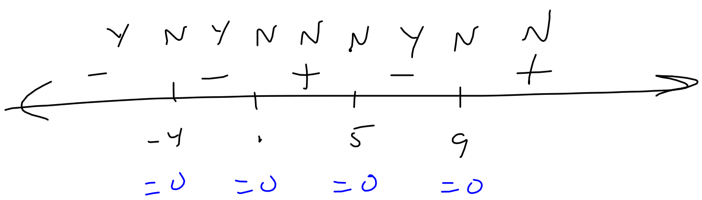
4. Suppose $f(x) = (x+4)^2(x-1)(x-5)(x-9) = x^5 - 7x^4 - 45x^3 + 187x^2 + 584x - 720$.

I'm showing you both factored and expanded form to help you answer the following:

a. (5 pts) Use a sign pattern to solve the inequality $f(x) < 0$.

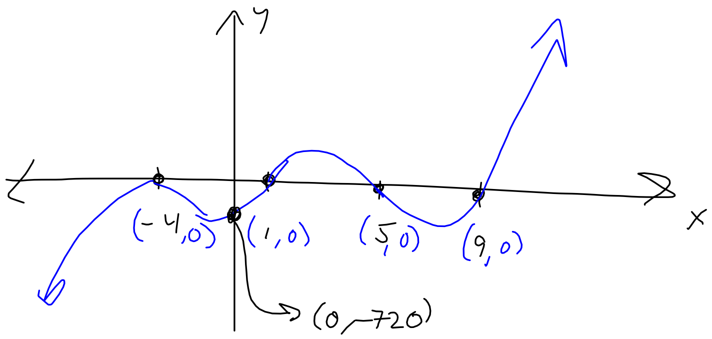
x^5

b. (5 pts) Provide a rough sketch of f, using its zeros, their respective multiplicities and its end behavior. Include x- and y-intercepts. Your graph should be smooth. Un-exaggerate the vertical for a better-quality graph.



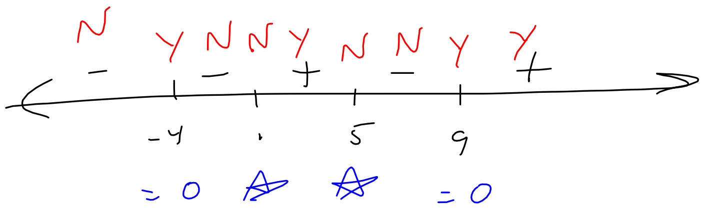
$f(x) < 0$

$\Rightarrow \boxed{x \in (-\infty, -4) \cup (-4, 1) \cup (5, 9)}$



c. (5 pts) What is the domain of $g(x)=\sqrt{\dfrac{(x-9)(x+4)^2}{(x-1)(x-5)}}$? $=\sqrt{\text{STUFF}}$

Need STUFF ≥ 0

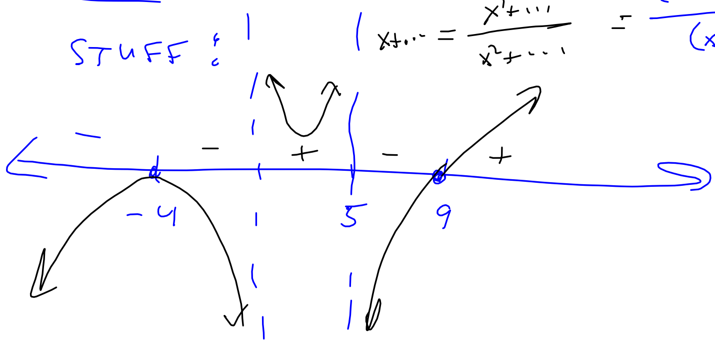


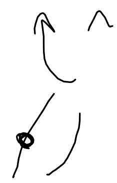
≥ 0

$\boxed{\mathcal{D} = \{-4\} \cup (1,5) \cup [9,\infty)}$

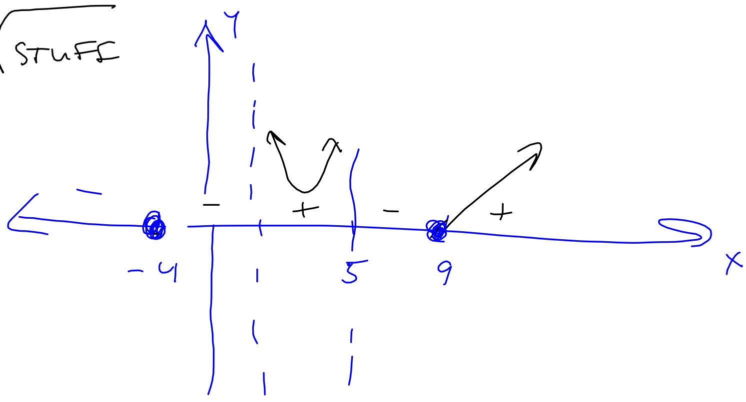
singleton

STUFF: $\quad \left(x+\dots = \dfrac{x^3+\dots}{x^2+\dots} \right. \quad = \dfrac{(x-9)(x+4)^2}{(x-1)(x-5)}$ SLANT ASYMPTOTE





$g(x)=\sqrt{\text{STUFF}}$



Teacher ain't tryin' to trick ya.

