§1.6 & a piece of 1.7, today?

Quadratic Equations

Factoring

Square Root Property (Absolute Value Equations)

Completing the square.

Solve by Factoring.

write much
Think little.

Muscle-build:

factor by grouping

$x^2+7x-3x-21$

$= x(x+7)-3(x+7)$

$= (x+7)(x-3)$

$\frac{-3x}{-3}=x$

$\frac{-21}{-3}=7$

$\frac{x(x+7)}{x+7}=x$

$\frac{-3(x+7)}{x+7}=-3$

Expand it back out:

$x^2-3x+7x-21$

$= x^2+4x-21$ If this is the question, if I could recapture the $x^2-3x+7x-21$, I could factor THAT by grouping.

Here's how to recapture the $-3x+7x$.

$ax^2+bx+c = x^2+4x-21$

$a=1, c=-21.$

$ac = -21 = -(3)(7)$

FIND a grouping of these factors that adds up to 4. (From the 4x middle term)

I suggest $(+7)(-3)=-21$

$+7-3 = 4$

So $x^2+4x-21$ becomes

Factor $6x^2 - 7x + 2$

$ac = (6)(2) = (2)(3)(2)$

4

3

Split so they sum to -7

$(-4)(-3) = +12$

$-4 - 3 = -7$

$6x^2 - 4x - 3x + 2$

$= 2x(3x-2) - 1(3x-2)$

$= (3x-2)(2x-1)$

Find factors of 12 that add up to -7 is what I meant.

$(-4)(-3) = 12$

$-4 - 3 = -7$

AB = 0 if and only if

A = 0 OR B = 0 Zero factor property
∴ product property.

Solve $6x^2 - 7x + 2 = 0$ by factoring

$\Longrightarrow (3x-2)(2x-1) = 0$

$\Longrightarrow 3x - 2 = 0$ OR $2x - 1 = 0$

$+2 = +2$ $\qquad$ $+1 = 1$

$3x = 2$ $\qquad$ $2x = 1$

$x = \frac{2}{3}$ OR $x = \frac{1}{2}$

$x \in \{ \frac{2}{3}, \frac{1}{2} \} = \{ \frac{1}{2}, \frac{2}{3} \}$

Should always check.

Better style
Left-to-Right

Absolute Value Equations

|☺| = 7 ⟹

☺ = 7 OR ☺ = −7

i.e.

☺ = ±7

(Inequalities
Doesn't work for |☺| < 7 and |☺| > 7)

$|x+2| = 7 \Rightarrow$

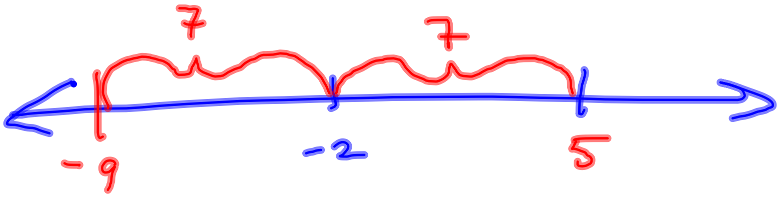
$x+2 = \pm 7 \Rightarrow$

$x = -2 \pm 7$ → $-2+7 = 5$, $-2-7 = -9$

$\Rightarrow$ $\boxed{x \in \{-9, 5\}}$

A visual: This question is asking us to find all #s that are 7 units fr -2.

$|x+2| = |x-(-2)|$



$$\sqrt{x^2} = |x|$$

$$\sqrt{3^2} = 3$$

$$\sqrt{(-3)^2} = 3$$

So to say

Square Root Property: $x^2 = 9$ is to say $x = \pm 3$

$$x^2 = 9$$

$$\sqrt{x^2} = \sqrt{9}$$

$$|x| = \sqrt{9} = 3$$

$$x = \pm 3$$

$$x^2 = 7 \longrightarrow$$

$$x = \pm\sqrt{7}$$

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USE SRP to solve

$x^2 - 5 = 0$

$x^2 = 5$

$x = \pm\sqrt{5}$

Twisted

$x^2 - 5 = 0$

$x^2 - (\sqrt{5})^2 = 0$

$(x - \sqrt{5})(x + \sqrt{5}) = 0$

$x = \sqrt{5}$ OR $x = -\sqrt{5}$

$\sqrt{5 \cdot 5 \cdot 3 \cdot 7 \cdot 7 \cdot 11}$

$5 \cdot 7\sqrt{3 \cdot 11} = 35\sqrt{33}$

$\left(x - \frac{1}{2}\right)^2 = \frac{25}{4}$

$x - \frac{1}{2} = \pm\sqrt{\frac{25}{4}} = \pm\sqrt{\frac{5^2}{2^2}} = \pm\frac{\sqrt{5^2}}{\sqrt{2^2}} = \pm\frac{5}{2}$

