1. (8 pts) Determine whether the relation below represents a function. What is the domain and what is the range?

$$\{(-3,6), (7,5), (4,9), (-5,6)\}$$

Yes, $D = \{-3, 7, 4, -5\}$
$R = \{6, 5, 9, 6\}$

But not $\boxed{\text{1-to-1}}$,
because $y=6$ corresponds
to $x=-3$ & $x=-5$
$f(-3) = f(-5)$, but $-3 \neq -5$
$y_1 = y_2$ $\therefore$ $x_1 \neq x_2$

2. (5 pts) Determine whether the equation $x^2 - y = 9$ defines y as a function of x. If it does *not*, show/explain why not.

$-y = -x^2 + 9$
$y = x^2 - 9$

Does one x-value give one y-value?
correspond to

$\boxed{\text{Yes}}$

$y^2 - x = 9$
$y^2 = x + 9$
$|y| = \sqrt{x+9}$
$y = \pm\sqrt{x+9}$

$\boxed{\text{No}}$

Two different y-values
for each x-value
If you think it
isn't, just guess @
an x-value that
gives more than one
y-value

$x = 1$: $y^2 - 1 = 9$
$y^2 = 10$
$y = \pm\sqrt{10}$ 2 y-values for $x = 1$.

Show that $3x+2 = f(x)$ is 1-to-1.

Algebraic.

Suppose $f(x_1) = f(x_2)$

Then $3x_1+2 = 3x_2+2$

$\Longrightarrow 3x_1 = 3x_2$

$\Longrightarrow x_1 = x_2$

$\Longrightarrow$ 1-to-1

$f(x) = \dfrac{x-4}{x+1}$ is 1-to-1.

Suppose $f(x_1) = f(x_2)$

$$\frac{x_1-4}{x_1+1} = \frac{x_2-4}{x_2+1}$$ Solve for x_1

Distributive law is big.

$(x_2+1)(x_1-4) = (x_2-4)(x_1+1)$ Cross-multiply

$\boxed{x_2x_1} - 4x_2 + x_1 - 4 = \boxed{x_2x_1} + x_2 - 4x_1 - 4$

$-4x_2 + x_1 = x_2 - 4x_1$

$5x_1 = 5x_2$

$x_1 = x_2$ $\blacksquare$

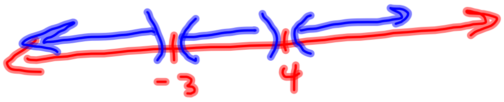
3. Let $f(x) = \dfrac{4}{x-4}$. Determine the following, if possible. If not possible, state why:

a. (2 pts) $f(2) = \dfrac{4}{2-4} = \dfrac{4}{-2} = -2$

b. (2 pts) $f(3) = \dfrac{4}{3-4} = \dfrac{4}{-1} = -4$

c. (2 pts) $f(4)$ $\nexists$ $4 \notin \mathcal{D}(f)$ ($x-4$ in denominator)

4. (7 pts) Find the domain of $g(x) = \dfrac{x^2 + 5x + 17}{x^2 - x - 12}$.



Need $x^2 - x - 12 \neq 0$

Solve

$x^2 - 4x + 3x - 12 = 0$

$x(x-4) + 3(x-4) = 0$

$x\Delta + 3\Delta = \Delta(x+3)$

$(x-4)(x+3) = 0$

$x \in \{-3, 4\}$

Throw 'em out!

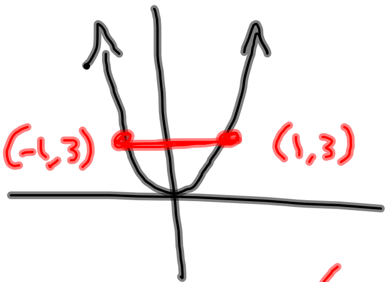
$\mathcal{D} = \{x \mid x \neq -3 \ \& \ x \neq 4\}$

$= \mathbb{R} \setminus \{-3, 4\}$

$= (-\infty, -3) \cup (-3, 4) \cup (4, \infty)$

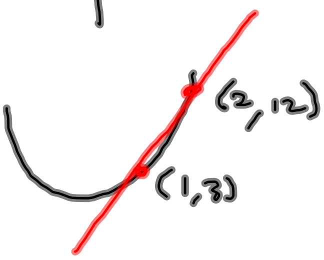
5. (4 pts) Let $f(x) = 3x^2$. Find the average rate of change of f from $x = -1$ to $x = 1$.

$$m = \frac{f(1) - f(-1)}{1 - (-1)} = \frac{3(1)^2 - 3(-1)^2}{2} = \frac{3-3}{2} = \frac{0}{2} = 0$$



How about from $x = 1$ to $x = 2$?

$$\frac{f(2) - f(1)}{2 - 1} = \frac{3(2)^2 - 3(1)^2}{1}$$



$$= \frac{12 - 3}{1} = 9$$

6. Let $f(x) = 2x - 6$ and $g(x) = \sqrt{x-3}$.

a. (5 pts) Determine the domain of *f*.

