Identify the vertex, axis of symmetry, y-intercept, x-intercepts, and opening of the parabola, then sketch the graph.

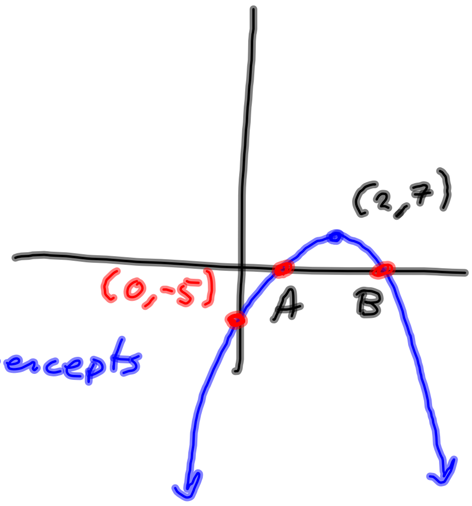
$y = -3x^2 + 12x - 5 \rightsquigarrow$ y-int $= (0,-5)$

$= -3(x^2 - 4x \quad) - 5$

$= -3(x^2 - 4x + 2^2 - 4) - 5$

$= -3(x-2)^2 - 3(-4) - 5$

$= -3(x-2)^2 + 7 \overset{\text{SET}}{=} 0$ for x-intercepts



$-3(x-2)^2 = -7$

$(x-2)^2 = \frac{7}{3}$

$|x-2| = \sqrt{\frac{7}{3}}$

$x - 2 = \pm\sqrt{\frac{7}{3}} = \pm \frac{\sqrt{7}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \pm \frac{\sqrt{21}}{3}$

$x = 2 \pm \frac{\sqrt{21}}{3} = \frac{2}{1} \cdot \frac{3}{3} \pm \frac{\sqrt{21}}{3} = \frac{6 \pm \sqrt{21}}{3}$

$$\boxed{\begin{aligned} \left(\frac{6-\sqrt{21}}{3}, 0\right) &= A \\ \left(\frac{6+\sqrt{21}}{3}, 0\right) &= B \end{aligned}}$$

Identify the vertex, axis of symmetry, y-intercept, x-intercepts, and opening of the parabola, then sketch the graph.

$y = -3x^2 + 12x - 5 \quad = f(x)$

$a = -3,\ b = 12,\ c = -5$

$\frac{-b}{2a} = -\frac{12}{2(-3)} = \frac{12}{6} = 2 = h$

vertex:
$(h, k) = (2, 7)$

$f(2) = -3(2)^2 + 12(2) - 5$
$= -12 + 24 - 5$
$= 7 = k$

$2\,\underline{|84}$
$2\,\underline{|42}$
$3\,\underline{|21}$
$\quad 7$

$b^2 - 4ac = 12^2 - 4(-3)(-5)$
$= 144 - 60$
$= 84$

$\sqrt{84} =$
$\sqrt{2 \cdot 2\, 3 \cdot 7} =$
$2\sqrt{21}$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-12 \pm 2\sqrt{21}}{2(-3)} = \frac{-12 \pm 2\sqrt{21}}{-6}$

$= \frac{2(-6 \pm 1\sqrt{21})}{-6} = \frac{-6 \pm \sqrt{21}}{-3}$

$= \frac{6 \mp \sqrt{21}}{3}$

$277 \div 3$

Quotient

$$\begin{array}{r} 92 \text{ r } 1 \\ 3\overline{)277} \\ -(270) \\ \hline 7 \\ -(6) \\ \hline 1 \end{array}$$

Remainder

Dividend

Divisor

$\frac{277}{3} = 92\frac{1}{3} = 92 + \frac{1}{3}$ is one way to view it

In 3.2, we'll look at it this way:

$\underline{277} = 3 \cdot 92 + 1$

Dividend = Divisor times quotient plus Remainder

Some books call this the "Division algorithm."

$$\frac{x^2+3x-3}{x-2}$$

Just look at the leading term.

$$
\begin{array}{r l}
 & x \quad +5 \quad \color{red}{r\ 7} \\
x-2 & \overline{)\, x^2+3x-3} \\
 & \color{red}{-(}\color{blue}{x^2-2x}\color{red}{)} \\
\hline
 & 5x-3 \\
 & \color{red}{-(}5x-10\color{red}{)} \\
\hline
 & 0+7
\end{array}
$$

$\frac{x^2}{x} = x$

$\frac{5x}{x} = 5$

$-3-(-10) = -3+10 = 7$

This says

$$x^2+3x-3 = (x-2)(x+5)+7$$

This says

$$x^2 + 3x - 3 = (x-2)(x+5) + 7$$

Let $f(x) = x^2 + 3x - 3$

Use division to find $f(2)$.

