After Test 3, there remain 16 sections in the book. After this week, there are 4 more weeks of class before the final week.

4 sections per week puts you in pretty good shape.

If you're 3 sections into Chapter 4, already, you're tip-top.

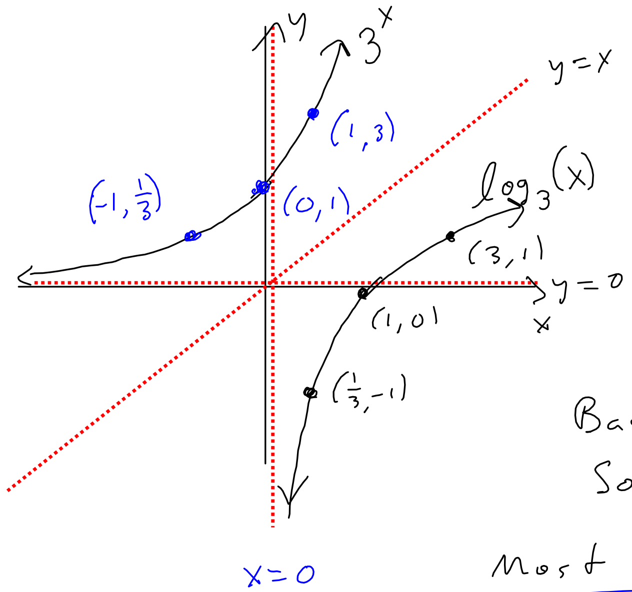
Final test has a ton of bonus from old stuff.

Section 1.7 - Always (and almost nobody gets it)

Completing the square (one with fractions in it, fer sherz)

Look at all the old Test 4s and old Finals. Test yourself on them. Check your work. Repeat. You're in TRAINING. So train.

HAVE YOU DONE YOUR TEST 3 CORRECTIONS, YET? THAT'S WHAT GOOD STUDENTS DO AFTER EVERY TEST.