$x - \frac{1}{2} = \pm\frac{5}{2}$

$x = \frac{1}{2} \pm \frac{5}{2}$

$\frac{1+5}{2} = \frac{6}{2} = 3$

$\frac{1-5}{2} = \frac{-4}{2} = -2$

$x \in \{-2, 3\}$

$\sqrt{-1} = i$ $i^2 = -1$

$(x-3)^2 = -20$ $\sqrt{-20} = i\sqrt{20}$

PRIMES → 2, 3, 5, 7, 11, 13, 17, 19,

$2 \,\lfloor 20$
$2 \,\lfloor 10$
5

$x-3 = \pm\sqrt{-20} = \pm 2i\sqrt{5}$

$i\sqrt{20} =$
$i\sqrt{2\cdot 2\cdot 5}$
$2i\sqrt{5}$

$x = 3 \pm 2i\sqrt{5}$

$x \in \{3 \pm 2i\sqrt{5}\}$ is OK by me, but a lawyer might want

$\{3-2i\sqrt{5}, 3+2i\sqrt{5}\}$

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Recall completing the square.

Solve by " " " :

$x^2 - 10x + 5 = 0$

$x^2 - 10x = -5$

$x^2 - 10x + 5^2 = -5 + 25$

$\frac{10}{2} = 5 \rightarrow 5^2$

$(x-5)^2 = 20$

Ghost in the machine:

$\sqrt{(x-5)^2} = \sqrt{20}$

$|x-5| = \sqrt{20}$

$x - 5 = \pm\sqrt{20} = \pm 2\sqrt{5}$

$2 \,|\, 20$

$2 \,|\, 10$

5

$+5 \qquad = +5$

$x = 5 \pm 2\sqrt{5}$

$x \in \{5 \pm 2\sqrt{5}\}$

$x \in \{5 - 2\sqrt{5}, 5 + 2\sqrt{5}\}$ Drew sez Course Compass wants it this way.

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See pp 141-2

Derivation of quadratic formula.

Assume $a > 0$

$ax^2 + bx + c = 0$

$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

$x^2 + \frac{b}{a}x = -\frac{c}{a}$

$\frac{\frac{b}{a}}{2} = \frac{b}{a} \cdot \frac{1}{2} = \frac{b}{2a} \longrightarrow \left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$

$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2}$

$\left(-\frac{c}{a}\right)\left(\frac{4a}{4a}\right) + \frac{b^2}{4a^2}$

$= \frac{-4ac + b^2}{4a^2}$

$= \frac{b^2 - 4ac}{4a^2}$

$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$

$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$

$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$

$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

See Strategy pg 143

① $ax^2 + c = 0$ No "x"-term
Square root prop

② EZ to factor, factor it

③ Hard. Quadratic Formula

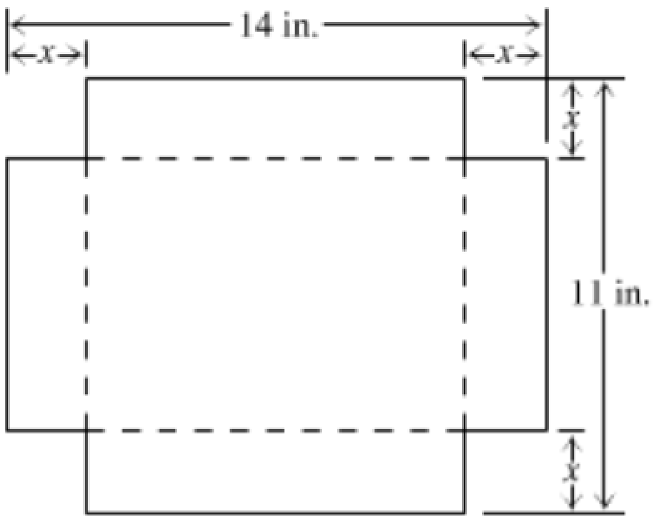
Graphing calculators can give insight or a quick check of your work
No graphing calculators or cell phones on tests.

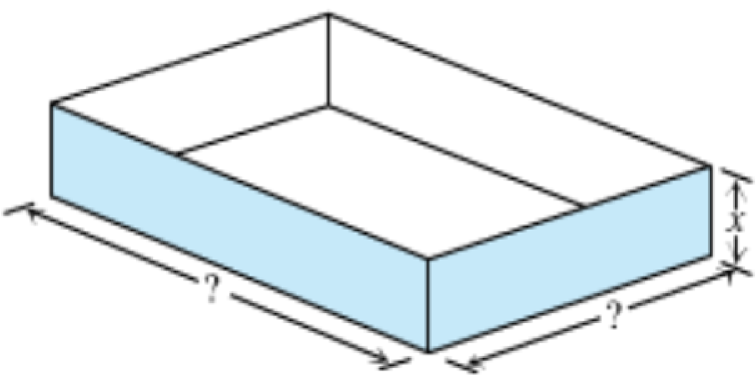
Let's do the standard "build a box" problem:

Imogene wants to make an open-top box for packing baked good by cutting equal squares from each corner of an 11 inch by 14 inch piece of cardboard. She figures that for versatility, the area of the bottom must be 80 in^2. What size squares should she cut from the corners?

1. Identify the variable(s) in words and units.
2. Draw a picture or two.
3. Construct any equations that might relate to the problem situation.
4. Solve and Check.

Let x = the width (and length) of the squares cut from the corners (in inches)





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