$f(2) = 7 =$ the remainder when $f(x)$ is divided by $x-2$

$f(x) = 3x^5 - 2x^4 + 5x^3 - 11x^2 + 4x - 7$

what's $f(2)$? Use division to find $f(2)$.

Use REMAINDER THEOREM to find $f(2)$

All the same question.

Synthetic Division $\qquad x+5 \ r\ 7$

Divide x^2+3x-3 by $x-2$

$$\begin{array}{r|rrr} +2 & 1 & 3 & -3 \\ & & 2 & 10 \\ \hline & 1 & 5 & 7 \\ & x^1 & x^0 & r \\ & & \text{constant} & \end{array}$$

This says

$x^2+3x-3 =$

$(x-2)(x+5)+7$

& $f(2)=7$

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$f(x) = 3x^5 - 2x^4 + 5x^3 - 11x^2 + 4x - 7$

what's $f(2)$? Use division to find $f(2)$.

Use REMAINDER THEOREM to find $f(2)$

Do $f(2)$ old-school I'll do it by division.

$$
\begin{array}{r|rrrrrl}
2 & 3 & -2 & 5 & -11 & 4 & -7 \\
 & & 6 & 8 & 26 & 30 & 68 \\
\hline
 & 3 & 4 & 13 & 15 & 34 & \boxed{61 = f(2)} \\
 & x^4 & x^3 & x^2 & x & c & r
\end{array}
$$

This says

$3x^5 - 2x^4 + 5x^3 - 11x^2 + 4x - 7 =$

$(x-2)(3x^4 + 4x^3 + 13x^2 + 5x + 34) + 61$

What we've just seen is the application of the Remainder Theorem.

Its brother is the Factor Theorem.

Let $f(x) = x^3 - 2x^2 - 13x - 10$

Find $f(5)$

$$\begin{array}{r|rrrr} 5 & 1 & -2 & -13 & -10 \\ & & 5 & 15 & 10 \\ \hline & 1 & 3 & 2 & 0 = f(5) \end{array}$$

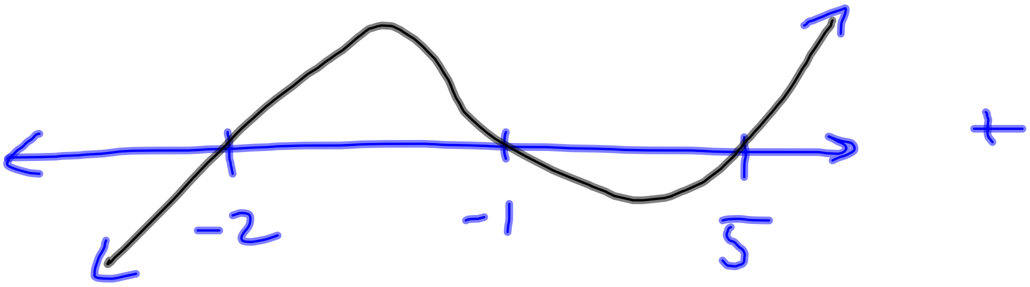
So,

$x^3 - 2x^2 - 13x - 10 = (x-5)(x^2+3x+2)$

We've "split off" a factor of $x-5$ from $x^3-2x^2-13x-10$. We're left with the "depressed" polynomial x^2+3x+2, which factors into $(x+2)(x+1)$, and so

$f(x) = x^3 - 2x^2 - 13x - 10 = (x-5)(x+2)(x+1)$

We've "split $f(x)$ into linear factors."



Why are we doing this?

To solve $f(x) = 0$, when $f(x)$ is a polynomial.

We saw already how having one zero ($x = 5$) creates a "depressed equation," meaning degree went from 3 to 2.

In this chapter, we'll learn how to guess zeros, and break $f(x)$ down until we get to quadratic factor.

$$f(x) = (x-5)\underbrace{(x^2+3x+2)}$$

we can kill these.

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Theorems for Chapter 3:

Remainder Theorem
Factor Theorem
How we break 'em down.

Rational Zeros Theorem
Descartes' Rule of Signs
Help us guess what the zeros are

Conjugate Pairs Theorem $a+bi \Longrightarrow a-bi$

Upper & Lower Bounds on Zeros.
Limit Search Pattern.
How far to look for zeros.

Vince's question from 3.1 showed two ways to attack those.

$f(x) = 1x^3 - 2x^2 - 13x - 10$

$x = 5, -1, -2$ are its zeros.

Rational zeros: $\frac{p}{q}$

Constant Term

p is a factor of -10 and

q " " " " 1

Leading Coefficient

$\frac{p}{q} = \pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{5}{1}, \pm\frac{10}{1}$

Descartes' Next Time.

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