$f(x) = 3^x$

$g(x) = \log_3(x) = f^{-1}(x)$

because

$\log_3(3^x) = x$

$3^{\log_3(x)} = x$

Basic idea: Solving equations

Most Basic

$3^x = 5$

Show me! $\log_3(3^x) = \log_3(5)$

$\frac{1}{3^x} = 81$

$3^{-x} = 81 = 3^4$

$\log_3(3^{-x}) = \log_3(3^4)$

$-x = 4$

$x = -4$

3 | 81
3 | 27
3 | 9
3

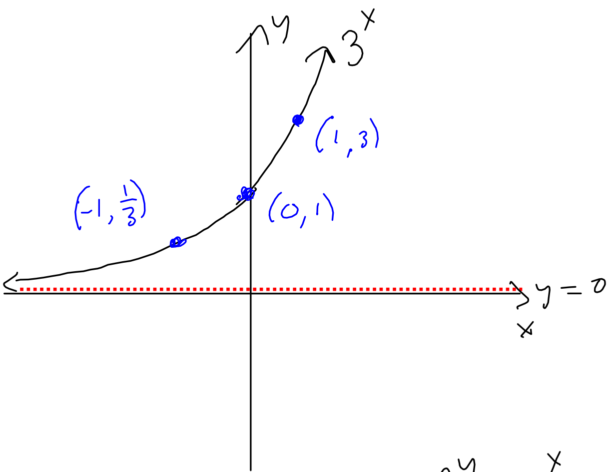
$\log_3(x) = 7$

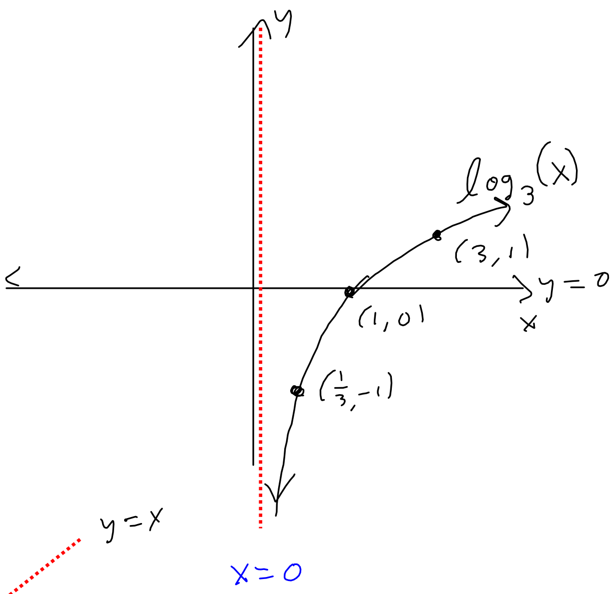
$3^{\log_3(x)} = 3^7$

$x = 3^7$

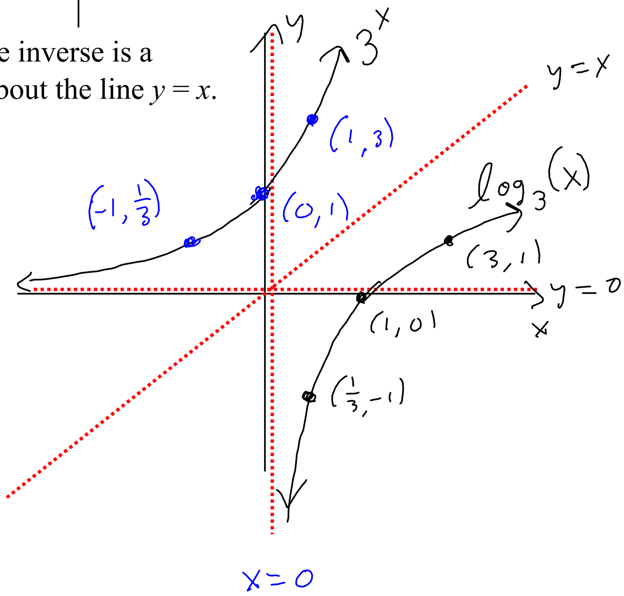
Graphing

$3 \cdot \log_5(4x - 24) + 7$





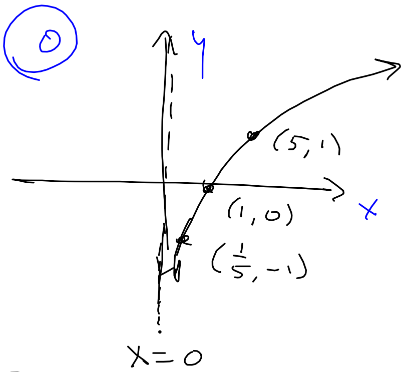
Graph of the inverse is a reflection about the line $y = x$.



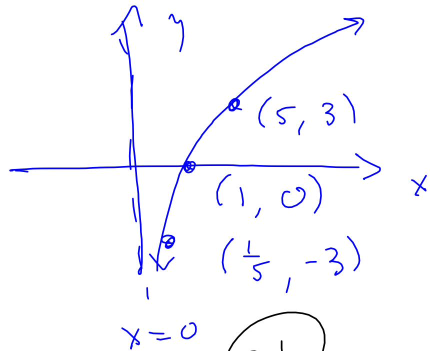
HAVE YOU DONE YOUR TEST 3 CORRECTIONS, YET? THAT'S WHAT GOOD STUDENTS DO AFTER EVERY TEST.

$3 \cdot \log_5 (4x - 24) + 7$

(0) $\log_5(x) = f(x)$



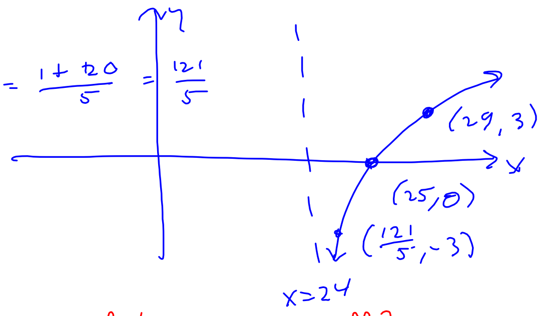
(1) $3\log_5(x) = 3f(x)$



(2a)

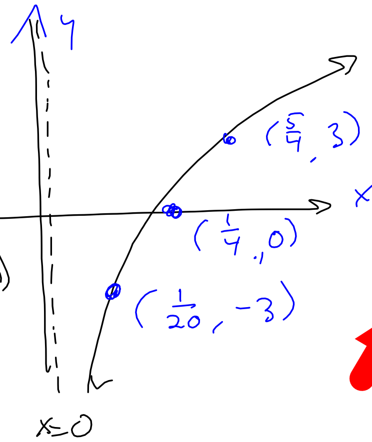
(M1) $3f(x-24) = 3\log_5(x-24)$

$\frac{1}{5} + 24 = \frac{1+120}{5} = \frac{121}{5}$



(2b)

(M2) $3f(4x) = 3\log_5(4x)$



$4x - 24$
$= 4(x-6)$

(3) M1 M2 $3f(4x-24) = 3f(4(x-6)) = 3\log_5(4x-24)$
$= 3\log_5(4(x-6))$

$x \to \frac{x}{4}$ $x \to x+6$

$\frac{1}{20} + 6 = \frac{1+120}{20} = \frac{121}{20}$

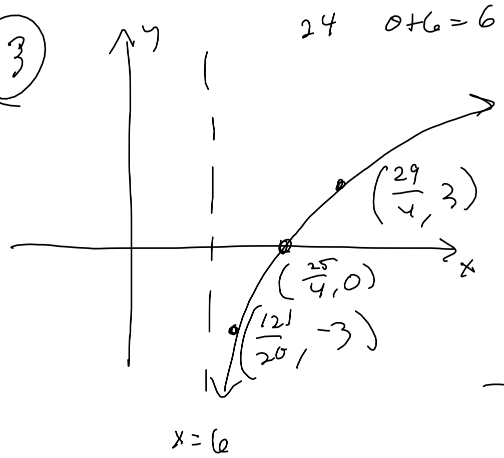
$\frac{\frac{121}{5}}{4} = \frac{121}{5} \cdot \frac{1}{4} = \frac{121}{20}$

