Problems out of order: 10 points off; Writing too faint: 10 points off; No margin in top left corner: 10 poi[n]
off.

1. Solve the equation $x^2+3x-28=0$ in three different ways:
   a. (10 pts) Factoring
   b. (10 pts) Completing the Square
   c. (10 pts) Quadratic Formula

Final Test– Spring, 2017

$2 \mid 28$, $2 \mid 14$, 7

$7-4=3$!

(a)

$x^2+3x-28$
$= x^2+7x-4x-28$
$= x(x+7)-4(x+7)$
$= (x+7)(x-4)=0$

$\boxed{x \in \{-7,4\}}$

(b)

$x^2+3x-28 = x^2+3x+\left(\frac{3}{2}\right)^2-\frac{9}{4}-\frac{112}{4}$
$= \left(x+\frac{3}{2}\right)^2-\frac{121}{4} \overset{SET}{=} 0 \longrightarrow$

$\left(x+\frac{3}{2}\right)^2 = \frac{121}{4}$

$x+\frac{3}{2} = \pm\sqrt{\frac{121}{4}} = \pm\frac{11}{2}$

$x = -\frac{3}{2} \pm \frac{11}{2}$ ↗ $\frac{8}{2}=4$ ↘ $\frac{-14}{2}=-7$ $\longrightarrow$ $\boxed{x \in \{-7,4\}}$

(c)

$x^2+3x-28=0$

$a=1,\ b=3,\ c=-28$

$b^2-4ac = 3^2-4(1)(-28) = 9+112 = 121$

$\sqrt{121}=11$

$\Rightarrow x = \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-3\pm 11}{2} =$ ↗ $\frac{8}{2}=4$ ↘ $\frac{-14}{2}=-7$

See?

Nov 30-8:25 AM

$|-5x-8|>4$

$|-5x-8|\le 4$

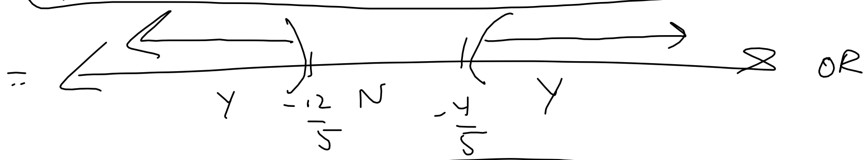
$|-5x-8|>4$

$-5x-8>4$ OR $-5x-8<-4$

$-5x>12$ OR $-5x<4$

$$\boxed{\left\{x \mid x<-\frac{12}{5} \text{ OR } x>-\frac{4}{5}\right\}}$$

=



$$= \boxed{\left(-\infty, -\frac{12}{5}\right) \cup \left(-\frac{4}{5}, \infty\right)}$$

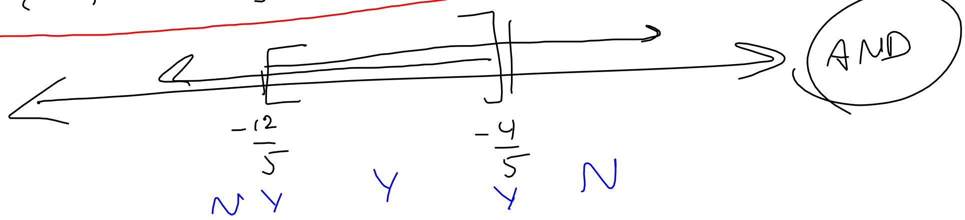
$|-5x-8| \leq 4$

$-5x-8 \leq 4$ AND $-5x-8 \geq -4$

$-5x \leq 12$ AND $-5x \geq 4$

$\boxed{\left\{ x \mid x \geq -\frac{12}{5} \text{ AND } x \leq -\frac{4}{5} \right\}}$

=



$= \boxed{\left[-\frac{12}{5}, -\frac{4}{5}\right]}$

. Let $f(x)=\sqrt{x+16}$ and $g(x)=x^2-4x-21$.

(a) $\mathcal{D}(f)$ Need $x+16\geq 0$ $\Rightarrow$ $x\geq -16$ $\rightarrow$ $\boxed{\mathcal{D}=[-16,\infty)}$

(b) $\mathcal{D}(g)=(-\infty,\infty)$ (B) $\mathcal{D}\left(\frac{f}{g}\right)=[-16,\infty)\cap\{x \mid g(x)\neq 0\}$

(c) $\frac{f}{g}=\frac{\sqrt{x+16}}{x^2-4x-21}$

$x^2-4x-21=0$
$(x-7)(x+3)=0$
$x\in\{-3,7\}$

Intersect "in both"

