Theorems for Chapter 3:

Let $f(x)$ be a polynomial with real coefficients.

Remainder Theorem

To find $f(2)$, divide by $x-2$. $f(2) =$ Remainder

Factor Theorem

Remainder Theorem when Remainder is zero.

$f(2)=0 \Longrightarrow (x-2)$ is a factor.

Rational Zeros Theorem

$3x^5 + \ldots\ldots + 2$

$p: 2$
$q: 3$ $\Longrightarrow \frac{p}{q} = \pm 1, \pm\frac{1}{3}, \pm 2, \pm\frac{2}{3}$ are all possible rational zeros.

Descartes' Rule of Signs

$f(x) = 5x^4 \underset{1}{-} 3x^3 \underset{2}{+} 2x^2 \underset{3}{-} x - 5$ There are 3 or 1 positive zeros.

$f(-x) = 5x^4 + 3x^3 + 2x^2 + x \underset{1}{-} 5$ There is ONE negative zero.

Feb 23-7:59 AM

Find all zeros of

$u(x) = x^4 - 4x^3 + x^2 + 12x - 12$, and write as the product of linear factors.

Rat'l Zeros:

p: 12 $\quad \frac{p}{q} = \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

q: 1

Descartes': 3 or 1 positive zeros.

$u(-x) = x^4 + 4x^3 + x^2 - 12x - 12$ 1 negative zero

Guess! $x = 1$

$$\begin{array}{r|rrrrr} 1 & 1 & -4 & 1 & 12 & -12 \\ & & 1 & -3 & -2 & 10 \\ \hline & 1 & -3 & -2 & 10 & -2 \end{array}$$

Nope

$$\begin{array}{r|rrrrr} 2 & 1 & -4 & 1 & 12 & -12 \\ & & 2 & -4 & -6 & 12 \\ \hline & 1 & -2 & -3 & 6 & 0 \end{array}$$

Sweet!

$$\begin{array}{r|rrrr} 2 & 1 & -2 & -3 & 6 \\ & & 2 & 0 & -6 \\ \hline & 1 & 0 & -3 & 0 \end{array}$$

Sweet!

This says $u(x) = (x-2)(x^3 - 2x^2 - 3x + 6)$ &

$x^3 - 2x^2 - 3x + 6 = 0$ is the new, depressed equation.

$u(x) = (x-2)^2(x^2 - 3)$

Finish:

$x^2-3=0$

$x^2=3$

$\sqrt{x^2}=\sqrt{3}$

$|x|=\sqrt{3}$

$x=\pm\sqrt{3}$

So, $u(x)=(x-2)^2(x-\sqrt{3})(x-(-\sqrt{3}))$ is split into linear factors, & $x=2, \pm\sqrt{3}$ are the zeros.

$x=2$ has multiplicity 2.

$x^2-(\sqrt{3})^2=(x-\sqrt{3})(x+\sqrt{3})$

$a=1,\ b=0,\ c=-3$

$b^2-4ac=0^2-4(1)(-3)$

$=12$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\dfrac{\pm 2\sqrt{3}}{2(1)}=\pm\dfrac{2\sqrt{3}}{2}=$

$=\pm\sqrt{3}$

$\sqrt{12}$:

2 | 12

2 | 6

3

$\sqrt{12}=\sqrt{2\cdot 2\cdot 3}$

$=2\sqrt{3}$

$g(x) = x^3 + 4x^2 + 12x + 9$

$\frac{p}{q}$: $\pm 1, \pm 3, \pm 9$

Descartes: No positive zeros

$g(-x) = -x^3 + 4x^2 - 12x + 9$ 3 or 1 negative zeros

$\quad\quad\quad\quad\ 1 \quad\ \ 2 \quad\ \ 3$

$4(-x)^2 =$

$4((-1)(x))^2$

$= 4(-1)^2(x)^2$

$= 4x^2$

$$\begin{array}{r|rrrr} -1 & 1 & 4 & 12 & 9 \\ & & -1 & -3 & -9 \\ \hline & 1 & 3 & 9 & 0 \\ & x^2 & x^1 & c & r \end{array}$$

$(x+1)(x^2+3x+9)$

Sweet!

$x^2 + 3x + 9 = 0$

$a=1,\ b=3,\ c=9$

$b^2 - 4ac = 3^2 - 4(1)(9)$

$= 9 - 36$

$= -27$

$\sqrt{-27} = i\sqrt{27}$

$= i\sqrt{3 \cdot 3 \cdot 3}$

$= \boxed{3i\sqrt{3}}$

$$\begin{array}{r|l} 3 & 27 \\ 3 & 9 \\ & 3 \end{array}$$

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31

$x = \cancel{\dfrac{-3 \pm \sqrt{3^2 - 4(1)(9)}}{2(1)}}$ icky

$= \dfrac{-3 \pm 3i\sqrt{3}}{2}$

$x = -1,\ \dfrac{-3 \pm 3i\sqrt{3}}{2}$

Split $g(x)$ into linear factors.

