what's the domain?

$g(x) = \sqrt{2x+6}$ $\qquad \mathcal{D}(g) = \{x \mid x \geq -3\}$

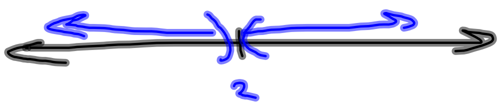
<u>Need</u> : $2x+6 \geq 0$ $\qquad [-3, \infty)$

$f(x) = \dfrac{1}{x-2}$ $\qquad \mathcal{D}(f) = \{x \mid x \neq 2\}$

Need: $x-2 \neq 0$ $\qquad = \{w \mid w \neq 2\}$

$x \neq 2$ $\qquad = \mathbb{R} \setminus \{2\}$

$\qquad = (-\infty, 2) \cup (2, \infty)$



See Monday Notes

for $(f \circ g)(x)$ & its domain.

§2.5

$f^{-1}(x)$ = Inverse function

Is NOT $(f(x))^{-1} = \frac{1}{f(x)}$

$f(x) = 2x-3$

$f^{-1}(x)$ has the property $(f \circ f^{-1})(x) = x$

$x = 2y-3$ Solve for y.

Switch & Solve Method.

$2y-3 = x$

$2y = x+3$

$y = \frac{x+3}{2} = g(x)$

$(f \circ g)(x) = f(g(x)) = f\left(\frac{x+3}{2}\right) = \cancel{2}\left(\frac{x+3}{\cancel{2}}\right) - 3$

$= x+3-3 = x$

Reversing Composition

Doesn't always work.

$f(x) = 2x-3$

$$x \xrightarrow[\text{times } 2]{} 2x \xrightarrow[\text{minus } 3]{} 2x-3$$

divide by 2 plus 3

$$\frac{x+3}{2} \longleftarrow x+3 \longleftarrow x$$

$f(x) = \frac{x+2}{x-3}$ Find $f^{-1}(x)$

$x = \frac{y+2}{y-3}$ Solve for y

$\frac{y+2}{y-3} = x$

$\left(\frac{y+2}{y-3}\right)(y-3) = x(y-3)$

$y+2 = xy - 3x$

$-xy \quad = -xy$

$-xy + y + 2 = -3x$

$-2 = -2$

$-xy + y = -3x - 2$

$y(-x+1) = -3x - 2$

$\boxed{y = \frac{-3x-2}{-x+1}} = \frac{3x+2}{x-1} = f^{-1}(x) = g(x)$

$y + 2 = 7y - 3(7)$

$y + 2 = 7y - 21$

$\boxed{-6y = -23}$

$y = \frac{23}{6}$

$f(x) = \frac{x+2}{x-3}$, $g(x) = f^{-1}(x) = \frac{3x+2}{x-1}$

$(f \circ g)(x) = f(g(x)) = f\left(\frac{3x+2}{x-1}\right)$ wow!

$$f\left(\boxed{\frac{3x+2}{x-1}}\right) = \frac{\boxed{\frac{3x+2}{x-1}}+2}{\boxed{\frac{3x+2}{x-1}}-3} =$$

D.L.

$$\frac{\frac{3x+2}{x-1} + \frac{2}{1}\cdot\frac{x-1}{x-1}}{\frac{3x+2}{x-1} - \frac{3}{1}\cdot\frac{x-1}{x-1}} = \frac{\frac{3x+2+2(x-1)}{x-1}}{\frac{3x+2-3(x-1)}{x-1}}$$

$$= \frac{\frac{3x+2+2x-2}{x-1}}{\frac{3x+2-3x+3}{x-1}} = \frac{\frac{5x}{x-1}}{\frac{5}{x-1}} = \left(\frac{\cancel{5}x}{\cancel{x-1}}\right)\cdot\left(\frac{\cancel{x-1}}{\cancel{5}}\right) = x$$

$\therefore$ $g(x)$ is the inverse of $f(x)$.

When this is the case, $(g \circ f)(x) = x$, also

Do this for practice

$f(x) = \sqrt{x}$. what's $f^{-1}(x)$?

$x = \sqrt{y}$ $\qquad\qquad$ $\sqrt{y^2} = |y|$

$(\sqrt{y})^2 = x^2$ $\qquad\qquad$ $(\sqrt{y})^2 = y$

$y = x^2 = f^{-1}(x)$

There's a slight problem with this.

Yes, they're inverses, but we have issues with the domain.

$g(x) = x^2$ what's $g^{-1}(x)$?

