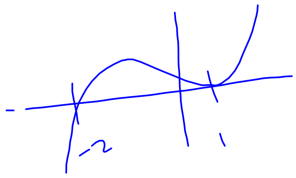
$$f(x) = 4x^5 - 12x^4 - x^3 + 44x^2 - 57x + 22.$$

$x=1$, $m=2$, $x=-2$ appear to be roots from the graph:



Try $x=1$, again!

or $x=-2$

$$\begin{array}{r|rrrrrr} 1 & 4 & -12 & -1 & 44 & -57 & 22 \\ & & 4 & -8 & -9 & 35 & -22 \\ \hline 1 & 4 & -8 & -9 & 35 & -22 & 0 \\ & & 4 & -4 & -13 & 22 & \\ \hline 1 & 4 & -4 & -13 & 22 & 0 & \\ & & 4 & 0 & -13 & & \\ \hline & 4 & 0 & -13 & \text{New P} & & \end{array}$$

sweet! A

sweet! ⟵ B

So try $x=-2$ on the depressed polynomial

$$\begin{array}{r|rrrr} -2 & 4 & -4 & -13 & 22 \\ & & -8 & 24 & -22 \\ \hline & 4 & -12 & 11 & 0 \end{array}$$

sweet! C

C $(x-1)^2(x+2)(4x^2-12x+11)$

A $(x-1)(4x^4-8x^3-9x^2+35x-22)=f(x)$

B $(x-1)(x-1)(4x^3-4x^2-13x+22)$

$$f(x) = 4x^5 - 9x^3 + 8x^2 - 9x + 6$$

1 2 3 4

5a Decartes: 4, 2, or 0 positive zeros

$f(-x) = -4x^5 + 9x^3 + 8x^2 + 9x + 6$

1

Exactly one negative.

x^4 spot

5b Rational zeros $\frac{p}{q} : \frac{6}{4}$

$\pm 1, \pm 2, \pm 3, \pm 6$

$\pm \frac{1}{2}, \pm \cancel{\frac{2}{2}}, \pm \frac{3}{2}, \pm \cancel{\frac{6}{2}}$

$\pm \frac{1}{4}, \pm \cancel{\frac{2}{4}}, \pm \frac{3}{4}, \pm \cancel{\frac{6}{4} = \frac{3}{2}}$

$(x+2)(4x^4 - 8x^3 + 7x^2 - 6x + 3)$

6

$$\begin{array}{r|rrrrrr} -1 & 4 & 0 & -9 & 8 & -9 & 6 \\ & & -4 & 4 & 5 & -13 & \\ \hline & 4 & -4 & -5 & 13 & -22 & \end{array}$$
Nope.

$$\begin{array}{r|rrrrrr} -2 & 4 & 0 & -9 & 8 & -9 & 6 \\ & & -8 & 16 & -14 & 12 & -6 \\ \hline 1 & 4 & -8 & 7 & -6 & 3 & 0 \\ & & 4 & -4 & 3 & -3 & \\ \hline 1 & 4 & -4 & 3 & -3 & 0 & \\ & & 4 & 0 & 3 & & \\ \hline & 4 & 0 & 3 & 0 & & \end{array}$$
sweet! sweet! sweet!

$\boxed{(x+2)(x-1)^2(4x^2+3)}$ $\boxed{x=-2,\ m=1 \quad x=1,\ m=2}$

$4x^2+3 = 4x^2+0x+3$

$a=4,\ b=0,\ c=3$

$b^2-4ac = 0^2-4(4)(3)$

$= -48 < 0$ So done with real zeros

7

$$\begin{array}{r|l} 2 & 48 \\ 2 & 24 \\ 2 & 12 \\ 2 & 6 \\ & 3 \end{array}$$

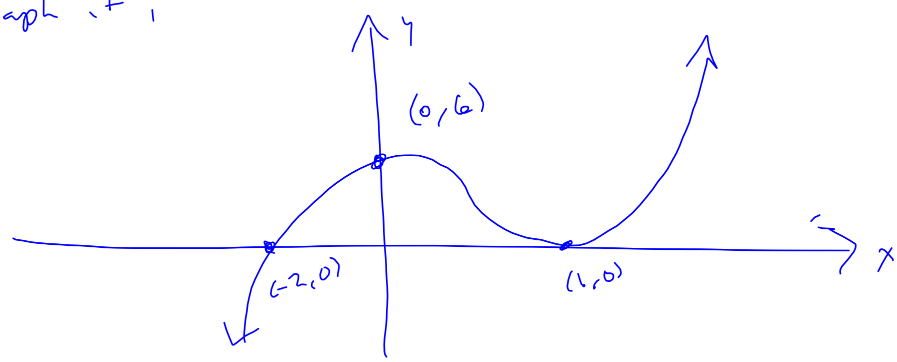
$\sqrt{-48} = 2\cdot 2\, i\sqrt{3} = 4i\sqrt{3} \Rightarrow$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a} = \dfrac{0 \pm 4i\sqrt{3}}{2(4)} = \dfrac{\pm 4i\sqrt{3}}{8} = \pm \dfrac{i\sqrt{3}}{2}$

So $f(x) = 4(x+2)(x-1)^2\left(x-\frac{i\sqrt{3}}{2}\right)\left(x+\frac{i\sqrt{3}}{2}\right)$

& it's split into linear factors

Graph it!



