MAT 121-G11 | Test 4 – Fall, 2018 | Name KEY

100 Points | Covers Chapter 4 and 5 | NO GRAPHING CALCULATORS!!!

1. (20 pts) Starting with $f(x)=\log_2(x)$, sketch the graph of $g(x)=-7\cdot\log_2(-5x+20)+3$ in 5 steps (counting $f(x)=\log_2(x)$ as the first step). Use $x=\frac{1}{2}, x=1$, and $x=2$ to find 3 points in the first graph, and show how these 3 points are moved around by each step in the transformation to $g(x)$.

2. (10 pts) Find the *exact* x- and y-intercepts for $g(x)$ from #1. That means no decimal approximations.

   a. x-intercept: $A=$

   b. y-intercept: $B=$

Label your final graph for #1 with the intercepts labeled with A and B.

3. (5 pts) Find the inverse, $g^{-1}(x)$, for $g(x)$ in #1. The moves are very similar to what you did in #2a.

4. Let $f(x)=\sqrt{x-11}$ and $g(x)=x^2-4x-21$.

   a. (5 pts) What is the domain of f?

   b. (5 pts) What is the domain of g?

   c. (5 pts) Determine $\left(\frac{f}{g}\right)(x)$. (Sometimes this is just called $\frac{f}{g}$ in the text.)

   d. (5 pts) What is the domain of $\left(\frac{f}{g}\right)(x)$?

   e. (5 pts) Determine $(f\circ g)(x)$ (Again, sometimes just called $f\circ g$).

   f. (5 pts) What is the domain of $f\circ g$?

5. The next two domain questions are very closely related.

   a. (5 pts) What is the domain of $\sqrt{\frac{(x-7)^2(x+4)^3}{(x-4)^2(x+2)^3}}$?

   b. (5 pts) What is the domain of $\log_{11}\left(\frac{(x-7)^2(x+4)^3}{(x-4)^2(x+2)^3}\right)$?

6. (10 pts) Solve $\ln(x-7)+\ln(x+3)=\ln(11)$. Give the exact solution(s).

7. Suppose the half-life of C-14 is 4550 years. (It isn't, quite, but just suppose…).

   a. (10 pts) Derive the exponential decay model, $A(t)=A_0e^{kt}$. The trick is to use the half-life to find the relative decay rate, k.

   b. (5 pts) How old is a sample of charcoal from a prehistoric fire pit, if 47% of the C-14 has decayed (i.e., 53% is left.) ? Round to the nearest year in your final answer. If it makes it easier for you, use an initial mass of 100 g of radioactive C-14 and a final mass of 53 g of the radioactive material. It's the same thing.

Bonus Answer up to four (4) 5-pointers. That's a total of 20 bonus points possible. Points to be had. Standards are high.

B 1 (10 pts) Solve the absolute value inequality: $|-3x-7|-8\le -5$. Yes, that's 10 points. Counts for 2 Bonus.

B 2 (5 pts) Re-write $f(x)=4x^2-3x-11$ in the form $a(x-h)^2+k$.

B 3 (5 pts) Solve the exponential equation $5\cdot 4^x=6\cdot \pi^x$. Give the exact answer in terms of natural logarithms.

B 4 (5 pts) Sketch the graph of $R(x)=\dfrac{(x-7)^2(x+4)^3}{(x-4)^2(x+2)^3}$.

B 5 (5 pts) Sketch the graph of $Q(x)=\sqrt{\dfrac{(x-7)^2(x+4)^3}{(x-4)^2(x+2)^3}}$

B 6 (5 pts) The population of Kokanee salmon at Dworshak reservoir was 1,000 Kokanee, when they first introduced the species on this date 1992. It's quite a coincidence that you'd be taking your test on the anniversary Since then, the population has grown exponentially, with a relative growth rate of 2% every year. What is the Kokanee population in Dworshak Reservoir, today? Round your answer to the nearest fish.

B 7 (5 pts) To the nearest year, when will (did) the population of Kokanee in the previous question reach 10,000?

B 8 (5 pts) Sketch the graph of $g(x)=-5\cdot 3^{3x+9}+11$.

B 9 (5 pts) Parents of newborn baby Jimmy want to make sure he can go to college, and they estimate it will cost \$100,000. They plan to set aside money every month and have a mutual fund that they believe will earn 5% interest, compounded monthly, over the long haul. How much should they set aside in order for little Jimmy to have \$100,000 in cold hard cash in his education fund on his 18th birthday? Assume they make their first payment a month after Jimmy's born.

