WP#3

① $f(x) = 6x^5 - 5x^4 - 44x^3 + 49x^2 + 84x - 108$

↙ . . . ↗

② $f(x) = 6x^5 - 5x^4 - 44x^3 + 49x^2 + 84x - 108$

1 2 3

3 or 1 positive zeros

$f(-x) = -6x^5 - 5x^4 + 44x^3 + 49x^2 - 84x - 108$

1 2

2 or 0 negative zeros.

③ The most painful Ratl Zeros exercise...

q: 6

p: 108

2 | 108
2 | 54
3 | 27
3 | 9
3

2 | 6
3

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 18, \pm 24, \pm 36, \pm 54, \pm 108$

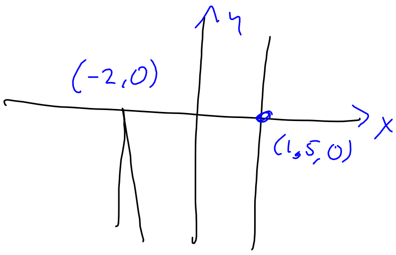
$\pm \frac{1}{2}, \pm \frac{2}{2}, \pm \frac{3}{2}, \pm \frac{4}{2}, \pm \frac{6}{2}, \pm \frac{8}{2}, \pm \frac{12}{2}, \pm \frac{18}{2}, \pm \frac{24}{2}, \pm \frac{36}{2}, \pm \frac{54}{2}, \pm \frac{108}{2}$

$\pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{3}{3}, \pm \frac{4}{3}, \pm \frac{6}{3}, \pm \frac{8}{3}, \pm \frac{12}{3}, \pm \frac{18}{3}, \pm \frac{24}{3}, \pm \frac{36}{3}, \pm \frac{54}{3}, \pm \frac{108}{3}$

$\pm \frac{1}{6}, \pm \frac{2}{6}, \pm \frac{3}{6}, \pm \frac{4}{6}, \pm \frac{6}{6}, \pm \frac{8}{6}, \pm \frac{12}{6}, \pm \frac{18}{6}, \pm \frac{24}{6}, \pm \frac{36}{6}, \pm \frac{54}{6}, \pm \frac{108}{6}$

I got 21 possibles.

④ Grapher:



Guess $x = -2, m = 2$
$x = \frac{3}{2}, m = 1$

−2	6	−5	−44	49	84	−108
		−12	34	20	−138	108
−2	6	−17	−10	69	−54	0
		−12	58	−96	54	
$\frac{3}{2}$	6	−29	48	−27	0	
		9	−30	27		
	6	−20	18	0		

$6x^2-20x+18=0 \Rightarrow 2(3x^2-10x+9)=0$

$a=3,\ b=-10,\ c=9$

$b^2-4ac = (-10)^2-4(3)(9) = 100-108 = -8$

~~So no more real zeros,~~

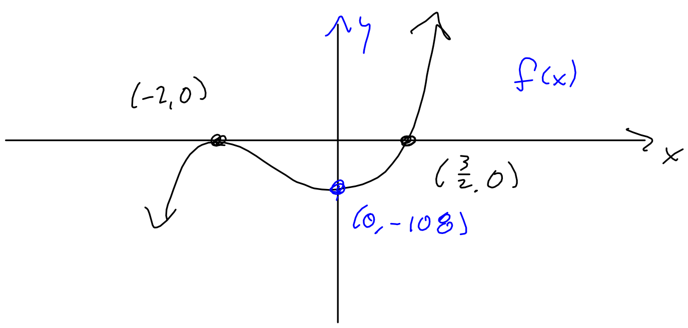
$$x=-2,\ m=2\ ;\ x=\tfrac{3}{2},\ m=1$$

$2\,|\,8$, $2\,|\,4$, 2

$\sqrt{-8} = 2i\sqrt{2}$

(5) $f(x) = (x+2)^2(x-\tfrac{3}{2})(6x^2-20x+18)$

(6)



