$$x^5 - 4x^4 + \frac{13}{2}x^3 + \frac{49}{2}x^2 - 128x + 138$$

$\boxed{\pm 2, \pm 3}, \pm 6, \pm 23, \pm 46, \pm 69, \pm 138$

$2 \,|\, 138$, $3 \,|\, 69$, 23

4, 2, 0 positive

$f(-x) = -x^5 - 4x^4 - \frac{13}{2}x^3 + \frac{49}{2}x^2 + 128x + 138$

$\boxed{\text{1 negative.}}$

$12 + \frac{13}{2} = \frac{24+13}{2} = \frac{37}{2}$

$\frac{49}{2} - 37 = \frac{49-74}{2} = -\frac{25}{2}$

-2	1	-4	$\frac{13}{2}$	$\frac{49}{2}$	-128	138
		-2	12	-37	25	Nope
	1	-6	$\frac{37}{2}$	$\frac{-25}{2}$	-103	

$\frac{42+13}{2} = \frac{55}{2}$

$\frac{-165+49}{2} = \frac{-116}{2} = -58$

-3	1	-4	$\frac{13}{2}$	$\frac{49}{2}$	-128	138
		-3	21	$\frac{-165}{2}$	174	-138
2	1	-7	$\frac{55}{2}$	-58	46	0
		2	-10	35	-46	
2	1	-5	$\frac{35}{2}$	-23	0	
		2	-6	23		
	$\boxed{1}$	$\boxed{-3}$	$\boxed{\frac{23}{2}}$	0		

$174 - 128 = 46$, $-3 \cdot 46 = -138$

$2 \cdot 58 = 116$, $58 \cdot 3 = 174$

$(x+3)(x-2)^2 \boxed{\left(x^2 - 3x + \frac{23}{2}\right)}$ irreducible

$1x^2 - 3x + \frac{23}{2} = 0 \Rightarrow$

$2x^2 - 6x + 23 = 0 \Rightarrow$

$a = 2, b = -6, c = 23$

$2 \,|\, 148$, $2 \,|\, 74$, 37

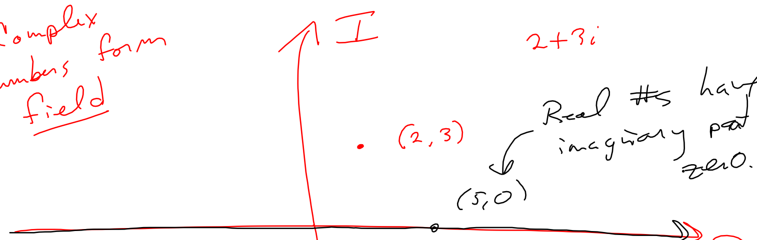
$\Rightarrow b^2 - 4ac = (-6)^2 - 4(2)(23) = 36 - 184 = \boxed{-148 < 0}$

$\sqrt{-148} = 2i\sqrt{37}$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{6 \pm 2i\sqrt{37}}{2(2)} = \frac{\cancel{2}(3 \pm i\sqrt{37})}{\cancel{2}(2)} = \frac{3 \pm i\sqrt{37}}{2}$$

$$1(x-2)^2(x+3)\left(x - \frac{3+i\sqrt{37}}{2}\right)\left(x - \frac{3-i\sqrt{37}}{2}\right)$$

Complex numbers form a field



FIELD $a \pm b$ is it

ab " " "

$\frac{a}{b}$ " " " if $b \neq 0$

Existence of 1

" " 0

Existence of inverses

$a, -a$

↑ additive inverse

$a, \frac{1}{a}$

↑ multiplicative inverse.

$a = 3+2i, \ -a = -3-2i$

$\frac{3+2i}{1}, \ \frac{1}{3+2i} = \frac{1}{3+2i} \cdot \frac{3-2i}{3-2i}$

$= \frac{3-2i}{9+4} = \frac{3-2i}{13}$

$\frac{1}{3+2i} = \frac{3}{13} - \frac{2}{13}i$

Every polynomial of degree n has n complex zeros.

$\mathbb{C}$ is algebraically closed field.

$\boxed{f(x) = x^2 + 1} \overset{\text{SET}}{=} 0$

$x^2 = -1$

$x = \pm\sqrt{-1}$ ain't real.

$\mathbb{R}$ is NOT algebraically closed

Irreducible* quadratic polynomial

* over the real number field

Easy points on slant asymptotes if you know your stuff.

$$R(x) = \frac{2x^3 - 7x^2 + 4x - 11}{x^2+3x+2} = \frac{- - - -}{(x+2)(x+1)}$$

Sketch Asymptotes. Assume no holes.

$$\begin{array}{r|rrrr} -1 & 2 & -7 & 4 & -11 \\ & & -2 & 9 & \text{No} \\ \hline & 2 & -9 & 13 & \end{array}$$

$$\begin{array}{r|rrrr} -2 & 2 & -7 & 4 & -11 \\ & & -4 & 22 & \text{Nah.} \\ \hline & 2 & -11 & 26 & \end{array}$$

No holes, confirmed.

So $x=-2$, $x=-1$ are vertical Asymptotes.

$$\begin{array}{r l} & 2x - 13 \\ x^2+3x+2 & \overline{)\,2x^3-7x^2+4x-11} \\ & -(2x^3+6x^2 \sim\sim) \\ \hline & -13x^2 \end{array}$$

$$\frac{2x^3}{x^2} = 2x$$

$$\frac{-13x^2}{x^2} = -13$$

$$\boxed{y = 2x - 13}$$

Taking the division all the way isn't needed for the slant asymptote, but here's what it looks like and what it means:

$$
\begin{array}{r}
2x - 13 \quad r\ 39x + 15 \\
x^2 + 3x + 2 \,\overline{\big)\, 2x^3 - 7x^2 + 4x - 11} \\
-\left(2x^3 + 6x^2 + 4x\right) \\
\hline
-13x^2 + 0x - 11 \\
-\left(-13x^2 - 39x - 26\right) \\
\hline
39x + 15
\end{array}
$$

This says $R(x) = \underline{2x - 13} + \dfrac{39x + 15}{x^2 + 3x + 2}$

$R(x)$ looks like $2x - 13$ the farther right/left you go.

$x \to \pm\infty$ $\quad 0$