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3) a) x-int: $g(x) = 0$

5pts

$$-7\log_2(-5x+20)+3=0$$
$$-7\log_2(-5x+20)=-3$$
$$\log_2(-5x+20)=\frac{3}{7}$$
$$2^{\log_2(\text{STUFF})}=2^{\frac{3}{7}}$$
$$-5x+20=2^{\frac{3}{7}}$$
$$-5x=2^{\frac{3}{7}}-20$$
$$x=\frac{2^{\frac{3}{7}}-20}{-5}=4-\frac{1}{5}\cdot 2^{\frac{3}{7}}$$

$$\boxed{A=\left(4-\frac{1}{5}\cdot 2^{\frac{3}{7}},0\right)} \approx$$

b) 5pts $g(0)=-7\log_2(-5(0)+20)+3$

$$=-7\log_2(20)+3$$

$$B=\left(0,-7\log_2(20)+3\right)\approx$$

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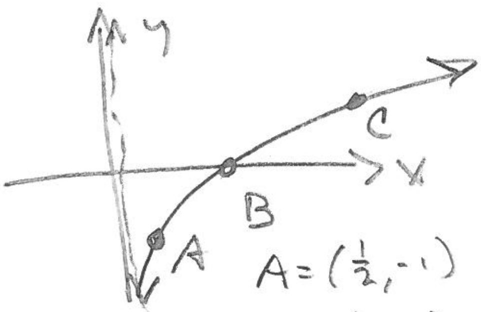
FALL, 2018

(1) $g(x) = -7\log_2(-5x+20)+3 = -7\log_2(-5(x-4))+3$

(0) $f(x) = \log_2(x)$



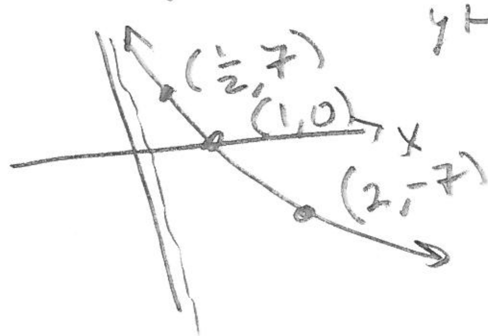
$A = (\frac{1}{2}, -1)$

$x=0$ $B = (1, 0)$

$C = (2, 1)$

(1) $-7\log_2(x)$

$y \mapsto -7y$

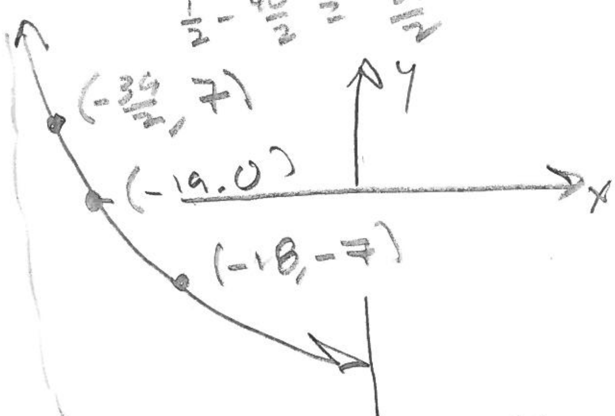


$x=0$

(2) (M1) $-7f(x+20)$

$x \mapsto x-20$

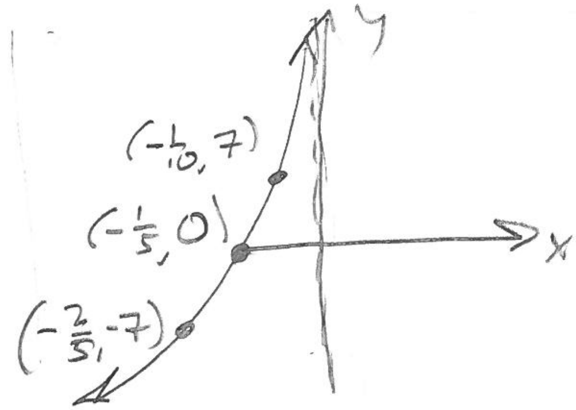
$\frac{1}{2} - \frac{40}{2} = -\frac{39}{2}$