$\mathcal{D}(f) = \mathbb{R} = (-\infty, \infty) = \{x \mid x \text{ is real}\}$

b. (5 pts) Determine the domain of *g*.

Need $x - 3 \geq 0$

$\mathcal{D} = \{x \mid x \geq 3\}$
$= [3, \infty) = \mathcal{D}(g)$

c. Find the following functions and state the domain of each.

i. (3 pts) $(f+g)(x) =$ $2x - 6 + \sqrt{x-3}$

$\mathcal{D}(f+g) = [3, \infty)$

ii. (3 pts) $(f \cdot g)(x) = (2x-6)(\sqrt{x-3})$

$\mathcal{D}(fg) = [3, \infty)$
$= \{x \mid x \geq 3\}$

$\left(\frac{f}{g}\right)(x) = \frac{2x-6}{\sqrt{x-3}}$

$\mathcal{D}\left(\frac{f}{g}\right) = (3, \infty)$
$= \{x \mid x > 3\}$

Also need $\sqrt{x-3} \neq 0$
$x - 3 \neq 0$
$x \neq 3$

$f(x) = 2x - 6$
$g(x) = \sqrt{x-3}$

Composition

$(f \circ g)(x) = f(g(x)) = 2g(x) - 6 = \boxed{2\sqrt{x-3} - 6}$

$\mathcal{D} = \{x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f)\}$

$= \{x \mid x \geq 3 \quad \text{and} \quad \text{no restriction}\}$

$= \{x \mid x \geq 3\}$

$(g \circ f)(x) = g(f(x)) = \sqrt{f(x) - 3}$

$= \sqrt{(2x-6) - 3}$

$\mathcal{D}(g \circ f) = \{x \mid x \in \mathcal{D}(f) \text{ and } f(x) \in \mathcal{D}(g)\}$

No Restriction

$\mathcal{D}(g) = \{x \mid x \geq 3\}$

Need $\{x \mid f(x) \geq 3\} = \{x \mid x \geq \frac{9}{2}\}$

$2x - 6 \geq 3$

$2x \geq 9$

$x \geq \frac{9}{2}$

Do vertical flip 1st

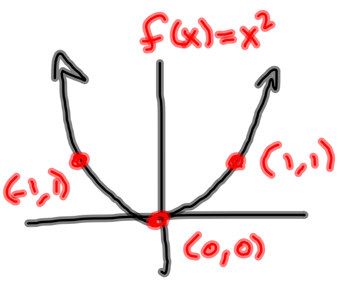
a. (6 pts) $g(x) = -(x-7)^2 - 11$

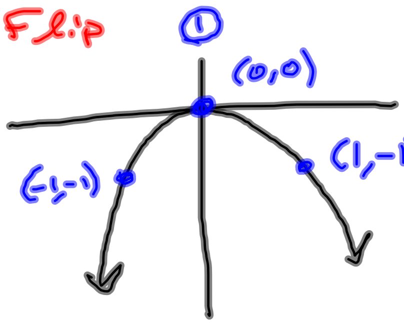
$f(x) = x^2 \longrightarrow \overset{①}{-x^2} \longrightarrow \overset{②}{-(x-7)^2} \longrightarrow \overset{③}{-(x-7)^2 - 11}$

$-f(x)$ Vertical Flip

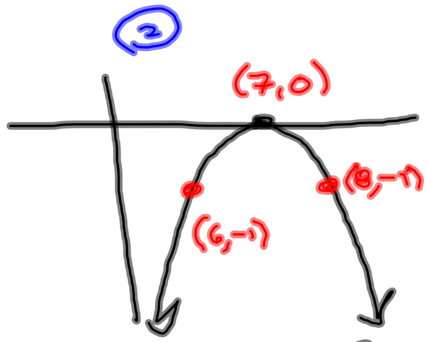
$-f(x-7)$ Right 7

$-f(x-7) - 11$ Down 11

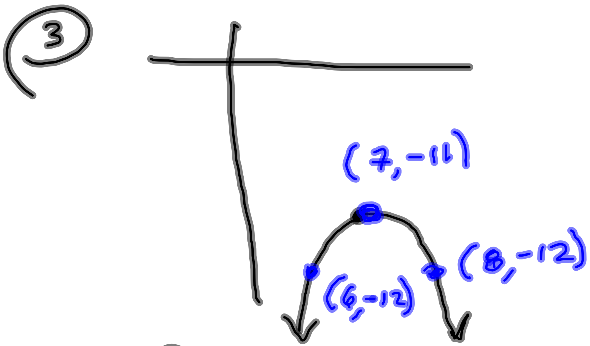




$-f(x) = -x^2$



$-f(x-7) = -(x-7)^2$

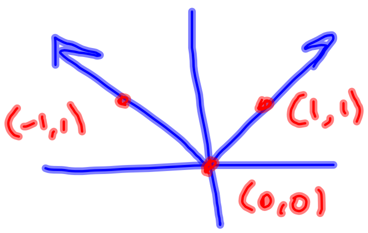


$-f(x-7) - 11$

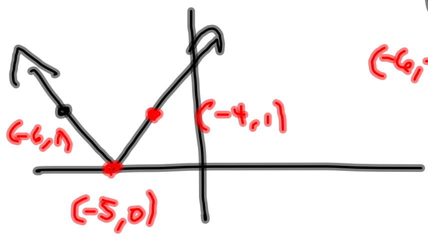
6 funcs
6 rubes
6^6 fun.