AND

-16 $\quad -3 \quad 7$

N Y Y N Y N Y

$\boxed{=[-16,-3)\cup(-3,7)\cup(7,\infty)}$

(d) $\boxed{f\circ g=}\,f(g(x))=\sqrt{g(x)+16}$

$\boxed{=\sqrt{x^2-4x-21+16}}=\sqrt{x^2-4x-5}=\sqrt{(x-5)(x+1)}$

(e) $\mathcal{D}(f\circ g)$. Need $x^2-4x-5\geq 0$

$+ \ 0 \ - \ 0 \ +$

$-1 \quad 5$

Y Y N Y Y

$\boxed{(-\infty,-1]\cup[5,\infty)=\mathcal{D}(f\circ g)}$

$\sqrt{x^2-21}$
Need $x^2-21\geq 0$
$(x-\sqrt{21})(x+\sqrt{21})\geq 0$

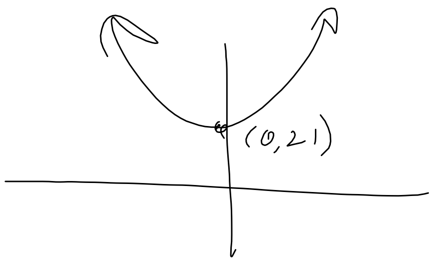
≥ 0 $\quad + \ - \ +$

$-\sqrt{21} \quad \sqrt{21}$

$(-\infty,-\sqrt{21}]\cup[\sqrt{21},\infty)$

What if it's $\sqrt{x^2+21}$?

Need $x^2+21\geq 0$ & it's ALWAYS ≥ 0



So $\mathcal{D}\left(\sqrt{x^2+21}\right)=\mathbb{R}$
$=(-\infty,\infty)$

x^2+21

$+$

What is the domain of $f(x)=\sqrt{(x+3)^{15}(x-1)^4(x-5)}$? 1

What is the domain of $g(x)=\log_{1.7}\left((x+3)^{15}(x-1)^4(x-5)\right)$? 2

What is the domain of $g(x)=\sqrt{\dfrac{(x+3)^{15}(x-1)^4}{(x-5)}}$ 3

1 Need $(x+3)^{15}(x-1)^4(x-5)\ge 0$ $x^{15+4+1} = x^{20}$

$+\ 0\ -\ 0\ -\ 0\ +$

$-3 \quad 1 \quad 5$ ≥ 0

Y Y N Y N Y Y

$\boxed{(-\infty,-3]\cup\{1\}\cup[5,\infty)}$

2 Need $(x+3)^{15}(x-1)^4(x-5) > 0$

$+\ 0\ -\ 0\ -\ 0\ +$

$-3 \quad 1 \quad 5$ > 0

Y N N N N N Y

$= (-\infty,-3)\cup(5,\infty)$

3 Need $\dfrac{(x+3)^{15}(x-1)^4}{x-5}\ge 0$

$+\ 0\ -\ 0\ -\ \cancel{0}\ +$

$-3 \quad 1 \quad 5$ ≥ 0

Y Y N Y N N N

$= \boxed{(-\infty,-3]\cup\{1\}\cup(5,\infty)}$

7. (10 pts) Use synthetic division to find $P(2)$, for $P(x) = 2x^5 - 5x^4 - 2x^2 + 6x - 13$

$0x^3$

$$\begin{array}{r|rrrrrr} 2 & 2 & -5 & 0 & -2 & 6 & -13 \\ & & 4 & -2 & -4 & -12 & -12 \\ \hline & 2 & -1 & -2 & -6 & -6 & \boxed{-25 = P(2)} \end{array}$$

#8 Geometric Series
Bonus only

10. (10 pts) What is the future value, in 10 years, of \$5,000 deposited at 5% annual percentage rate, compounded weekly? (Recall, there are 52 weeks in a year.).

$t = 10$, $P = 5000$, $r = .05$, $m = 52$

$A = P(1+\frac{r}{m})^{mt} = 5000(1+\frac{.05}{52})^{52 \cdot 10}$

$\approx 8241.626224 \approx \boxed{\$8241.63}$

New Wrinkle.

Same everything, except you want to end up with \$10000 in 10 years. How much did you start with?

$P(1+\frac{r}{m})^{mt} = A$, but we want P = Present Value,

$P(1+\frac{.05}{52})^{52 \cdot 10} = 10{,}000 \Rightarrow \frac{10000}{(1+\frac{.05}{52})^{520}} = 10000(1+\frac{.05}{52})^{-520}$

$P = A(1+\frac{r}{m})^{-mt}$

```
5000(1+.05/52)^(
52*10)
        8241.626224
10000(1+.05/52)^
-(52*10)
        6066.763845
```

start with \$5000 what's A?

End with \$10000 what's P?

From the 2011 Final. Some things have changed. We mark up what's shifted to Bonus.

1. Solve the equation $x^2 - 2x - 15 = 0$ in three different ways:

part a (10 pts) Factoring

All Good Stuff

part b (15 pts) Completing the square

part c (15 pts) Quadratic formula

2. Solve the absolute value inequality. Give your answer in set-builder *and* interval notation.

part a (10 pts) $|7x+2| \geq 4$ **part b** (10 pts) $|2x-7| < 4$

3. Let $f(x) = \sqrt{x-14}$ and $g(x) = x^2 - 3x - 14$

part a (15 pts) What's the domain of $f(x)$? Give the answer in set-builder and interval notation.

part b (15 pts) Determine $(f \circ g)(x)$. Simplify your answer.

part c (5 pts) What's the domain of $(f \circ g)(x)$? Give your answer in set-builder and interval notation.