$x^6 - x^5 - x^4 + x^3 - 12x^2 + 12x$

$= x(x^5 - x^4 - x^3 + x^2 - 12x + 12)$

1 2 3 4

$g(x) = x^5 - x^4 - x^3 + x^2 - 12x + 12$

Descartes! 4, 2, 0 pos.

$g(-x) = -x^5 - x^4 + x^3 + x^2 + 12x + 12$

1

1 neg.

§3

$$\frac{3x^3 + 11x^2 - 114x - 80}{x^3 + 12x^2 + 11x - 168} = \frac{(3x+2)(x-5)(x+8)}{(x-3)(x-7)(x+8)} = R(x)$$

Hole: $(-8, R^*(-8))$

$$R^*(-8) = \frac{(-14)(-13)}{(-11)(-15)} = \frac{182}{65}$$

$(-8, \frac{182}{65})$ = HOLE.

$$\Rightarrow R^*(x) = \frac{(3x+2)(x-5)}{(x-3)(x-7)}$$

Bonus SET $= 3$

$$3x^2 - 15x + 2x - 10 = 3(x^2 - 7x - 3x + 21)$$

$$3x^2 - 13x - 10 = 3(x^2 - 10x + 21)$$

$$\cancel{3x^2} - 13x - 10 = \cancel{3x^2} - 30x + 63$$

$$-13x - 10 = -30x + 63$$

$$17x = 73$$

$$x = \frac{73}{17}$$

$(\frac{73}{17}, 3)$ is where $R(x)$ intersects its horizontal asymptote $y=3$

Define $f(t) \equiv r$

End Behavior of Rational functions

$\dfrac{\text{Lower Degree}}{\text{Higher Degree}}$ is Proper $y=0$ is Horizontal Asymptote

$\dfrac{\text{SAME DEGREE}}{\text{SAME DEGREE}}$ Look at leading terms

$\dfrac{5x^5 + \ldots \text{smaller}}{7x^5 + \ldots}$ $\xrightarrow{x \to \pm\infty}$ $\boxed{\dfrac{5}{7} = y}$

$\dfrac{\text{Higher Degree}}{\text{Lower Degree}}$: Tough. Need Long Division.

OBLIQUE ASYMPTOTE (SLANT)

deg = 3

deg = 2

3 > 2.

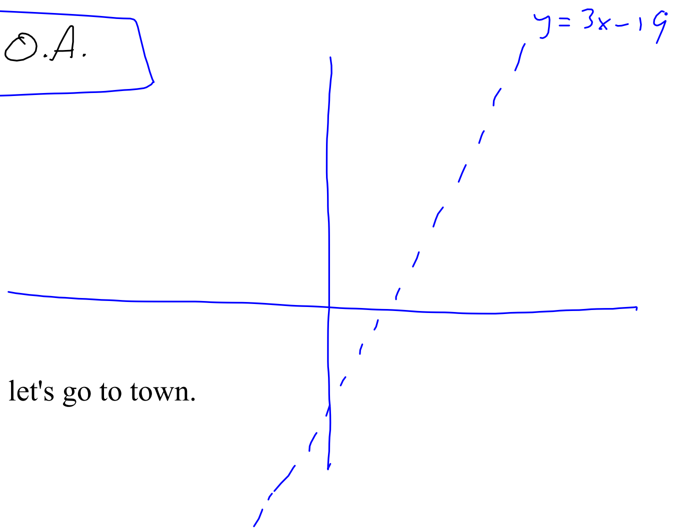
$$\frac{3x^3+5x^2-4x-4}{x^2+8x+7} = \frac{(3x+2)(x-1)(x+2)}{(x+1)(x+7)}$$

$$\begin{array}{r} 3x - 19 \\ x^2+8x+7 \,\big)\, \overline{3x^3+5x^2-4x-4} \\ -\,(3x^3+24x^2+21x) \\ \hline -19x^2-25x-4 \end{array}$$

$\dfrac{3x^3}{x^2} = 3x$

$\dfrac{-19x^2}{x^2} = -19$

$\boxed{y = 3x-19 \text{ is O.A.}}$



With the O.A. in our hip pocket, let's go to town.

$$R(x) = \frac{3x^3 + 5x^2 - 4x - 4}{x^2 + 8x + 7} = \frac{(3x+2)(x-1)(x+2)}{(x+1)(x+7)}$$

D:d $\boxed{\text{O.A.: } y = 3x - 19}$

$\mathcal{D} = \mathbb{R} \setminus \{-1, -7\}$

$\boxed{\text{V.A.: } x = -1,\ x = -7}$

x-ints: $(-2,0)$, $(-\frac{2}{3},0)$, $(1,0)$

y-int: $(0, -\frac{4}{7})$

$A = (-7,0)$

$B = (-2,0)$

$C = (-1,0)$

$D = (-\frac{2}{3},0)$

$E = (0,-\frac{4}{7})$

$F = (1,0)$

$G = ?\ (\frac{19}{3},0)$

$H = (0,-19)$

$3x - 19 = 0$

$3x = 19$

$x = \frac{19}{3}$