$|x+5| - 8$

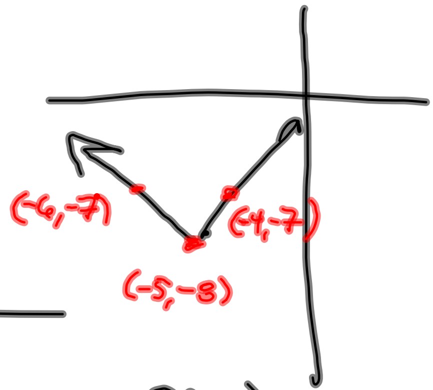
left 5, down 8



$f(x) = |x|$



$f(x+5) = |x+5|$



$f(x+5) - 8$

$= |x+5| - 8$

$$F = \frac{K m_1 m_2}{r^2}$$ K = Konstant of proportionality.

F is jointly proportional to m_1 & m_2 and inversely proportional to the square of r.

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y is proportional to x

$y = Kx$ for some K.

If $y = 7$ when $x = 5$, what's y when $x = 11$?

$\therefore 7 = 5K$

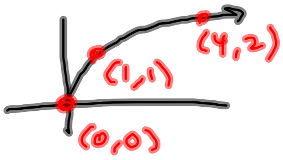
$\frac{7}{5} = K$

$y\Big|_{x=11} = \frac{7}{5} \cdot 11 = \boxed{\frac{77}{5} = y}$

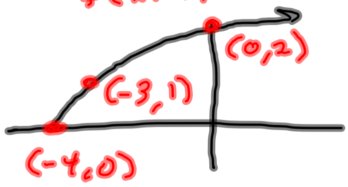
Vertical Reflections first.
Horizontal Reflections last works.

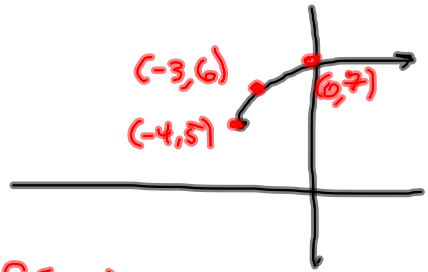
$g(x) = \sqrt{4-x} + 5 = \sqrt{-x+4} + 5$

1 $f(x) = \sqrt{x} \longrightarrow$ 2 $\sqrt{x+4} \longrightarrow$ 3 $\sqrt{x+4} + 5$



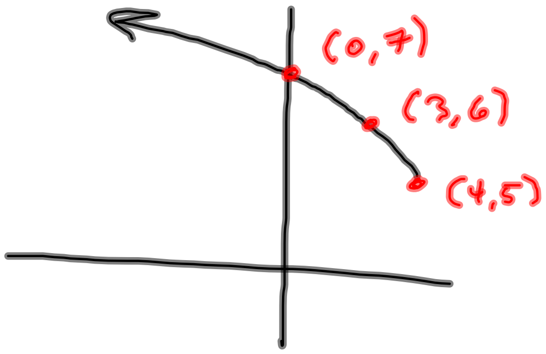
$f(x+4)$





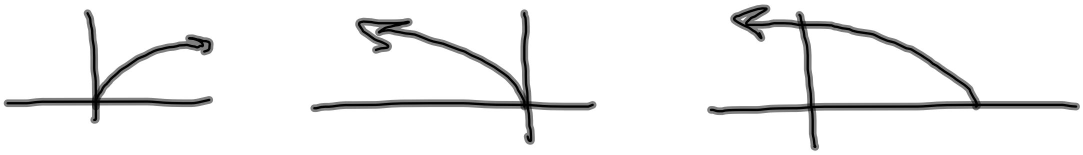
$f(x+4)+5$
$=$

$\longrightarrow$ 4 $\sqrt{-x+4} + 5$

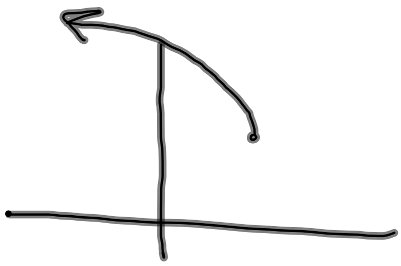


$f(-x+4)+5$

$\sqrt{x} \longrightarrow \sqrt{-x} \longrightarrow \sqrt{-(x-4)}$



$\longrightarrow \sqrt{-(x-4)} + 5$



Doing
$4-x$ as a
$-(x-4)$ in
this version.
$-x \longrightarrow -(x-4)$

Feb 14-10:10 AM