$x=-20$

(2) (M2) $-7f(-5x)$

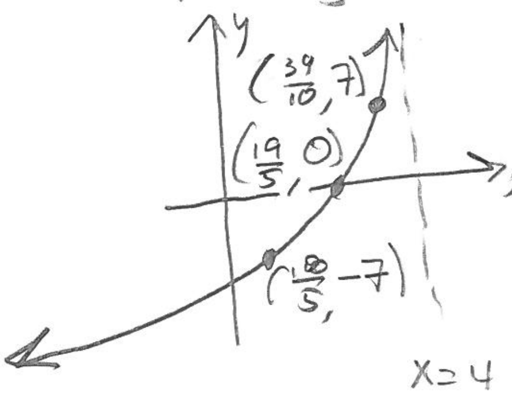
$x \mapsto -\frac{1}{5}x$



$x=0$

(3) $\overset{M1}{-7f(-5x+20)} = \overset{M2}{-7f(-5(x-4))}$

$x \mapsto -\frac{1}{5}x$ $x \mapsto x+4$



$x=4$

M1:

$-\frac{39}{2} \cdot (-\frac{1}{5})$

$\frac{-19}{-5}$

$\frac{-18}{-5}$

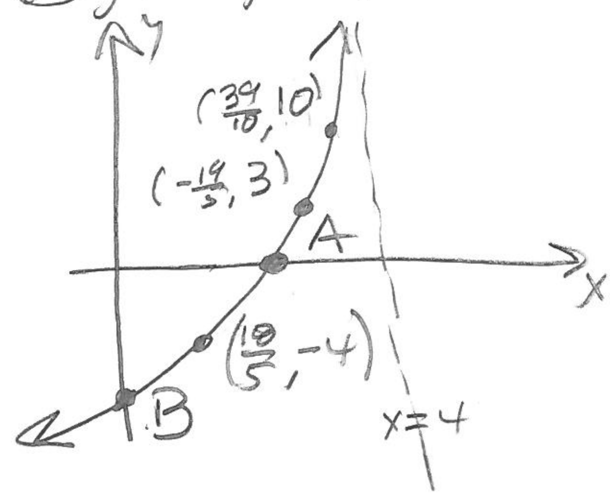
M2:

$-\frac{1}{10}+4$

$-\frac{1}{5}+4$

$-\frac{2}{5}+\frac{20}{5}$

(4) $g(x)$ $y \mapsto y+3$



121 T4

3 $-7\log_2(-5y+20)=x$

$\log_2(-5x+20) = -\frac{1}{7}x$

$2^{\text{left}} = 2^{\text{right}}$

$-5x+20 = 2^{-\frac{1}{7}x}$

$-5x = 2^{-\frac{1}{7}x} - 20$

$$x = \boxed{\frac{2^{-\frac{1}{7}x}-20}{-5} = g^{-1}(x)}$$

or $\boxed{g^{-1}(x) = 4 - \frac{1}{5}\cdot 2^{-\frac{1}{7}x}}$

4 (a) $\mathcal{D}(f)$: Need $x-11 \geq 0 \Longrightarrow x \geq 11$

5pts $\longrightarrow \boxed{\mathcal{D} = [+11, \infty)}$

5pts (b) $\boxed{\mathcal{D}(g) = (-\infty, \infty)}$ (It's a polynomial!)

(c) 5pts $\frac{f}{g} = \frac{\sqrt{x-11}}{x^2-4x-21}$ Need $x \geq 11$ and $g(x) \neq 0$

$g(x) = x^2-4x-21 =$
$= (x-7)(x+3) = 0$
$\Longrightarrow x = -3, 7$ THROW OUT OF DOMAIN
$\Longrightarrow \mathcal{D}\left(\frac{f}{g}\right) = \{x \mid x \geq 11 \text{ and } x \neq -3 \text{ or } 7\}$

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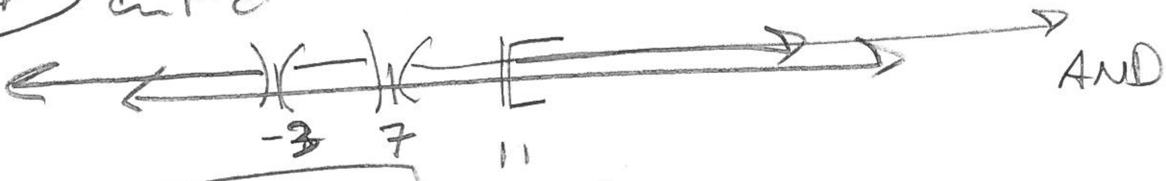
4d cont'd