8. (5 pts) You don't need to graph $R(x)=\dfrac{2x^3-3x^2-3x+2}{x^2-x-6}$, here, but I do want to see you graph its asymptotes. Hint: This function has no holes. Hint: Don't waste time trying to factor the numerator.

$\mathcal{D} = \mathbb{R} \setminus \{-2, 3\}$

$(x-3)(x+2) = x^2-x-6$

Top is tough to factor.

$x=-2, 3$ are either holes or vertical asymptotes.

$\boxed{\text{V.A.: } x=-2, x=3}$

S.A.: Only if you think teacher's tryin' to trick ya.

$2x-1 = y = \text{S.A.}$

$x^2-x-6 \,\overline{)\, 2x^3-3x^2-3x+2}$

$\frac{2x^3}{x^2}=2x$ $\quad -(2x^3-2x^2-12x)$

$-x^2$

$\frac{-x^2}{x^2}=-1$

$-2 \,|\; 2 \quad -3 \quad -3 \quad 2$
$\quad\quad -4 \quad 8 \quad -10$
$\quad\; 2 \quad -4 \quad 5 \quad$ No

$3 \,|\; 2 \quad -3 \quad -3 \quad 2$
$\quad\quad 6 \quad 9 \quad 18$
$\quad\; 2 \quad 3 \quad 6 \quad$ No

See if a factor of $x+2$ is hiding upstairs. So they're asymptotes. No holes.

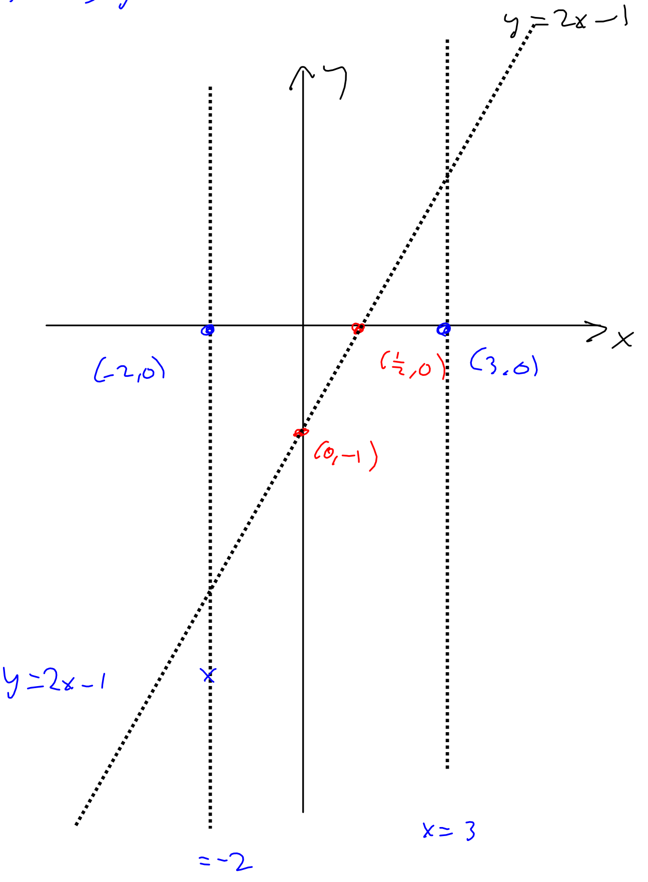
$x=-2, x=3, y=2x-1$

$(0,-1)$

$(\frac{1}{2}, 0)$ $\quad 2x-1=0$

$2x=1$

$x=\frac{1}{2}$



$\lim_{x\to 3} (5x+7) = 22$

Proof.

Let $\varepsilon > 0$ be given. Define $\delta = \frac{\varepsilon}{5}$. Then $0 < |x-3| < \delta$

$\Rightarrow |5x+7-22| = |5x-15| = 5|x-3| < 5\delta = 5\cdot\frac{\varepsilon}{5} = \varepsilon$ ▨

$\lim_{x\to 4} (x^2-5x+6) = 2$

Scratch:

$x^2-5x+6-2 = x^2-5x+4$

$= \underset{<\delta}{(x-4)}\boxed{(x-1)}$

Needs a bound

Assume $\delta \le 1$.

Then $|x-4| < \delta \le 1 \Rightarrow$

$-1 < x-4 < 1$

$+3 < x < 5$

$2 < x-1 < 4$

so, $|x-1| < 4$

Proof

Let $\varepsilon > 0$. Define $\delta = \min\{1, \frac{\varepsilon}{4}\}$

Then $0 < |x-4| < \delta \Rightarrow$

$|x^2-5x+6-2| = |x^2-5x+4|$

$= |x-1||x-4| < 4\cdot\frac{\varepsilon}{4} = \varepsilon$ ▨