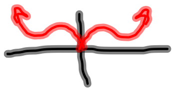
$f(2) = 2^2 - 2(2) - 1 = -1 < 0$

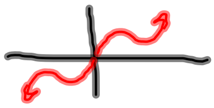
$f(3) = f(3) = 3^2 - 2(3) - 1 = 2 > 0$

$\therefore$ by IVT, $\underbrace{\text{there is a}}_{\exists}$ $c \in (2,3)$ $\underbrace{\text{such that}}_{\ni}$ $f(c) = 0$

$\therefore$, by IVT, $\exists\ c \in (2,3) \ni f(c) = 0$.

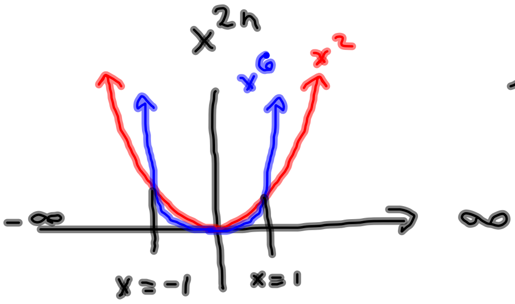
Symmetry of function graphs

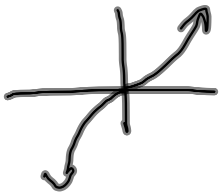
Even $f(-x) = f(x)$  Sym. w.r.t. y-axis

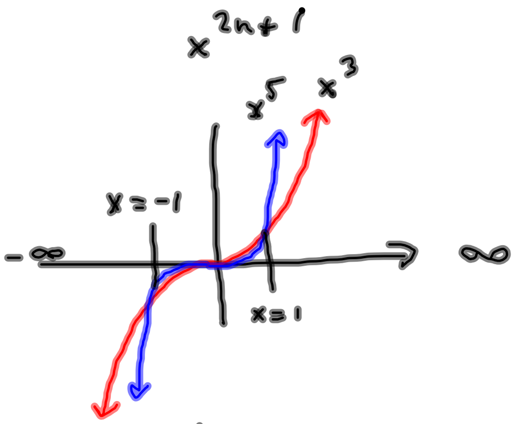
odd $f(-x) = -f(x)$  Sym. w.r.t. origin

Handy for graphing. Get one half of the picture for free.

Power functions







$$\left(\frac{1}{2}\right)^6 = \frac{1^6}{2^6} = \frac{1}{64}$$

$$\left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

End Behavior:

$$\lim_{x\to\infty} x^{2n} = \infty$$

$$\lim_{x\to-\infty} x^{2n} = \infty$$

End Behavior

$$\lim_{x\to\infty} x^{2n+1} = \infty$$

$$\lim_{x\to-\infty} x^{2n+1} = -\infty$$

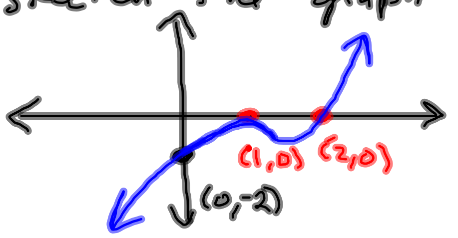
Behavior Near Zeros.

If $x=c$ is zero of multiplicity k, then $(x-c)^k$ is a factor of $f(x)$.

If k is even, f touches the x-axis at $x=c$

If k is odd, f crosses the x-axis at $x=c$

Sketch the graph of $f(x)=(x-2)^1(x-1)^2$



$f(0)=(-2)(-1)^2=-2$

End Behavior:

It's a 3rd degree

Trick for factored form

$(x)^1(x)^2=x^3$

x^3 dominates $f(x)$ as $x\longrightarrow\pm\infty$.

$f(x) = x^3+2x^2-6x-12$ factors by grouping

$= x^2(x+2)-6(x+2)$

$= (x+2)(x^2-6)$

$= (x+2)(x-\sqrt{6})(x+\sqrt{6})$

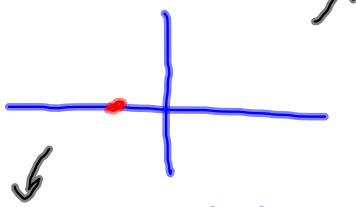
$x^2-6=0$

$x^2=6$

$x=\pm\sqrt{6}$

Graph:

End behavior: x^3

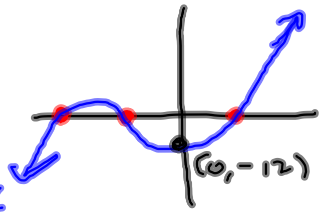


Zeros

$x=-\sqrt{6}$ cross

$x=-2$ cross

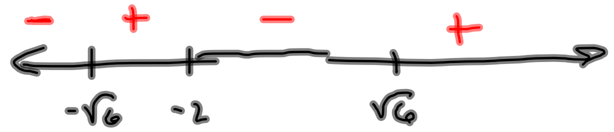
$x=\sqrt{6}$ cross



Used the end behavior as a starting point & then touch/cross criteria for the rest.