$= [11, \infty)$! $= \mathcal{D}\left(\frac{f}{g}\right)$

4e 5pts $f \circ g = \sqrt{x^2 - 4x - 21 - 11}$

4f 5pts Need $x \in \mathcal{D}(g) = \mathbb{R} = (-\infty, \infty)$

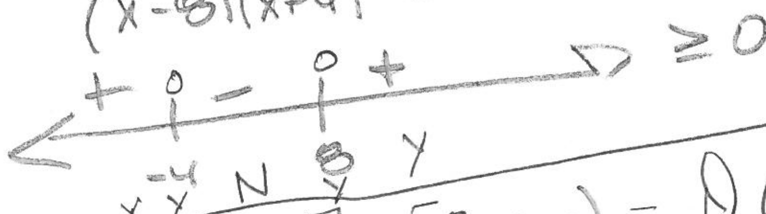
AND

Need $g(x) \in \mathcal{D}(f)$

$x^2 - 4x - 21 \geq 11$

$x^2 - 4x - 32 \geq 0$

$(x-8)(x+4) \geq 0$



$= (-\infty, -4] \cup [8, \infty) = \mathcal{D}(f \circ g)$

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5 a Need $\dfrac{(x-7)^2(x+4)^3}{(x-4)^2(x+2)^3} \ge 0$

Critical
7, −4, 4, −2

$\dfrac{x^5}{x^5} = 1 = y = H.A.$

$+ \quad 0 \quad - \quad ☆ \quad + \quad ☆ \quad + \quad 0 \quad +$ $\quad \ge 0$

$-4 \qquad -2 \qquad 4 \qquad 7$

5pts

Y Y N N Y N Y Y Y

$= \boxed{(-\infty,-4] \cup (-2,4) \cup (4,\infty) = D}$

b 5pts Need $\dfrac{(x-7)^2(x+4)^3}{(x-4)^2(x-2)^3} > 0$

$+ \quad 0 \quad - \quad ☆ \quad + \quad ☆ \quad + \quad 0 \quad +$

$-4 \qquad -2 \qquad 4 \qquad 7$

Y N N N Y N Y N Y

$= \boxed{(-\infty,-4) \cup (-2,4) \cup (4,7) \cup (7,\infty)}$

121 J4

6 10pts $\ln(x-7)+\ln(x+3)=\ln(11)$

$\ln((x-7)(x+3))=\ln(11)$

$e^{\text{left}} = e^{\text{right}}$

$x^2-4x-21 = 11$

$x^2-4x-32=0$

$(x-8)(x+4)=0$

$x \in \{-4, 8\}$ → x tha

$\mathcal{D}(\text{problem})$: NEED $x-7>0$ & $x+3>0$

$x>7$ & $x>-3$

$\Rightarrow x>7$

$\Rightarrow$ $\boxed{x \in \{8\}}$

121 T4

7 $\frac{1}{2}$ life = 4550

a 10 pts $\boxed{A(t) = A_0 e^{kt}}$

$A(4550) = A_0 e^{4550k} = \frac{1}{2} A_0$

$e^{4550k} = \frac{1}{2}$

ln(left) = ln(right)

$4550k = \ln(\frac{1}{2}) = -\ln(2)$

$\boxed{k = \frac{-\ln(2)}{4550}} \approx -1.5234004 \times 10^{-4}$

$= \boxed{-0.00015234004 \approx k}$

b 5 pts 53% of C-14 remains.

$A_0 e^{kt} = .53 A_0$

$e^{kt} = .53$

ln(left) = ln(right)

$kt = \ln(.53)$

$t = \frac{\ln(.53)}{k} = \frac{\ln(.53)}{\frac{-\ln(2)}{4550}} = \frac{-4550 \ln(.53)}{\ln(2)}$

