Factoring Quadratics.

$3x^2+x-10=0$

$a=3, b=1, c=-10$

$b^2-4ac = 1^2-4(3)(-10)$

$=1+120=121 \Rightarrow \sqrt{121}=11$

$11 \overline{)121}$
11

$\boxed{2, 3, 5, 7, 11, 13,}$ 17, 19, 23

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-1\pm 11}{2(3)}$

$\rightarrow \frac{10}{6}=\frac{5}{3}$

$\rightarrow \frac{-12}{6}=-2$

$3\left(x-\frac{5}{3}\right)(x+2)$

$=\boxed{(3x-5)(x+2)}$

$|A| < B \Rightarrow$

$A < B$ AND $A > -B$

$|A| > B \Rightarrow$

$A > B$ OR $A < -B$

$\boxed{B2}$ 5 $|2x+7| + 8 < 9$

$-8 = -8$

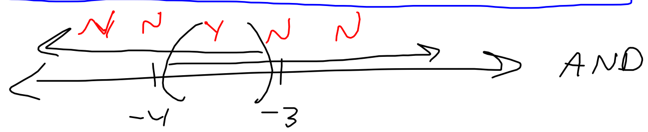
$|2x+7| < 1$

$2x+7 < 1$ AND $2x+7 > -1$

* $2x < -6$ $2x > -8$

$\frac{2x}{2} < \frac{-6}{2}$ $\frac{2x}{2} > \frac{-8}{2} = -4$

$\boxed{\{x \mid x < -3 \text{ AND } x > -4\}}$

= 

$= \boxed{(-4, -3)}$

$$|3x+11| + 19 > 10$$
$$\underline{\quad -19 \quad = -19 \quad}$$
$$|3x+11| > -9 \qquad (-\infty, \infty)$$

ALWAYS,

because $|A| \geq 0$ Always.

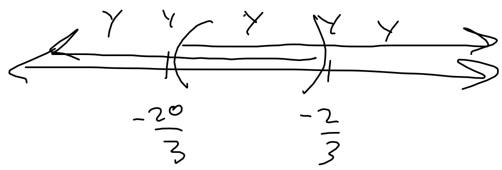
$|A| \geq 0 > -9$ so ...

$$|3x+11| > -9$$

$3x+11 > -9$ OR $3x+11 < 9$

$3x > -20$ OR $3x < -2$

$$\left\{ x \;\middle|\; x > -\frac{20}{3} \text{ OR } x < -\frac{2}{3} \right\}$$

$=$ 

OR

$= (-\infty, \infty)$

AND: $\left(-\frac{20}{3}, -\frac{2}{3}\right)$

But Not an "AND."

Recall 1-to-1 means:

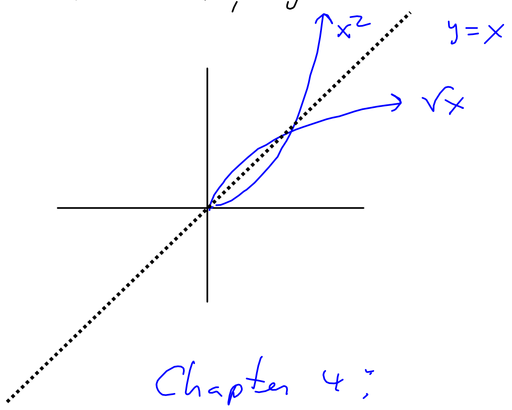
Func: Each x is paired with one y

1-to-1: Each y is paired with one x

1-to-1 ensures that there is an inverse function.

$(f \circ g)(x) = x$ if f & g are inverses.

$f(x) = \sqrt{x}$, $g(x) = x^2$ (restricted to $x \geq 0$)



We solve

$\sqrt{x} = 7$

by using its inverse x^2:

$(\sqrt{x})^2 = 7^2$

$\boxed{x = 49}$

Chapter 4:

3^x & $\log_3(x)$ are inverses!

$3^x = 81$

$\log_3(3^x) = \log_3(81)$

$x = 4$

$\log_3(x)$ says to x: "Show me your power!"

81 = 3 to what power?

