15 pts on Test 1? No way! You guys must be lying to me!!!

E1 E2 E2P2 E2P3 P2+P3 WS

$(\text{WS }\%)\left(\tfrac{1}{2}\right)(\text{Missed}) + E2 = E2C$

Steve needs to revisit Test 1 scores.

Steve found the following in his notes, from the day the Test 1's were handed back:

Test 1 Add 15 pts

I told you you guys were lying. And this proves... uh.... OK. Add 15 points to Test 1 scores. I'm making the change in the gradebook, now.

Feb 25-10:14 AM

$\sqrt{x^2} = |x|$

$\sqrt{x}^{\,2} = x$ and the assumption is $x \geq 0$.

$x - 7\sqrt{x} + 12 = 0$ is quadratic in form.

Let $u = \sqrt{x}$, Then

$u^2 = x$, and so

$u^2 - 7u + 12 = 0$

$u^2 - 4u - 3u + 12 = 0$

$u(u-4) - 3(u-4) = 0$

$(u-4)[u-3]$

Factors of $(1)(12) = 12$ whose sum is -7.

$\left.\begin{array}{l}(-4)(-3) = 12\\ -4-3 = -7\end{array}\right\}$ Sweet!

$u = 4$ or $u = 3$

$\sqrt{x} = 4$ or $\sqrt{x} = 3$

$(\sqrt{x})^2 = 4^2$ or $(\sqrt{x})^2 = 3^2$

$x = 16$ or $x = 9$

$x - 7\sqrt{x} + 12 = 0$

$16 - 7\sqrt{16} + 12 \stackrel{?}{=} 0$

$16 - 7(4) + 12 =$

$28 - 28 = 0$ ✓

$9 - 7\sqrt{9} + 12 =$

$= 9 - 7(3) + 12$

$= 21 - 21 = 0$ ✓

Feb 25-8:54 AM

$x^4 - 7x^2 + 12 = 0$

Let $u = x^2$. Then

$u^2 - 7u + 12 = 0 \Longrightarrow$

$u = 3$ or $u = 4$

$x^2 = 3$ or $x^2 = 4$

$\sqrt{x^2} = |x| = \sqrt{3}$ $\quad\vdots$

$x = \pm\sqrt{3}$ $\quad x = \pm 2$

<u>Check:</u>

$(\sqrt{3})^4 - 7(\sqrt{3})^2 + 12 =$

$9 - 7(3) + 12 = 0$ ✓

$(-\sqrt{3})^4 - 7(-\sqrt{3})^2 + 12 =$

$9 - 21 + 12$

$2^4 - 7(2)^2 + 12 =$

$16 - 28 + 12 = 0$ ✓

Same for -2

$$|2x^2 - x - 2| = 1$$

Absolute Values on next test(s), for sure.

$2x^2 - x - 2 = 1$ OR $2x^2 - x - 2 = -1$

$2x^2 - x - 3 = 0$ OR $2x^2 - x - 1 = 0 \Rightarrow$

Factors of -6 whose sum is -1

$\left.\begin{array}{l}(-3)(2) = -6 \\ -3 + 2 = -1\end{array}\right\} \checkmark$

$2x^2 - 3x + 2x - 3 =$

$x(2x-3) + 1(2x-3) =$

$(2x-3)(x+1) = 0$

$x = \frac{3}{2}$ OR $x = -1$

$(2x+1)(x-1) = 0 \Rightarrow$

$x = -\frac{1}{2}$ OR $x = 1$

$$\boxed{x \in \{-1, -\tfrac{1}{2}, 1, \tfrac{3}{2}\}}$$

Feb 25-9:18 AM

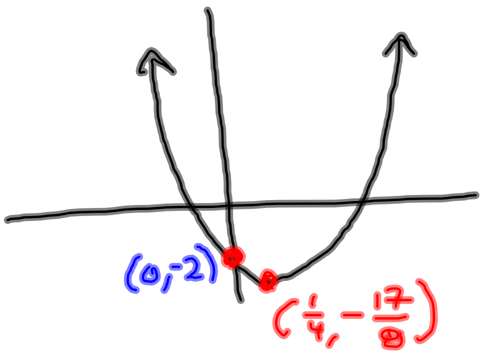
Graph of $2x^2 - x - 2$

$$-2 - \frac{1}{8} = \frac{-16-1}{8} = \frac{-17}{8}$$

$$f(x) = 2x^2 - x - 2$$

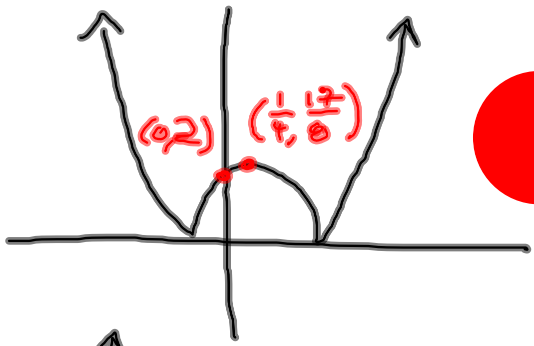
$$= 2\left(x^2 - \frac{1}{2}x + \left(\frac{1}{4}\right)^2\right) - 2 - 2\left(\frac{1}{16}\right)$$