$x = y^2$

$y^2 = x$

$\sqrt{y^2} = \sqrt{x}$

$|y| = \sqrt{x}$

$y = \pm\sqrt{x}$ is NOT A FUNCTION!

$x = 1$: $y = \pm 1$. This gives $(1,1)$ & $(1,-1)$ as pairs in the relation. Two different y-values paired the single input $x = 1$.

That's why we talk about 1-to-1 functions.

f is 1-to-1 if

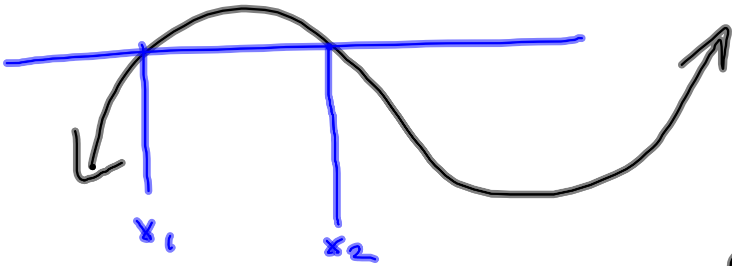
$$\boxed{x_1 \neq x_2 \implies f(x_1) \neq f(x_2)}$$

$$\boxed{f(x_1) = f(x_2)} \implies \underline{x_1 = x_2}$$

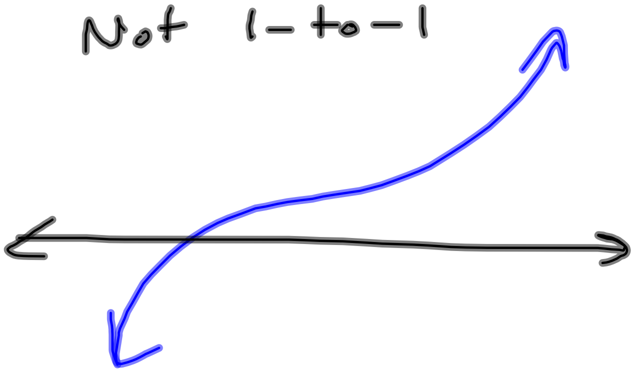
To be a function, each x is paired with one y.

To .. .. 1-to-1 function, it's a function, <u>and</u> each y is paired with one x.

One-to-one functions have the property that their inverse relation is <u>a function</u>.



Not 1-to-1



The inverse relation is not a function.

$x_1 \neq x_2$ but

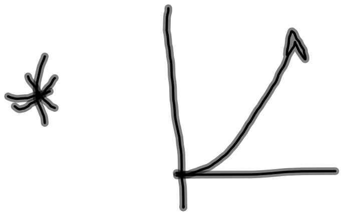
$f(x_1) = f(x_2)$

$\boxed{\text{Horizontal Line Test}}$

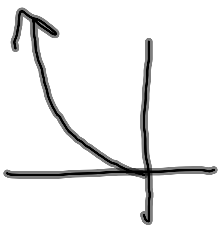
$\sqrt{x} = f(x)$, $x^2 = g(x)$

we can <u>make</u> $g(x)$ 1-to-1

Restrict its domain!



$\underbrace{g(x) = x^2 , \; x \geq 0}$ is 1-to-1, with inverse $f(x) = \sqrt{x}$