(7) Keep going $\quad x = \dfrac{-b\pm\sqrt{b^2-4ac}}{2a} = \dfrac{10\pm 2i\sqrt{2}}{2(3)}$

$= \dfrac{\cancel{2}(5\pm i\sqrt{2})}{\cancel{2}(3)} = \dfrac{5\pm i\sqrt{2}}{3}$

So, $f(x) = 6(x+2)^2(x-\tfrac{3}{2})\left(x-\left(\tfrac{5+i\sqrt{2}}{3}\right)\right)\left(x-\left(\tfrac{5-i\sqrt{2}}{3}\right)\right)$

-2	6	-5	-44	49	84	-108
		-12	34	20	-138	108
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		-12	58	-96	54	
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So no more real zeros.

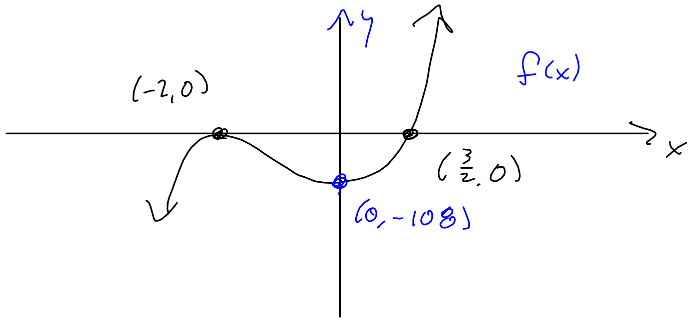
$x=-2,\ m=2\ ;\ x=\frac{3}{2},\ m=1$

$8 = 2\cdot 2\cdot 2$

$\sqrt{-8} = 2i\sqrt{2}$

(5) $f(x) = (x+2)^2(x-\frac{3}{2})(6x^2-20x+18)$

(6)



(7) Keep going $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{10\pm 2i\sqrt{2}}{2(3)}$

$= \frac{2(5\pm i\sqrt{2})}{2(3)} = \frac{5\pm i\sqrt{2}}{3}$

So, $f(x) = 6(x+2)^2(x-\frac{3}{2})\left(x-\left(\frac{5+i\sqrt{2}}{3}\right)\right)\left(x-\left(\frac{5-i\sqrt{2}}{3}\right)\right)$

(8) $R(x) = \dfrac{3x^2 - 13x - 10}{x^2 + 4x - 21} = \dfrac{(3x+2)(x-5)}{(x+7)(x-3)}$

If you don't know how I got those factored forms, then you're still not grokking Chapter 1, wp#1 & need to ask.

$\mathcal{D} = \mathbb{R} \setminus \{-7, 3\}$ NOTHING CANCELS, SO

V.A.: $x = -7$, $x = 3$

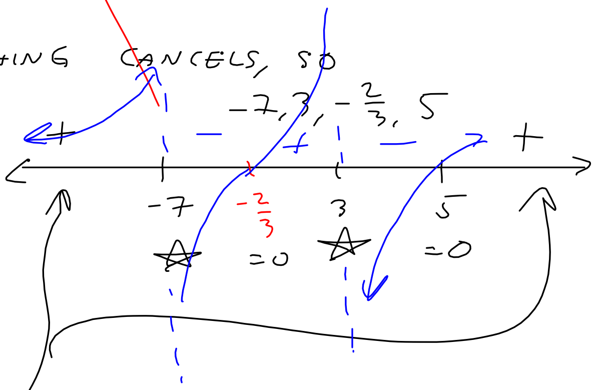
x-ints $x = -\frac{2}{3}, 5$

$\rightsquigarrow (-\frac{2}{3}, 0), (5, 0)$

y-int: $R(0) = \frac{-10}{-21}$

$(0, +\frac{10}{21})$

End Behavior: $\frac{3x^2}{x^2} = 3 = y$ H.A.!