$$= 2\left(x - \frac{1}{4}\right)^2 - \frac{17}{8} = f(x)$$



what about

$$|2x^2 - x - 2| = |f(x)|$$



Our question

$$|2x^2 - x - 2| = 1$$

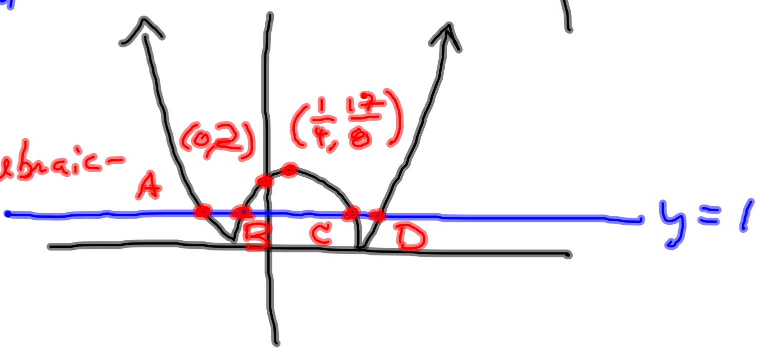
we found, algebraically, that

$A = (-1, 1)$

$B = (-\frac{1}{2}, 1)$

$C = (1, 1)$

$D = (\frac{3}{2}, 1)$



$$x \in \left\{-1, -\frac{1}{2}, 1, \frac{3}{2}\right\}$$

Feb 25-9:25 AM

$$\left(\frac{b-5}{6}\right)^2 - 6 = \frac{b-5}{6}$$

Let $u = \frac{b-5}{6}$

$u^2 - 6 = u$

$u^2 - u - 6 = 0$

$(u-3)(u+2) = 0$

$u = -2$ OR $u = 3$

$\frac{b-5}{6} = -2$ $\qquad$ $\frac{b-5}{6} = 3$

$b-5 = -12$ $\qquad$ $b-5 = 18$

$b = -7$ $\qquad$ $b = 23$

Check

$$\boxed{b \in \{-7, 23\}}$$

Feb 25-9:35 AM

$$\sqrt{x+1} = x-5$$

$$\left(\sqrt{x+1}\right)^2 = (x-5)^2$$

$$x+1 = x^2 - 10x + 5^2$$

$$x^2 - 10x + 25 = x + 1$$

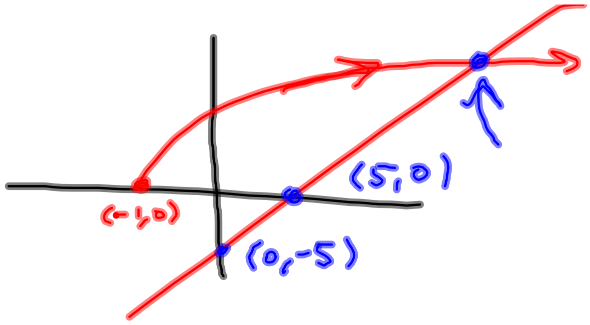
$$x^2 - 11x + 24 = 0$$

$$(x-8)(x-3) = 0$$

$x = 3$ or $\boxed{x = 8}$

$x = 3 \rightarrow$ No

$$\boxed{x \in \{8\}}$$



check:

$\sqrt{8+1} = 8-5$?

$\sqrt{9} = 3$ ✓

$\sqrt{3+1} = 3-5$

$\sqrt{4} = -2$ No!

Squaring both sides can introduce extraneous roots.

