MAT 121-G81-G82 Test 3 – Fall, 2018 Name________________

70 Points Covers Chapter 3 NO GRAPHING CALCULATORS!!!

Remember to use separate paper for everything except your name. Leave a margin in the top left corner. Spread out your work. Use only one column for your work. *Submit problems in order!!!*

1. (10 pts) Form a polynomial of *minimal degree* in *factored form* that has real coefficients (after expanding) and will have the given zeros. Do *not* expand your polynomial. Leave it factored! If you run out of room, you're doing it wrong!

$x = 5 - 2i$, multiplicity 1; $x = -5$, multiplicity 4; $x = 2$, multiplicity 2.

2. (10 pts) Use synthetic division to find $P(3)$ if $P(x) = 3x^5 - 4x^4 + 8x^2 - 10x + 24$

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3. (5 pts) Represent the work you just did on the previous problem by writing $P(x)$ in the form $\textit{Dividend} = \textit{Divisor} \bullet \textit{Quotient} + \textit{Remainder}$.

4. Suppose $f(x) = (x-1)\,(x+3)^2\,(x-5)\,(x+2) = x^5 + 2x^4 - 22x^3 - 68x^2 - 3x + 90$. I'm showing you both factored and expanded form to help you answer the following:

   a. (10 pts) Solve the inequality $f(x) < 0$. Your sign pattern for this one will be helpful in the next two. You just have to interpret what you're seeing.

   b. (10 pts) Provide a rough sketch of *f*, using its zeros, their respective multiplicities and its end behavior. Include *x*- and *y*-intercepts. Your graph should be smooth. Un-exaggerate the vertical for a better quality graph.

   c. (5 pts) What is the domain of $g(x) = \sqrt{\dfrac{(x+3)^2(x-5)}{(x-1)(x+2)}}$?

5. Let $f(x) = 4x^5 + 32x^4 + 83x^3 + 55x^2 - 75x - 99$

   a. (5 pts) Use Descartes' Rule of Signs to determine the possible number of positive and negative zeros of *f*.

   b. (5 pts) List all possible rational zeros of *f*.

   c. (Bonus 5 pts) Find the smallest possible integer bounds on positive and negative zeros.

6. (10 pts) Find the *real* zeros of $f(x) = 4x^5 + 32x^4 + 83x^3 + 55x^2 - 75x - 99$. Then factor *f* over the set of **real numbers**. This should involve an irreducible quadratic factor.

(If things go haywire, come up with a *plausible*-looking polynomial, in factored form, with the right number of real roots and 2 nonreal roots. The 2 nonreal roots will still be living inside the irreducible quadratic factor, so you'll have to make up a quadratic factor with nonreal zeros).

7. (5 pts) Find the remaining (nonreal) zeros of f and factor f over the set of **complex numbers**. This step requires breaking down the quadratic piece that's irreducible over the real numbers. The fundamental theorem tells us that *nothing* is irreducible over the complex numbers.

   (You can still get full credit for this one, even if things went haywire, in #6, if you solve the depressed equation, correctly, and display your *plausible*-looking follow-up to your *plausible*-looking answer to #6. The more you know about what you're pointing towards, the more points you'll earn.)

8. (5 pts) You don't need to graph $R(x)=\dfrac{2x^3+7x^2-24x-45}{3x^2+x-10}$, here, but I do want to see its asymptotes.

   Hints: This function has no holes. Also, do not expect nice integer coefficients in your result.

9. (10 pts) Sketch the graph of $F(x)=\dfrac{2x^2-3x-9}{3x^2+x-10}$. Show all asymptotes and intercepts.

ANSWER ANY TWO (2) OF THE FOLLOWING, FOR UP TO 20 BONUS POINTS!!!

B1 (10 pts) Form a polynomial of *minimal degree* in *factored form* that has ***rational*** coefficients (after expanding) and will have the given zeros. Do *not* expand your polynomial. Leave it factored! If you run out of room, you're doing it wrong.

Zeros: $x=2+\sqrt{2}$, multiplicity 2;
$x=1-4i$, multiplicity 1;
$x=7$, multiplicity 5.

B2 Solve both of the following absolute value inequalities.

a. (5 pts) $|2x+7|+8<9$

b. (5 pts) $|3x+11|+19>10$

B3 (10 pts) Sketch the graph of $R(x)=\dfrac{2x^3+7x^2-24x-45}{3x^2+x-10}$

Hints:

a. You already found $R(x)$'s asymptotes in #8.

b. One of $R(x)$'s x-intercepts is $(3,0)$.

B4 (10 pts) Sketch the graph of $G(x)=\dfrac{2x^3+7x^2-24x-45}{3x^3+16x^2-5x-50}$. Hint: $G(x)$ looks exactly like $F(x)$, from #9, except it has a hole.

B5 If $f(x)=\sqrt{x+11}$ and $g(x)=\dfrac{2}{x-6}$, what is the domain of $f\circ g$?