≈ 4167.507595 yrs

$\approx$ $\boxed{4168 \text{ yrs old}}$

121 TY

B1 10pts $|-3x-7|-8 \le -5$

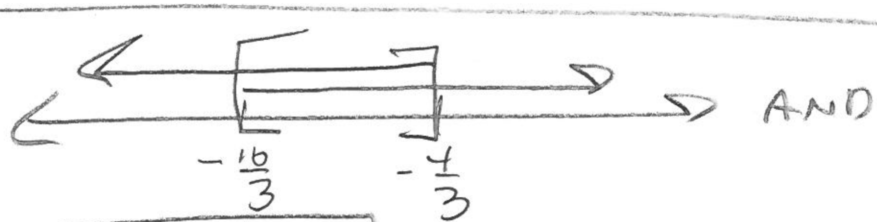
$|-3x-7| = |3x+7| \le 3$

$3x+7 \le 3$ AND $3x+7 \ge -3$

$3x \le -4$ $3x \ge -10$

$\boxed{\{x \mid x \le -\frac{4}{3} \text{ AND } x \ge -\frac{10}{3}\}}$

=



$= \boxed{[-\frac{10}{3}, -\frac{4}{3}]}$

B1 Method 2 $|-3x-7|-8 \le -5$

$|-3x-7| \le 3$

$-3x-7 \le 3$ AND $-3x-7 \ge -3$

$-3x \le 10$ $-3x \ge 4$

$\{x \mid x \ge -\frac{10}{3} \text{ AND } x \le -\frac{4}{3}\}$ = SAME.

121 T4

B2 5pts

$$f(x) = 4x^2 - 3x - 11$$

$$-160$$
$$-16$$
$$-176 \quad -9$$

$$= 4\left(x^2 - \frac{3}{4}x \qquad\right) - 11$$

$$= 4\left(x^2 - \frac{3}{4}x + \left(\frac{3}{8}\right)^2\right) - 11 - 4\left(\frac{9}{64}\right)$$

$$\left(-11 - \frac{9}{16} = \frac{(-11)(16) - 9}{16} = \frac{-176 - 9}{16} = \frac{-185}{16}\right)$$

$$= \boxed{4\left(x - \frac{3}{8}\right)^2 - \frac{185}{16} = f(x)}$$

B3 5pts

$$5 \cdot 4^x = 6 \cdot \pi^x$$

$$\ln(\text{left}) = \ln(\text{right})$$

$$\ln(5 \cdot 4^x) = \ln(6 \cdot \pi^x)$$

$$\ln(5) + \ln(4^x) = \ln(6) + \ln(\pi^x)$$

$$\ln(5) + (\ln(4))x = \ln(6) + (\ln(\pi))x$$

$$a = \ln 5, \; b = \ln 4, \; c = \ln 6, \; d = \ln \pi$$

$$\Rightarrow a + bx = c + dx$$

$$\Rightarrow bx - dx = c - a$$

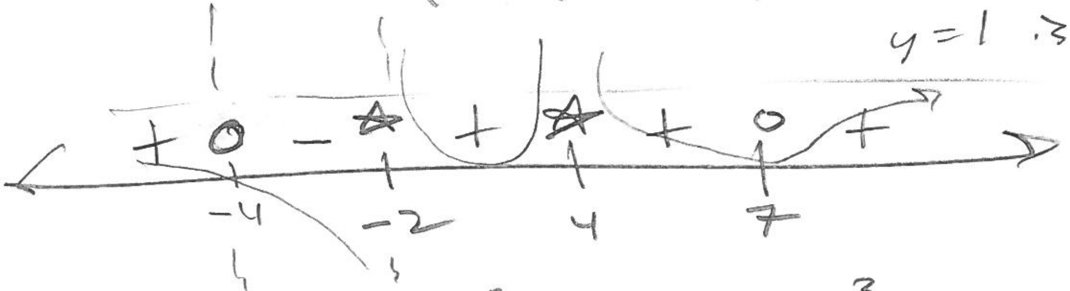
$$\Rightarrow x(b - d) = c - a$$

$$\Rightarrow x = \frac{c - a}{b - d} = \boxed{\frac{\ln(6) - \ln(5)}{\ln(4) - \ln(\pi)} = x}$$