Check: y-intercept is (0,−12)

Test values can also help confirm.



$(x+2)(x^2-6)$

The zeros divide the x-axis into intervals.
The function does not change sign on each interval.

	Test
$(-\infty, -\sqrt{6})$	-3
$(-\sqrt{6}, -2)$	-2.1
$(-2, \sqrt{6})$	0
$(\sqrt{6}, \infty)$	3

$f(-3) = (-3+2)((-3)^2-6) = (-1)(9-6) = -3 < 0$

You can test each interval this way. But why bother.

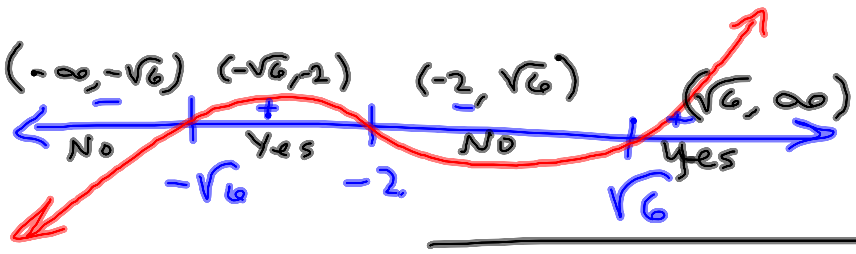
One sign & behavior near zeros & end behavior gives you a rough graph and a $\boxed{\text{sign pattern.}}$

Solve

$x^3 + 2x^2 - 6x - 12 > 0$

$f(x) = x^3 + 2x^2 - 6x - 12$
$= (x+2)(x-\sqrt{6})(x+\sqrt{6})$

Zeros: $x = -2, \pm\sqrt{6}$



want > 0, so $\boxed{x \in (-\sqrt{6}, -2) \cup (\sqrt{6}, \infty)}$

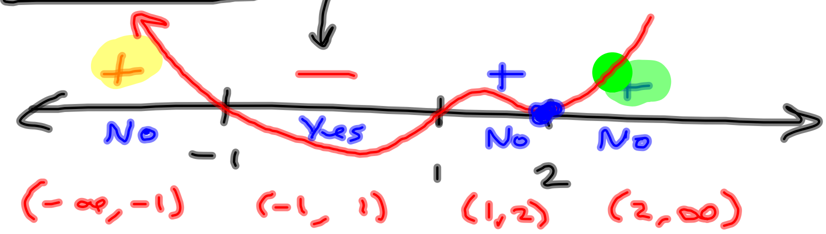
Most common student screwup

$x+2>0$	$x-\sqrt{6}>0$	$x+\sqrt{6}>0$
$x>-2$	$x>\sqrt{6}$	$x>-\sqrt{6}$

Feb 28-9:43 AM

Solve $(x-2)^2(x+1)^1(x-1)^1 \le 0$

$\underline{f(0)} = (-2)^2(1)(-1) = 4(-1) = -4 \ \underline{\le 0}$



Check E.B.: $(x)^2(x)(x) = x^4$ ↖ .... ↗

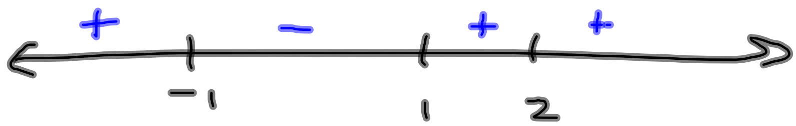
$x \in (-1, 1)$ No. $f(x) = 0$ is OK, for $\le$

$x \in [-1, 1]$ No $f(x) = 0$ @ $x = 2$, dummy, and "$=0$" is good.

$x \in [-1, 1] \cup \{2\}$

Solve $\dfrac{(x-2)^2(x-1)}{x+1} \le 0$

SAME EXACT SIGN Pattern as before



parenthesis

parentheses

$x \in [-1, 1] \cup \{2\}$ No.

$x \in (-1, 1] \cup \{2\}$ $-1 \notin \mathcal{D}$, Steve.