$2^x = 9$

$\log_2(2^x) = \log_2(9)$

$x \cdot \log_2(2) = \log_2(9)$

$x = \log_2(9) = \dfrac{\ln(9)}{\ln(2)} \approx 3.169925001 \approx \boxed{3.1699}$

```
ln(9)/ln(2)
      3.169925001
log(9)/log(2)
      3.169925001
```

Common mistake:

$\dfrac{\ln(9)}{\ln(2)} = \boxed{\ln\left(\frac{9}{2}\right) \text{ FALSE}} = \ln(4.5)$ is bad

$\log\left(\frac{A}{B}\right) = \log(A) - \log(B)$

All comes from 055/099 properties of exponents.

$A^B A^C = A^{B+C}$

$\log(AB) = \log(A) + \log(B)$

$\dfrac{A^B}{A^C} = A^{B-C}$

$(A^B)^C = A^{BC}$

$\log(A^B) = B\log(A)$

$\log(A^B)$

GUARANTEED TEST QUESTION LIKE THIS ONE.

RADIOMETRIC DATING IS THE BIG APPLICATION FOR US.

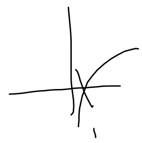
The half-life of Millsium is 50 yrs. How old is a sample of Millsium that's decayed to the point where only 20% of the radioactive Millsium remains? $A(t) = A_0 e^{kt}$

A_0 = initial amount
k = Growth/decay rate

THE $\frac{1}{2}$-life equation:

A_0 = starting amount

$$A_0 e^{50k} = \frac{1}{2} A_0$$



$A(50) =$

$$e^{50k} = \frac{1}{2}$$

Check it out!

$\ln(\frac{1}{2}) = \ln(2^{-1})$

$-1 \ln(2)$

$$\ln(e^{50k}) = \ln(\tfrac{1}{2})$$

$$50k = \ln(\tfrac{1}{2})$$

$$k = \frac{\ln(\frac{1}{2})}{50} = \boxed{-\frac{\ln(2)}{50}} \approx .013862946$$

MODEL: $A(t) = A_0 e^{kt}$.

```
ln(9)/ln(2)
         3.169925001
log(9)/log(2)
         3.169925001
ln(2)/50
         .0138629436
```

So, when does it get to 20%?

$A(t) = .2A_0$?

$A(t) = A_0 e^{-\frac{\ln(2)}{50}t}$

$$A_0 e^{kt} = .2A_0$$

$\approx A_0 e^{-.013862946t}$

$$e^{kt} = .2$$

Don't do dad

```
         3.169925001
log(9)/log(2)
         3.169925001
ln(2)/50
         .0138629436
50ln(.2)/ln(2)
        -116.0964047
```

$$\ln(e^{kt}) = \ln(.2)$$

$$kt = \ln(.2)$$

$$\boxed{t = \frac{\ln(.2)}{k}} = \frac{\ln(.2)}{-\frac{\ln(2)}{50}} = \frac{-50\ln(.2)}{\ln(2)}$$

Guesstimate with $\frac{1}{2}$-lives:
Start w/ 100%

≈ 116.0964047 seems not very long. But it slots where it should.

0	100%
50	50%
100	25%
150	12.5%

$.2A_0$ is 20% of what you started with.

$.2A_0$ = 20% remaining fits where it should @ 116 years.

POPULATION IS GROWING AT $\boxed{\text{4\% ANNUALLY.}}$

IF THE POPULATION IS 500,000, TODAY, WHAT WILL IT BE IN 20 YEARS?

ASSUME POPULATION OBEYS THE LAW OF UNINHIBITED GROWTH.

(THAT MEANS EXPONENTIAL GROWTH!)

$K = .04$

$$A(t) = A_0 e^{kt} = A_0 e^{.04t}$$

NICE WHEN THEY HAND YOU THE GROWTH RATE. I MAKE YOU WORK FOR IT.

t = number of years from now.

A_0 = initial amt / # of people = 500,000, so

$$A(t) = 500{,}000\, e^{.04t}$$

```
ln(2)/50
        .0138629436
50ln(.2)/ln(2)
       -116.0964047
500000e^(20*.04)

        1112770.464
```

In 20 years:

$$A(20) = 500000 e^{.04 \cdot 20} \approx$$

$$\approx 1112770$$

$$\approx 1{,}112{,}770 \text{ people.}$$

What's Doubling Time?

$$A(t) = 2A_0$$

$$A_0 e^{kt} = 2A_0$$

$$A_0 e^{.04t} = 2A_0$$

$$e^{.04t} = 2$$

$$\ln(e^{.04t}) = \ln(2)$$

$$.04t = \ln(2)$$

$$t = \frac{\ln(2)}{.04} \approx 17.32867951$$

or about 17 years.