$(x-(-1))\left(x - \left(\frac{-3+3i\sqrt{3}}{2}\right)\right)\left(x - \left(\frac{-3-3i\sqrt{3}}{2}\right)\right)$

Conjugate Pairs Theorem $a+bi \Longrightarrow a-bi$

If f(x) has REAL coefficients and $3+2i$ is a zero, then so is $3-2i$.

$(a-bi)(a+bi)$
$= a^2+b^2$

$(x-(3-2i))(x-(3+2i))$

$= x^2 - x(3+2i) - (3-2i)x + (3-2i)(3+2i)$

$= x^2 - 3x - 2ix - (3x - 2ix) + 9 + 6i - 6i - 4i^2$

$= x^2 - 3x - 2ix - 3x + 2ix + 9 + 6i - 6i + 4$

$= x^2 - 6x + 13$

write a polynomial of degree 3 with zeros at $x = 1, 2, 2-3i$.

$(x-1)(x-2)(x-(2-3i))$

write a polynomial of degree 3 with real coefficients, with zeros at $x=1,2,2-3i$

$(x-1)(x-2)(x-(2-3i))(x-(2+3i))$

CAN'T BE DONE by C.P.T.

Must be at least degree 4.

Upper & Lower Bounds on Zeros.

+ 0 + 0 + +

$$f(x) = (x-2)(x^4 + 5x^2 + 3x + 1) + 2$$

If $x > 2$, then $f(x) > 0$

In particular $f(x) \neq 0$

In the course of dividing, you see this:

$$\begin{array}{r|rrrrrr} 2 & & & & & & \\ \hline & 1 & 0 & 5 & 3 & 1 & 2 \end{array}$$

All positive or zero means $x=2$ is an upper bound on real zeros.
Don't have to try anything bigger.

Lower bounds on real zeros

Book way Dividing by $(x+2)$

$$\begin{array}{r|cccccccc} -2 & & & & & & & & \\ \hline & 1 & -2 & 0 & -3 & 1 & -1 & 0 & 0 \end{array}$$

Alternates in sign (or has a 0)

This says -2 is a lower bound on real zeros.

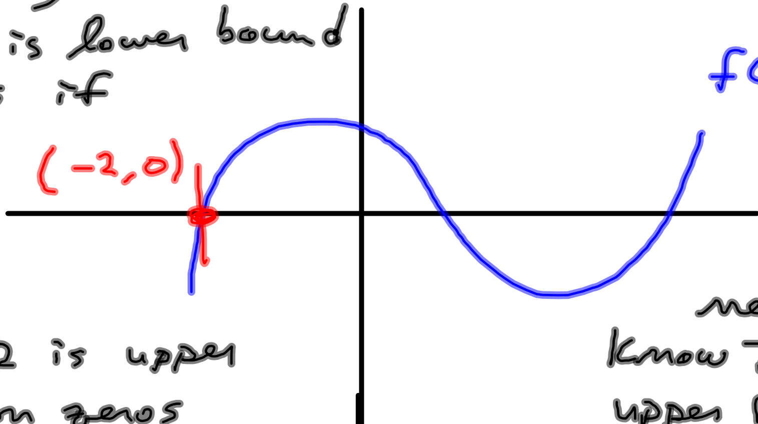
Fundamental Theorem of Algebra:

EVERY Polynomial of degree n has exactly n complex zeros, and can be split into a product of linear factors.

real numbers live inside complex #s.

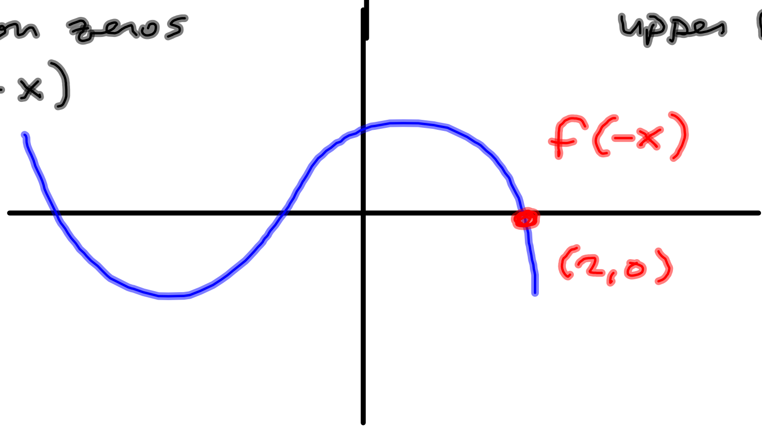
My way for lower bounds:

x = -2 is lower bound on zeros if



This way, you only need to know Theorem on upper Bounds

x = +2 is upper bound on zeros for f(-x)



Kalista's picture is better.