$A = (-7, 0)$

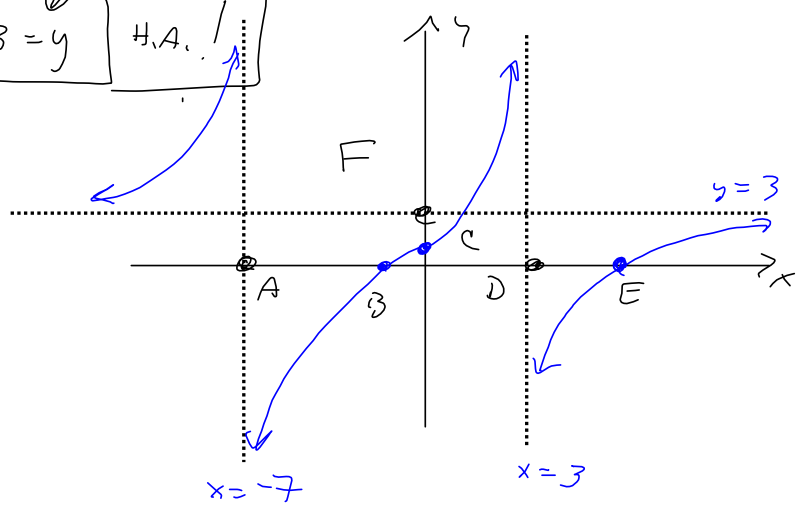
$B = (-\frac{2}{3}, 0)$

$C = (0, \frac{10}{21})$

$D = (3, 0)$

$E = (5, 0)$

$F = (0, 3)$ oops!



#9 we do #8 with a hole in it

$$Q(x) = \frac{3x^3 + 11x^2 - 114x - 80}{x^3 + 12x^2 + 11x - 168} = \frac{(3x+2)(x-5)(x-?)}{(x+7)(x-3)(x-?)}$$

Easiest way to find "?"

$$x^3 + 12x^2 + 11x - 168 = (x+7)(x-3)(x+8)$$

-7	1	12	11	-168
		-7	-35	168
3	1	5	-24	0
		3	24	
	1	8		

Check if there's $x+8$ upstairs:

-8	3	11	-114	-80
		-24	104	80
	3	-13	-10	0

Yep.

So $$\frac{(3x+2)(x-5)(x+8)}{(x+7)(x-5)(x+8)} = Q(x)$$

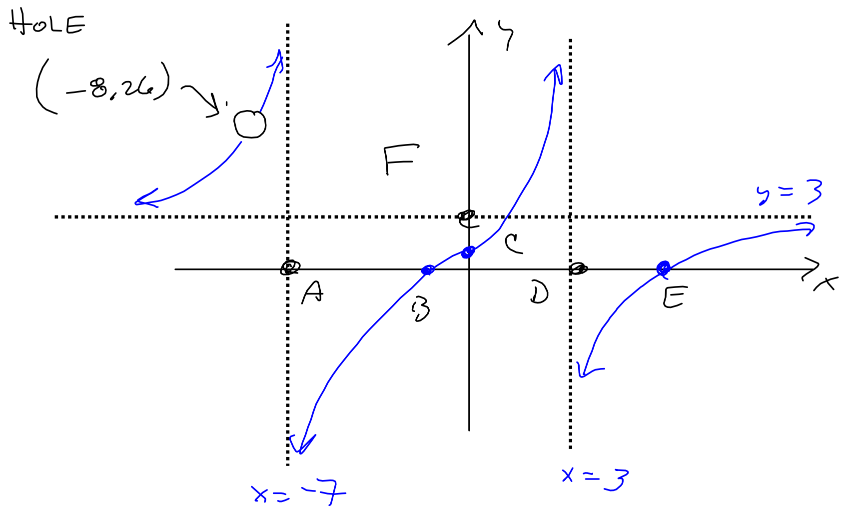
$Q(x) = R(x)$ everywhere except $x = -8$, so

$$R(-8) = \frac{(3(-8)+2)(-8-5)}{(-8+7)(-8-3)} = \frac{(-24+2)(-13)}{(-1)(-11)} =$$

$$\rightarrow = \frac{(\cancel{-22}^{\,2})(-13)}{(-1)(\cancel{-11}_{\,1})} = 26 \longrightarrow$$