121 TV

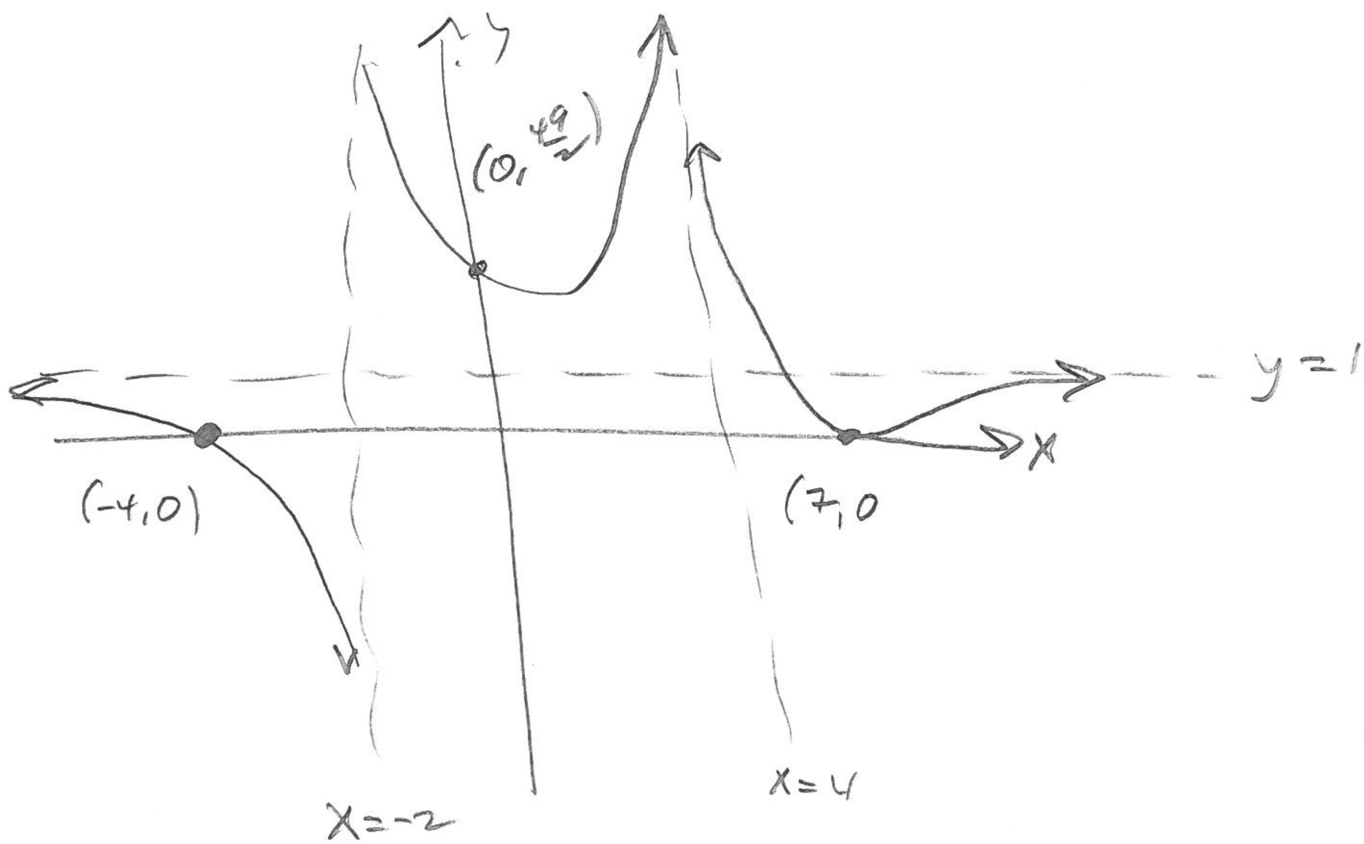
B4 $\quad R(x) = \dfrac{(x-7)^2(x+4)^3}{(x-4)^2(x+2)^3} = \dfrac{x^5+\cdots}{x^5+\cdots} \xrightarrow{|x|\to\infty} 1$

$y = 1$ is H.A.



$$R(0) = \frac{(-7)^2(4)^3}{(-4)^2(2)^3} = \frac{49\cdot 4^3}{4^2\cdot 2^3} = \frac{49\cdot 4}{2\cdot 2\cdot 2}$$