$\frac{1}{4} + 6 = \frac{1+24}{4} = \frac{25}{4}$

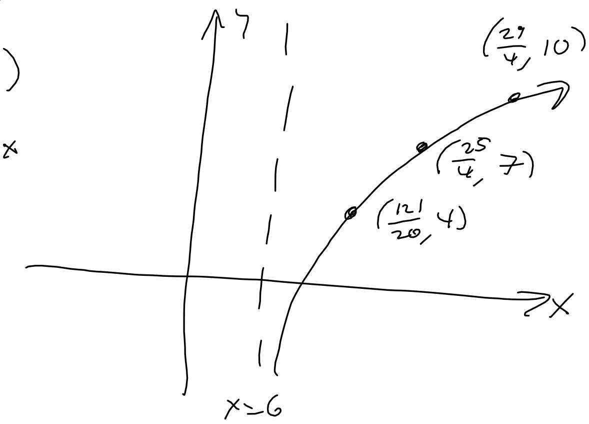
$\frac{25}{4}$ $\frac{29}{4}$ $\frac{24}{4} = 6$ $\frac{5}{4} + 6 = \frac{5+24}{4} = \frac{29}{4}$

24 $0 + 6 = 6$

(3)



(4) $g(x)$ up 7 :



The half-life of Millsium is 50 yrs.

How old is a sample of Millsium that's decayed to the point where only 20% of the radioactive Millsium remains?

THE $\frac{1}{2}$-life equation:

A_0 = starting amount

$$A(50) = A_0 e^{50k} = \frac{1}{2} A_0$$

$$e^{50k} = \frac{1}{2}$$

$$\ln(e^{50k}) = \ln\left(\frac{1}{2}\right)$$

$$50k = \ln\left(\frac{1}{2}\right)$$

$$k = \frac{\ln\left(\frac{1}{2}\right)}{50}$$

MODEL: $A(t) = A_0 e^{kt}$.

So, when does it get to 20%?

$$A(t) = .2A_0 \ ?$$

$$A_0 e^{kt} = .2A_0$$

$$e^{kt} = .2$$

$$\ln(e^{kt}) = \ln(.2)$$

$$kt = \ln(.2)$$

$$\boxed{t = \frac{\ln(.2)}{k}}$$

$$\log(AB) = \log(A) + \log(B)$$

$$\log(A^B) = B\log(A)$$

$$\log\left(\frac{x^2\sqrt[5]{y}}{z^7}\right) = \log(x^2) + \log(y^{\frac{1}{5}}) - \log(z^7)$$

$$= 2\log(x) + \frac{1}{5}\log(y) - 7\log(z)$$

$$A^{B\cdot C} = (A^B)^C = (A^C)^B$$

$$e^{5\ln(7)} = e^{\ln(7)\cdot 5} = \left(e^{\ln(7)}\right)^5 = 7^5$$

$$e^{\ln(7)} = 7$$

$$5^{2\cdot 3} = (5^2)^3 = (5^3)^2$$

Logs turn powers into products & products into sums. Computers love them, because computers are, at heart, just adding machines

$10^6 \cdot 5^7$:

$\log(10^6 \cdot 5^7)$

$= 6\log(10) + 7\log(5)$

Then do

$10^{6\log 10 + 7\log(5)}$

for $10^6 \cdot 5^7$

Computer can carry a table of logarithm values in a data base. So it turns big multiplications into a table search and a couple of additions.

$3^4 \cdot 5^3 = 81(125) = 10{,}125$

$\log(\quad) = 4\log(3) + 3\log(5)$

$\approx 1.908485019 + 2.096910013$

$= 4.005395032$

```
    1.908485019
3log(5)
    2.096910013
Ans+4log(3)
    4.005395032
10^Ans
          10125
```

& $10^{4.005395032} \approx 10{,}125$!

Big numbers? Logs knock 'em down to size.

$$\log_2(\sqrt{20}) = \log_2\left(20^{\frac{1}{2}}\right) = \frac{1}{2}\log_2(20) = \frac{1}{2}\log(2^2 \cdot 5)$$

$$= \frac{1}{2}\log_2(4 \cdot 5) = \frac{1}{2}\left[\log_2(4) + \log_2(5)\right]$$

$$= \frac{1}{2}\log_2(2 \cdot 2) + \frac{1}{2}\log_2(5)$$

$$\frac{1}{2}\log_2(2) + \frac{1}{2}\log_2(2) + \frac{1}{2}\log_2(5)$$

$$= \log_2(2) + \frac{1}{2}\log_2(5)$$