4. (15 pts) Solve $(x-2)^3(x+1)(x-4)^2 > 0$. Give the solution set in interval notation.

5. (10 pts) What is the domain of $h(x) = \sqrt{(x-2)^3(x+1)(x-4)^2}$?

6. (10 pts) What is the domain of $\sqrt{\dfrac{(x-2)^3(x+1)}{(x-4)^2}}$? The hard part's done...

7. (10 pts) Use synthetic division to find $f(3)$ for $f(x) = x^4 - 5x^3 - 3x^2 + 43x - 60$

Bonus Material from Chapter 8:

8. (10 pts) Determine a, r and n for the finite geometric sequence $2, \frac{4}{5}, \frac{8}{25}, \ldots, \frac{256}{78125}$

Use a, r, and n to determine the sum by the formula $\sum_{k=1}^{n} a \cdot r^{k-1} = a\left(\frac{1-r^n}{1-r}\right)$. A fractional answer is better, but I'll give you most of the points if you provide a decimal answer that is accurate to 4 decimal places.

9. (10 pts) Find the sum of the infinite series $\sum_{k=1}^{\infty} 4 \cdot \left(\frac{3}{4}\right)^{k-1} = 4 + 4 \cdot \frac{3}{4} + 4 \cdot \left(\frac{3}{4}\right)^2 + \ldots\ldots$

Will Count:

10. (10 pts) What's the future value, in 10 years, of \$10,000 deposited into a savings account, earning 4.3% annual percentage rate, compounded daily?

This is Bonus:

11. (10 pts) An annnuity consists of monthly payments of \$600 into an account earning 8.4% annual interest, compounded monthly, for 10 years. There are two ways to ask this question:

First way: How much does JG Wentworth feel that this annuity is worth?

Second way: If the annuity described is actually your monthly loan payments, how much did you borrow in the first place?

Totally legit question. I can ask about ANY basic function, here, and NOT just an exponential or a logarithmic function.

12. (15 pts) Sketch the graph of $g(x) = 3 \cdot 2^{2x-4} - 5$ by transforming the basic function $f(x) = 2^x$. Show x- and y-intercepts.

Bonus (10 pts) Use Pascal's Triangle (Binomial Theorem!) to help you expand $(x-2)^4$. Expanding without using this technique will not earn any points.

I think I will be making Pascal's Triangle question a non-bonus, regular question.

8.4 Permutations

$P(n,k)$ means choose and arrange k things out of n things.

$\{a,b,c,d,e,f,g\}$ Choose and arrange 3 things out of seven things

$$P(7,3) = \frac{7!}{(7-3)!} = \frac{7!}{4!} = \frac{7\cdot6\cdot5\cdot\cancel{4\cdot3\cdot2}}{\cancel{4\cdot3\cdot2}} = 210$$

$\{a,b,c,d\}$ $P(4,2)$

ab, ac, ad, ba, bc, bd, ca, cb, cd, da, db, dc
1 2 3 4 5 6 7 8 9 10 11 12

$$P(4,2) = \frac{4!}{(4-2)!} = \frac{4!}{2!} = 4\cdot3 = 12$$

$P(7,3)$: $\underline{7} \cdot \underline{6} \cdot \underline{5}$

Combos : ORDER MATTERS NOT.

Divide $P(n,k)$ by the # of ways to rearrange things

$\underline{3} \cdot \underline{2} \cdot \underline{1} = 3! = 6$ ways to arrange 3 things

$$P(7,3) = \frac{7!}{(7-3)!}$$

$$C(7,3) = \text{"7 choose 3"} = \frac{7!}{(7-3)!\,3!}$$

$$P(n,k) = \frac{n!}{(n-k)!}$$

$$C(n,k) = \frac{n!}{(n-k)!\,k!}$$

$C(4,2) = \binom{4}{2}$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

Bonus (10 pts) Use Pascal's Triangle (Binomial Theorem!) to help you expand $(x-2)^4$. Expanding without using this technique will not earn any points.

$1x^4 + 4x^3(-2)^1 + 6(x^2)(-2)^2 + 4x^1(-2)^3 + (-2)^4$

$C(n,k) = \binom{n}{k}$ $(x+y)^n$

$$\sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k = x^n + \binom{n}{1}x^{n-1}y + \binom{n}{2}x^{n-2}y^2 + \dots + \binom{n}{n}x^0 y^n.$$

$$\binom{4}{0}x^4(-2)^0 + \binom{4}{1}x^{4-1}(-2)^1 + \binom{4}{2}x^{4-2}(-2)^2 + \binom{4}{3}x^{4-3}(-2)^3 + \binom{4}{4}x^{4-4}(-2)^4$$

$1\,x^n \cdot y^0$ $1\,x^0 \cdot y^n$