$3 \,\underline{|\,81}$
$3 \,\underline{|\,27}$
$3 \,\underline{|\,9}$
$\;\;\;3$

$81 = 3^4$

FORMALLY:

$\log_b(x) = y$ means

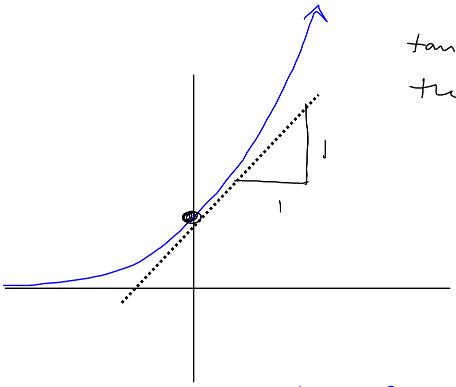
"log to the base b equals y"

means $x = b^y$

Check your calculator. Most only do $\log_{10}(x)$ and $\log_e(x) = \ln(x)$

$e \approx 2.7128...$

e^x is the unique function whose height and slope are identical!



tan line has slope = to the height of $f(x) = e^x$

$e^0 = 1$

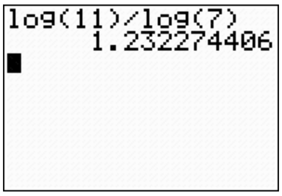
Euler's magic "e"

Differential Equations

To use calculator for something like $\log_7(11)$, do this:

CHANGE OF BASE FORMULA:

$$\log_7(11) = \frac{\log_{10}(11)}{\log_{10}(7)} \approx 1.232274406$$



$2^x = 256 = 2^8$

$\boxed{x = 2}$

2 | 256
2 | 128
2 | 64
2 | 32
2 | 16
2 | 8
2 | 4
2

$\log_2(2^x) = \log_2(256)$

$= \frac{\log(256)}{\log(2)}$

$\log(x)$ means $\log_{10}(x)$.

& $\log_{10}(x)$ means write x as a power of 10.

$\log_3(27910)$

$\approx$

Also, to solve logs:

$$\log_3(2x+1) - 11 = -15$$

$$\log_3(2x+1) = -4$$

$$3^{\log_3(2x+1)} = 3^{-4}$$

$$2x+1 = 3^{-4} = \frac{1}{3^4} = \frac{1}{81}$$

$$2x = \frac{1}{81} - 1 = \frac{1-81}{81} = -\frac{80}{81}$$

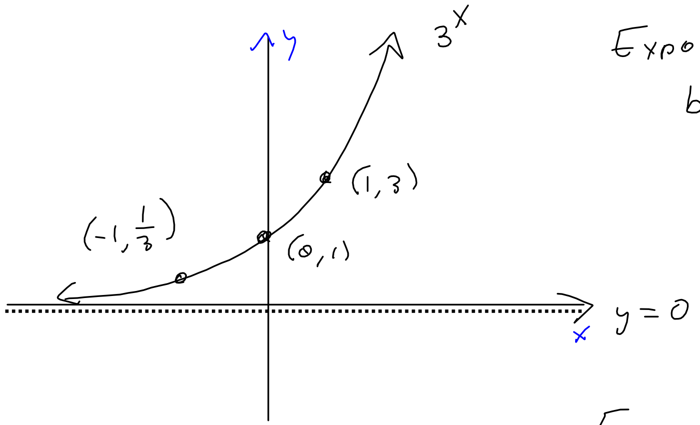
$$\frac{2x}{2} = \frac{-\frac{80}{81}}{\frac{2}{1}} = -\frac{80}{81} \cdot \frac{1}{2} = \boxed{-\frac{40}{81} = x}$$

Exponentials un-do logs.
Logs un-lock exponentials

Just like x^2 & $\sqrt{x}$,
" " x^5 & $\sqrt[5]{x}$.

$$\log_3(x) = \frac{\ln(x)}{\ln(3)} = \frac{\log(x)}{\log(3)} = \frac{\log_7(x)}{\log_7(3)} = \dots$$

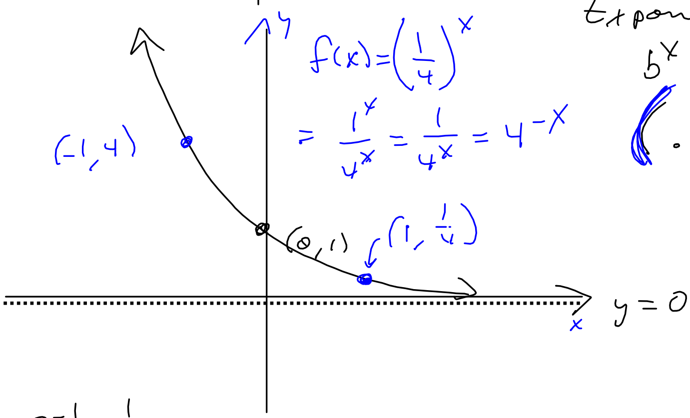
$$= \frac{\log_\pi(x)}{\log_\pi(3)}$$



Exponential Growth

b^x when $b > 1$

(e^x, 10^x, 3^x, 1.001^x, ...)



Exponential Decay

b^x when $0 < b < 1$

($.9^x$, $\left(\frac{1}{3}\right)^x$, $.5201^x$,

OR $3^{-x} = (3^{-1})^x = \left(\frac{1}{3}\right)^x$) !

$3^{-1} = \frac{1}{3}$