$\Longleftrightarrow$

$$A = B \Longrightarrow A^2 = B^2$$

But

$$A^2 = B^2 \not\Longrightarrow A = B$$

$(-3)^2 = 3^2$ but $-3 \neq 3$

$$\sqrt{x} + \sqrt{x-36} = 2$$

$\sqrt{a}\sqrt{b} = \sqrt{ab}$

if $a, b \geq 0$

$$\left(\sqrt{x} + \sqrt{x-36}\right)^2 = 2^2$$

$$\left(\sqrt{x}\right)^2 + 2\sqrt{x}\sqrt{x-36} + \left(\sqrt{x-36}\right)^2 = 4$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$x + 2\sqrt{x(x-36)} + x - 36 = 4$$

Isolate the radical

$$2x + 2\sqrt{x(x-36)} - 36 = 4$$

$$2\sqrt{x(x-36)} = -2x + 40$$

$$2^2\left(\sqrt{x(x-36)}\right)^2$$

$$\left(2\sqrt{x(x-36)}\right)^2 = (-2x+40)^2 = (2x-40)^2$$

$$4(x(x-36)) = 4x^2 - 2(2x)(40) + 40^2$$

$$4(x^2 - 36x) = 4x^2 - 160x + 1600$$

$$4x^2 - 144x = 4x^2 - 160x + 1600$$

$$+160x \qquad +160x$$

$$16x = 1600$$

$$x = 100$$

$$\sqrt{x} + \sqrt{x-36} = 2$$

$$\sqrt{100} + \sqrt{100-36} = 2\ ?$$

$$10 + \sqrt{64} = 2\ ?$$

$$10 + 8 = 2\ ?$$

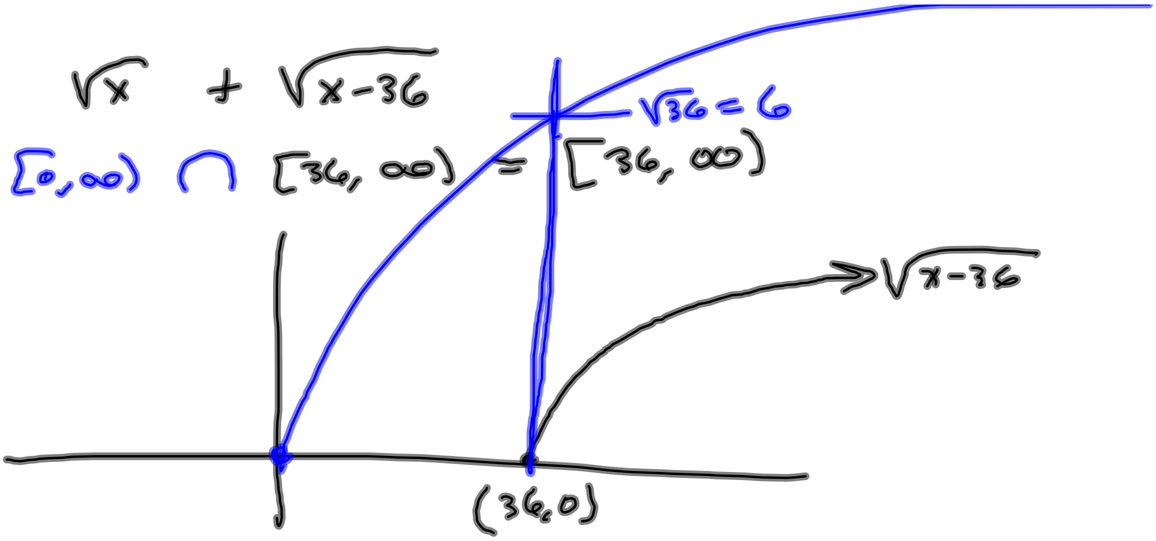
Nope.

Either I messed up, or there's no solution.

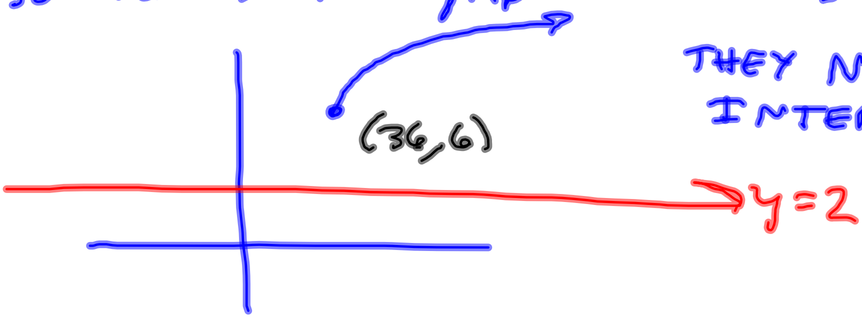
Extraneous roots, only.

$\sqrt{x} + \sqrt{x-36}$

$[0,\infty) \cap [36,\infty) = [36,\infty)$



So here's the graph of the sum:



THEY NEVER INTERSECT.