$$= \frac{49}{2} \rightarrow \left(0, \frac{49}{2}\right) \text{ y-int}$$



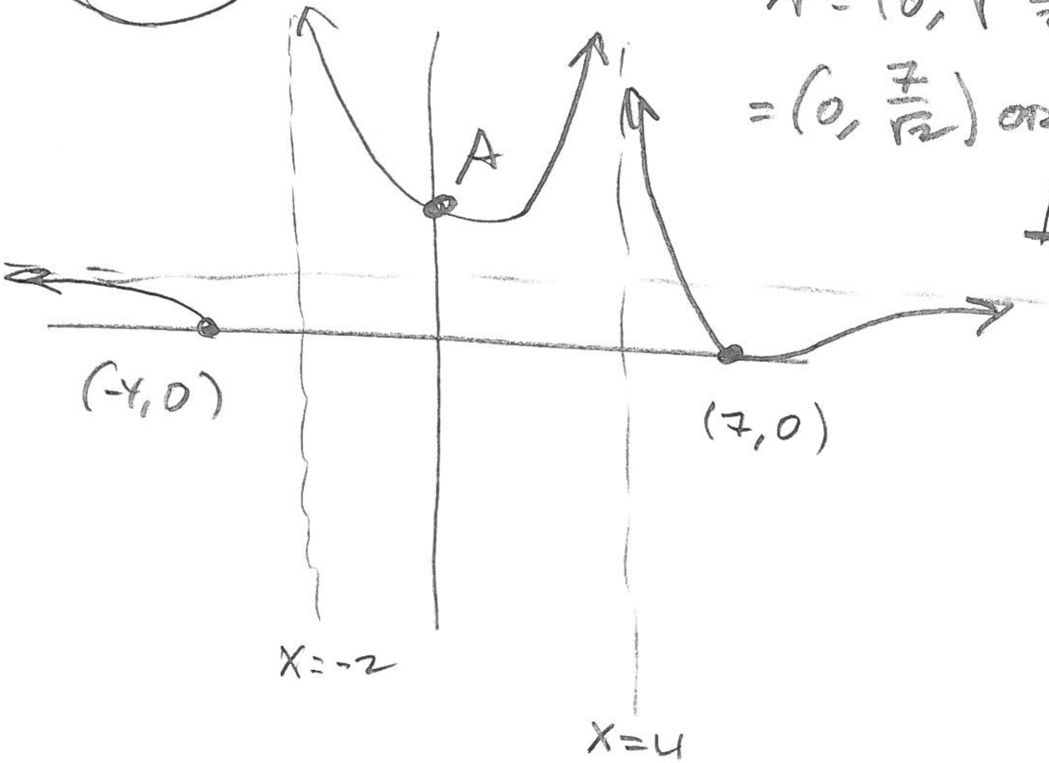
121 T4

B5

5pts

$A = \left(0, \sqrt{\tfrac{49}{2}}\right)$

$= \left(0, \tfrac{7}{\sqrt{2}}\right)$ or $\boxed{\left(0, \tfrac{7\sqrt{2}}{2}\right) = A}$