Hole in $Q(x)$ is $(-8, 26)$

So $Q(x)$ graph is JUST like $R(x)$ graph, except for hole @ $(-8, 26)$


HOLE
(-8, 26)
F
y = 3
A
B
C
D
E
x
y
x = -7
x = 3

(10) $T(x) = \dfrac{x^3+12x^2+11x-168}{3x^2-13x-10} = \dfrac{\qquad}{(3x+2)(x-5)}$

On a time-controlled test, this'd be a little much, but on a take-home? Not at all.

On the sit-down test, I'll be wanting to see the slant (oblique) asymptote and any vertical asymptotes, but won't require you to factor another 3rd-degree polynomial. Just... too... much.

V.A.: $x = -\frac{2}{3}$ & $x = 5$, (assuming nothing cancels)

We can check that:

$-\frac{2}{3}\big|$ 1 12 11 −168; −2/3 −68/9 No; 1 34/3 31/9 way

$5\big|$ 1 12 11 −168; 5 85 Naw; 1 17 96

So no cancellation
So no holes
So only asymptotes

Slant asymptote?

$$\frac{1}{3}x + \frac{49}{9}$$

$3x^2-13x-10 \,\big)\, x^3+12x^2+11x-168$

$-\left(x^3 - \frac{13}{3}x^2 - \frac{10}{3}x\right)$

$\frac{49}{3}x^2$

$\frac{x^3}{3x^2} = \frac{1}{3}x$

$\frac{36+13}{3} = \frac{49}{3}$

$\frac{\frac{49}{3}x^2}{3x^2} = \frac{49}{9}$

Slant Asymptote:

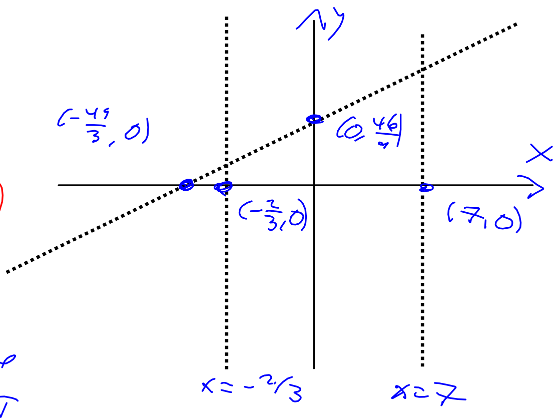
$y = \frac{1}{3}x + \frac{49}{9}$

x	y
0	49/9
−49/3	0

SET 0 ⟹ $\frac{1}{3}x + \frac{49}{9} = 0$

$\frac{1}{3}x = -\frac{49}{9}$

$x = -\frac{49}{3} \rightsquigarrow \left(-\frac{49}{3}, 0\right)$



This is as much as I'd ask on the in-class test, & I generally don't make you check for holes, like I would on a take-home.

Continuing the Take-Home

$$\frac{x^3+12x^2+11x-168}{(3x+2)(x-5)}$$

$$= \frac{(x+3)(x-7)(x+8)}{(3x+2)(x-5)}$$ if you remember #s 8&9 idiot.

2 | 168
2 | 84
2 | 42
3 | 21
7

$\mathcal{D} = \mathbb{R} \setminus \{-\frac{2}{3}, 5\}$

V.A. = $x = -2/3$, $x = 5$

x-int : $(-8, 0)$, $(-\frac{2}{3}, 0)$, $(5, 0)$

y-int : $(0, \frac{168}{5})$