Half-life:

$$A_0 e^{kt} = \frac{1}{2} A_0$$

```
50ln(.2)/ln(2)
     -116.0964047
500000e^(20*.04)

      1112770.464
ln(2)/.04
      17.32867951
```

POPULATION WAS 100,000 IN 1980

IT'S 400,000, TODAY. WHEN WILL IT HIT A MILLION?

Let t = the # of years after 1980

$1980 \longrightarrow t=0$. $A_0 = 100{,}000$

TODAY: $2018 \longrightarrow 2018-1980 = 38$

So $A(t) = A_0 e^{kt} = 100{,}000 e^{kt}$

And $A(38) = 100{,}000 e^{38k} = 400{,}000$

$$\Longrightarrow e^{38k} = \frac{400\,000}{100\,000} = 4$$

$$\ln(e^{38k}) = \ln(4)$$

$$38k = \ln(4)$$

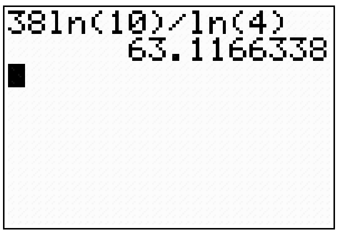
$$\boxed{k = \frac{\ln(4)}{38}} \approx$$

Final piece: when will it hit 1,000,000?

$$A_0 e^{kt} = 1{,}000{,}000$$

$$100\,000 e^{kt} = 1{,}000{,}000$$

$$e^{kt} = 10$$



$$\ln(e^{kt}) = \ln(10)$$

$$kt = \ln(10)$$

$$t = \frac{\ln(10)}{k} = \frac{\ln(10)}{\frac{\ln(4)}{38}} = \boxed{\frac{38\ln(10)}{\ln(4)} = t}$$

The I don't have a calculator answer.

$$\approx 63.1166338 \text{ yrs} \approx \boxed{63 \text{ years}}$$

71. *Dating a Bone* A piece of bone from an organism is found to contain 10% of the carbon-14 that it contained when the organism was living. If the half-life of carbon-14 is 5730 years, then how long ago was the organism alive?
HINT First find the rate of decay for carbon-14.

72. *Old Clothes* If only 15% of the carbon-14 in a remnant of cloth has decayed, then how old is the cloth?
HINT Use the decay rate for carbon-14 from the previous problem.

$$A_0 e^{kt} = A(t)$$

$\frac{1}{2}$-life is 5730

$$A_0 e^{5730k} = \frac{1}{2} A_0$$

$$e^{5730k} = \frac{1}{2}$$

$$\ln(e^{5730k}) = \ln\left(\frac{1}{2}\right) = -\ln(2)$$

$$5730k = -\ln(2)$$

$$\boxed{k = \frac{-\ln(2)}{5730}}$$

73. *Dating a Tree* How long does it take for 12 g of carbon-14 in a tree trunk to be reduced to 10 g of carbon-14 by radioactive decay?

$A = A(t)$ is the amount of radioactive C-14, in g.

$$12e^{kt} = 10$$

$$e^{kt} = \frac{10}{12} = \frac{5}{6}$$

$$\ln(e^{kt}) = \ln\left(\frac{5}{6}\right)$$

I left this out

$$kt = \frac{5}{6}$$

Nope

```
38ln(10)/ln(4)
        63.1166338
5*5730/(6*ln(2))
        6888.86882
```

$$t = \frac{5}{6k} = \frac{5}{6\left(-\frac{\ln(2)}{5730}\right)} = -\frac{5 \cdot 5730}{6\ln(2)}$$

Too many years! ~~Grow~~ Decaying from 12g to 10g is less than a half-life. Shoot, it'll be down to 6g in 5730 years! So should take *less* time to reach 10g!

Fix:

$$e^{kt} = \frac{5}{6}$$

$$\ln(e^{kt}) = \ln\left(\frac{5}{6}\right)$$

$$kt = \ln\left(\frac{5}{6}\right)$$

$$t = \frac{\ln\left(\frac{5}{6}\right)}{k} = \frac{\ln\left(\frac{5}{6}\right)}{\frac{-\ln(2)}{5730}} = \frac{-5730\ln\left(\frac{5}{6}\right)}{\ln(2)}$$

```
        63.1166338
5*5730/(6*ln(2))
        6888.86882
5730*ln(5/6)/ln(
2)
       -1507.187145
```

$$\approx 1507.187145$$

$$\approx \boxed{1508 \text{ yrs}}$$

Let $u = \log(z)$.

Then $\log(z^2) = 2\log(z)$ & so

$$(\log(z))^2 = 2\log(z)$$

$\Rightarrow$ $u^2 = 2u$

$u^2 - 2u = u(u-2) = 0 \Rightarrow$

$u = 0$ or $u = 2$

$\log(z) = 0$ | $\log(z) = 2$

$10^{\log(z)} = 10^0$ | $10^{\log(z)} = 10^2$

$z = 1$ | $z = 100$