121 T4

B6

$A_0 e^{kt}$, $A_0 = 1000$

$\boxed{t = \text{\# of years after } 1992}$

$k = +0.02$

5 pts

$\boxed{A(t) = \text{Kokanee pop. in Dworshak as function of } t.}$

$2018 - 1992$

$= 26 = t$

$1000e^{.02(26)} \approx 1682.02765$

$\approx$ $\boxed{1682 \text{ Kokanee in } 2018}$

B7 5 pts

Want $A(t) = 10{,}000$

$1000\,e^{.02t} = 10000$

$e^{.02t} = 10$

$.02t = \ln(10)$

$t = \dfrac{\ln(10)}{.02} \approx 115.129546$

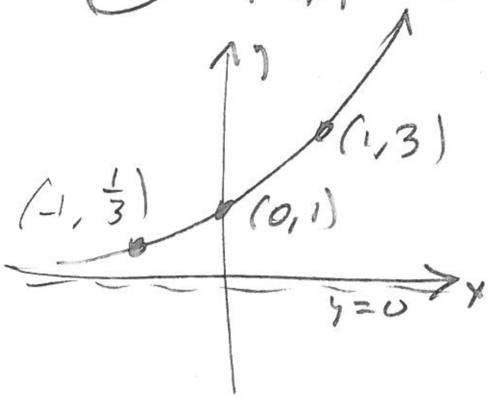
$\approx 115 \approx t \longrightarrow$

$1992 + 115 = 2107$ is when pop reaches 10,000 Kokanee

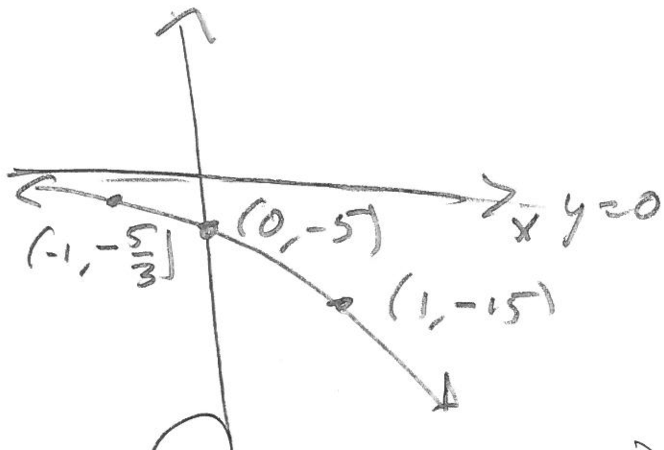
121 T4

B 8 $g(x) = -5 \cdot 3^{3x+9} + 11$

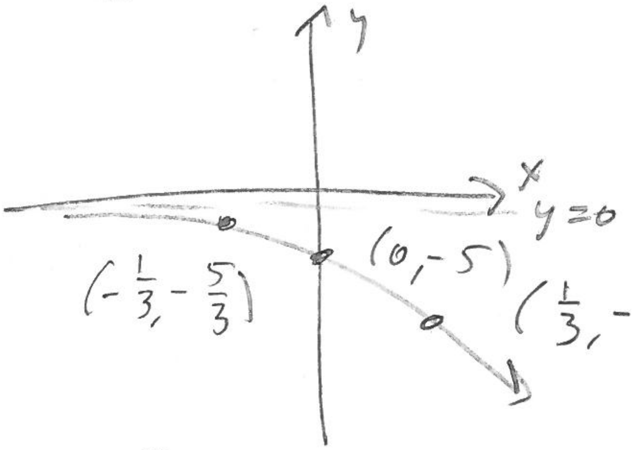
0 $f(x) = 3^x$



1 $-5f(x)$



2 $-5f(3x)$

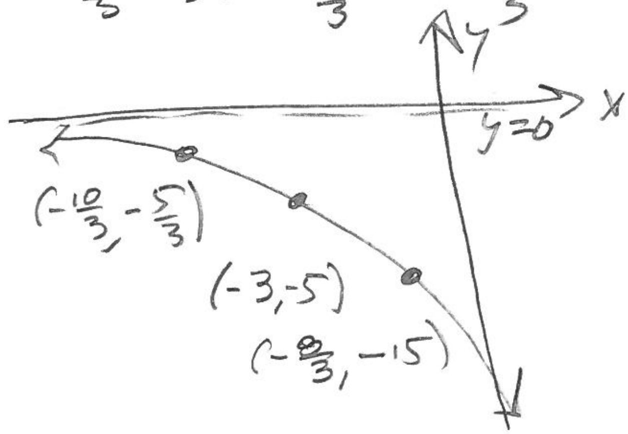


3 $-5f(3(x+3))$

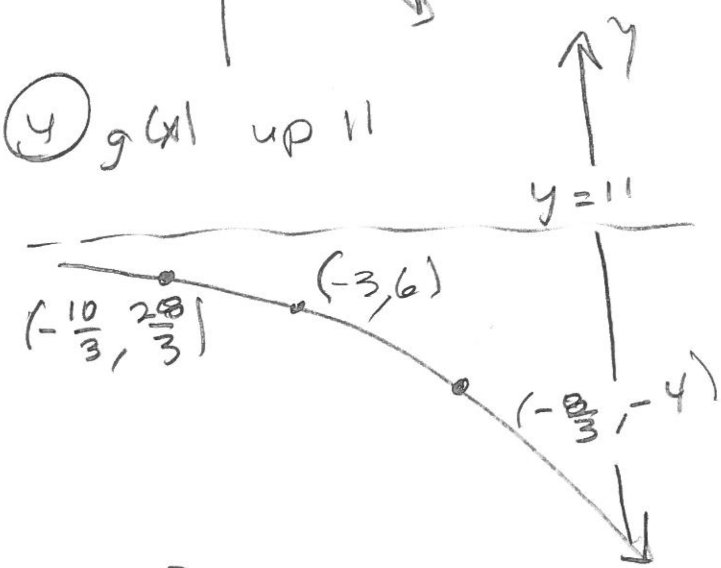
$-\frac{1}{3} - 3 = -\frac{1}{3} - \frac{9}{3} = -\frac{10}{3}$

$0 - 3 = -3$

$\frac{1}{3} - 3 = \frac{1-9}{3} = -\frac{8}{3}$



4 $g(x)$ up 11



$\frac{-5+33}{3}$

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T4

B9 FUTURE VALUE $= R\left[\frac{(1+i)^n-1}{i}\right] = F$

We know F

$r = .05 \quad t = 18$

$m = 12$

$FV = 100{,}000$

$n = mt = 12 \cdot 18 =$

$i = \frac{r}{m} = \frac{.05}{12}$

WANT R %

$R = \frac{Fi}{(1+i)^n - 1}$

$= \frac{(100{,}000)\left(\frac{.05}{12}\right)}{\left(1+\frac{.05}{12}\right)^{12\cdot 18} - 1}$

≈ 286.3671853

$\approx \boxed{\$286.37 \text{ per month} \approx R}$