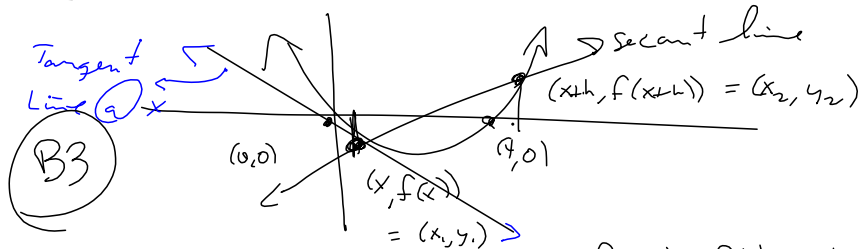


④  $f(x) = x^2 - 4x = x(x-4) \stackrel{\text{SET}}{=} 0 \Rightarrow x = 0, 4$



⇒ Difference Quotient is  $\frac{f(x+h) - f(x)}{(x+h) - x} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{(x+h)^2 - 4(x+h) - (x^2 - 4x)}{h}$        $(x+h)^2 = x^2 + 2xh + h^2$

$= \frac{x^2 + 2xh + h^2 - 4x - 4h - x^2 + 4x}{h} = \frac{2xh + h^2 - 4h}{h} = \frac{h(2x + h - 4)}{h}$

$= 2x + h - 4 \quad (h \neq 0)$

Then Bonus 1 :

$2x + h - 4 \xrightarrow{h \rightarrow 0} 2x - 4 = f'(x)$

Cool follow up: Find the equation of the tangent line at  $x=1$  :

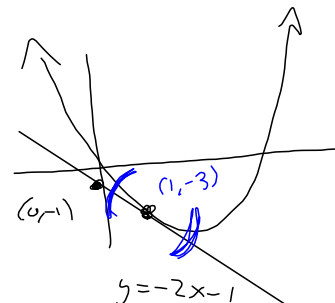
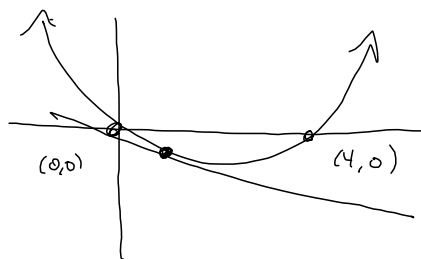
$f'(1) = m_{\text{tan}} = 2(1) - 4 = -2 = m$

$f(1) = 1^2 - 4(1) = -3 \Rightarrow (1, -3) = (x_1, y_1)$

$\Rightarrow y = m(x - x_1) + y_1$

$= -2(x - 1) - 3 = -2x + 2 - 3 = -2x - 1$

Devin hates me.



Secant Line - Average Slope

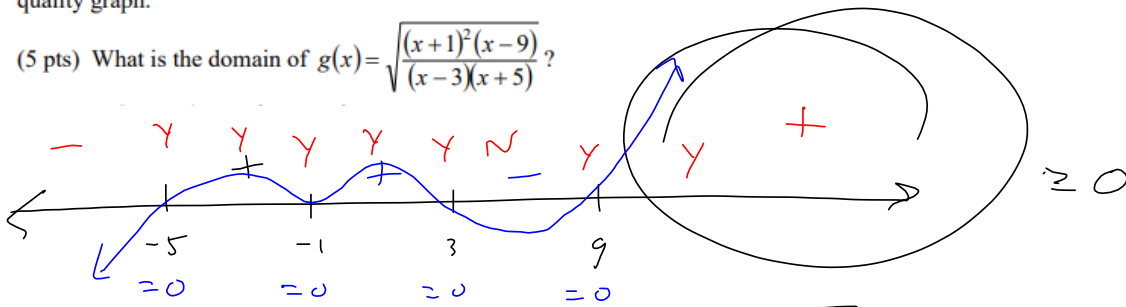
$\xrightarrow{h \rightarrow 0}$

Tangent Line - EXACT slope at a single point.

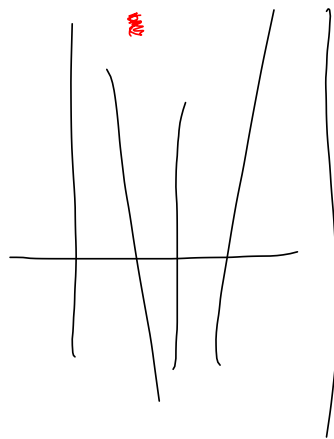
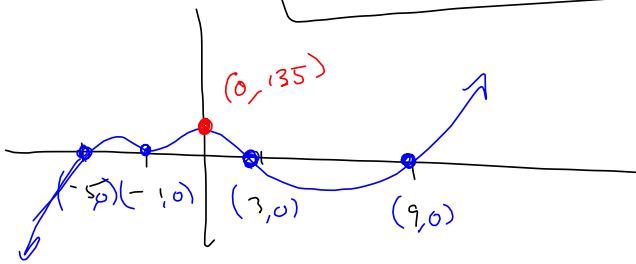
4. Suppose  $f(x) = (x+5)(x+1)^2(x-3)(x-9) = x^5 - 5x^4 - 46x^3 + 62x^2 - 237x + 135$ . I'm showing you both factored and expanded form to help you answer the following:  $-5, -1, 3, 9$

- a. (10 pts) Solve the inequality  $f(x) \geq 0$ . Your sign pattern for this one will be helpful in the next two. You just have to interpret what you're seeing.
- b. (10 pts) Provide a rough sketch of  $f$ , using its zeros, their respective multiplicities and its end behavior. Include  $x$ - and  $y$ -intercepts. Your graph should be smooth. Un-exaggerate the vertical for a better quality graph.

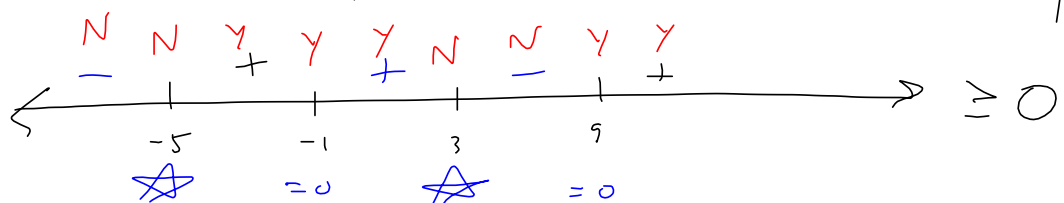
c. (5 pts) What is the domain of  $g(x) = \sqrt{\frac{(x+1)^2(x-9)}{(x-3)(x+5)}}$ ?



$$\Rightarrow x \in [-5, 3] \cup [9, \infty)$$



ⓐ Domain: Need  $\frac{(x+1)^2(x-9)}{(x-3)(x+5)} \geq 0$



$$\Rightarrow x \in (-5, -1] \cup (-1, 3) \cup [9, \infty) = (-5, 3) \cup [9, \infty)$$