$g(x) = x^2$, $x \leq 0$ is 1-to-1 with inverse $f(x) =$

Try $f(x) = \sqrt{x}$

$(g \circ f)(x) = g(\sqrt{x})$ but $\sqrt{x} \geq 0$

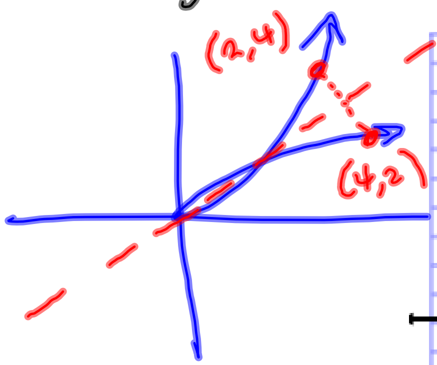
$\sqrt{x} \notin \mathcal{D}(g)$

Then try $f(x) = -\sqrt{x}$

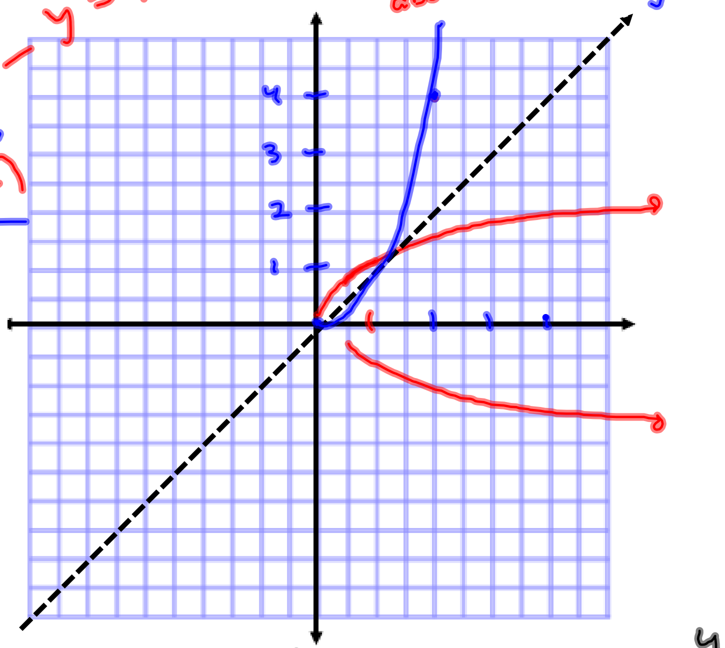
$g(-\sqrt{x}) = (-\sqrt{x})^2 = x$ ✓

$g(x) = x^2, \quad x \geq 0$

$g^{-1}(x) = \sqrt{x}$

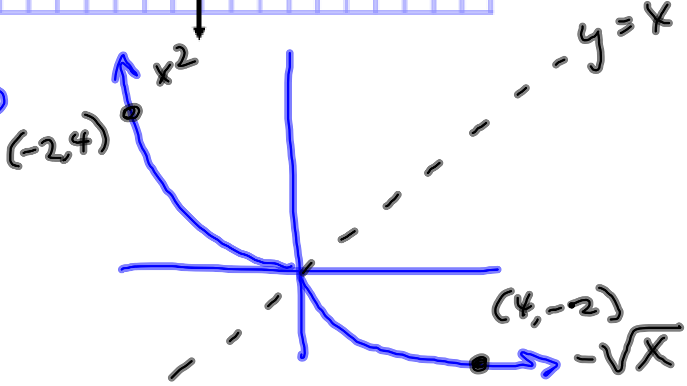


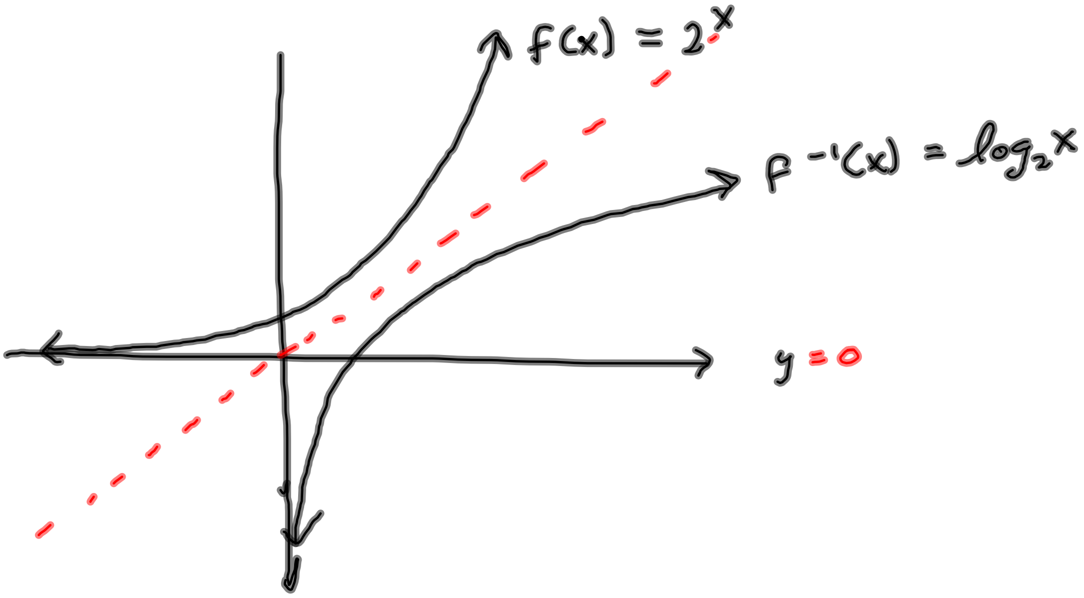
Supposed to show symmetry about y = x



$g(x) = x^2, \quad x \leq 0$

$g^{-1}(x) = -\sqrt{x}$





$f(x) = 2^x$
$f^{-1}(x) = \log_2 x$
$y = 0$