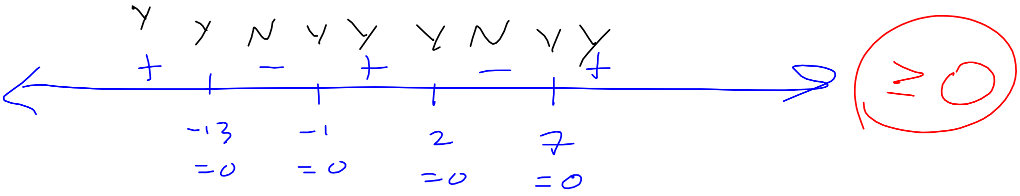
$(x-2)(x+1)(x-7)(x+13) \geq 0$ $\quad x^4 + \dots$



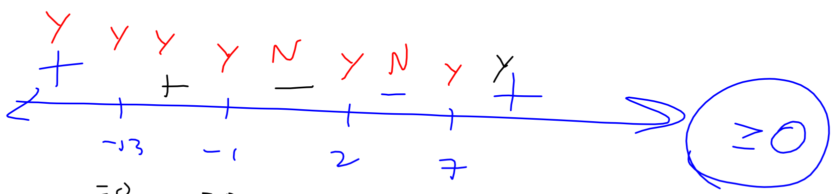
$\Rightarrow x \in (-\infty,-13] \cup [-1,2] \cup [7,\infty)$

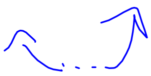
$(x-2)^3(x+1)^5(x-7)^1(x+1)^{1719} \geq 0$

Same solution.

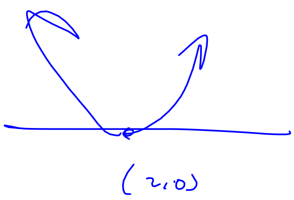
ALL ODD POWERS, so sign changes @ each boundary

$(x-2)^2(x+1)(x-7)^3(x+13)^{18} \geq 0$ $\quad x^2 \cdot x \cdot x^3 \cdot x^{18} + \dots = x^{24} + \dots$





$(x-2)^2(x+1)(x-7)^3(x+13)^{18}$



Find domain of

$$\sqrt{(x-2)^2(x+1)(x-7)^3(x+13)^{18}} = \sqrt{Mess}$$

Need Mess ≥ 0

$\Rightarrow x \in (-\infty,-1] \cup \{2\} \cup [7,\infty)$

How would this change the domain question?

Find domain of

$$\frac{5x^2 - 18x + 298{,}996}{\sqrt{(x-2)^2(x+1)(x-7)^3(x+13)^{18}}}$$

This adds Division by zero concerns

New Y/N pattern. Same signs.

Y N Y N N N N N Y

$+ \quad + \quad - \quad - \quad +$

$-13 \quad -1 \quad 2 \quad 7$

$=0 \quad =0 \quad =0 \quad =0$

≥ 0

> 0

$\Rightarrow x \in (-\infty,-13) \cup (-13,-1) \cup (7,\infty)$

$\Rightarrow x \in (-\infty,-1] \cup \{2\} \cup [7,\infty)$ from previous.

We basically threw out the boundary points

NOTE: ≥ 0 and $\neq 0$ means > 0

Look at old Test 3s (and 4s and 5s) for this type of question.

Chapter 3 Polynomials & Rational Functions.

$$P(x) \qquad \frac{P(x)}{Q(x)}$$

Synthetic Division

Long Division

$$\begin{array}{r} 36 \text{ r } 1 \\ 3\overline{)109} \\ -\ 90 \\ \hline 19 \\ -\ 18 \\ \hline 1 \end{array}$$

The interpretations:

This says

$$\frac{109}{3} = 36 + \frac{1}{3}$$

OR

$$\boxed{109 = 3(36) + 1}$$

Crucial underpinning of the Remainder Theorem

Dividend = Divisor · Quotient + Remainder

$$\frac{x^2 - 5x + 6}{x+1}$$

$$\begin{array}{r} x - 6 \text{ r } 12 \\ x+1\overline{)x^2 - 5x + 6} \\ -\ (x^2 + x) \\ \hline -6x + 6 \\ -\ (-6x - 6) \\ \hline 12 \end{array}$$

$$\frac{x^2}{x} = 1 \qquad \frac{-6x}{x} = -6$$

This says $\dfrac{x^2 - 5x + 6}{x+1} = x - 6 + \dfrac{12}{x+1}$

" " $x^2 - 5x + 6 = (x+1)(x-6) + 12$

$f(x) = \text{Div} \cdot \text{Quot} + \text{Rem}$

$f(-1) = 0 \cdot \text{Quot} + \text{Rem} = \text{Rem}$!

$f(x) = x^2 - 5x + 6$

What's $f(-1)$? $f(-1) = 12$!

Divide Synthetically:

$$\begin{array}{r|rrr} -1 & 1 & -5 & 6 \\ & & -1 & 6 \\ \hline & 1 & -6 & r\ 12 \\ & x^1 & x^0 & \end{array}$$

This says

$f(x) = (x+1)(x-6) + 12$

$f(x) = x^3 + 3x^2 - 5x + 11$

what's $f(7)$?

$$\begin{array}{r|rrrr} 7 & 1 & 3 & -5 & 11 \\ & & 7 & 70 & 455 \\ \hline & 1 & 10 & 65 & \boxed{466 = f(7)} \end{array}$$

$$\begin{array}{r} \overset{3}{6}5 \\ \times\ 7 \\ \hline 455 \end{array}$$

$7^3 + 3(7)^2 - 5(7) + 11$ is slow & won't get full credit.

That IS the Remainder Theorem. And it's the lead-in to the Factor Theorem.

When trying to factor, you're looking for a remainder of zero!!!!

Synthetic Division

$f(x) = x^5 - 5x^4 + 3x^3 + 11x^2 - x + 1$

What's $f(2)$? Use synthetic division.

$2 \rightarrow x-2$

2	1	−5	3	11	−1	1
		2	−6	−6	10	18
	1	−3	−3	5	9	$\boxed{19 = f(2)}$
	x^4	x^3	x^2	x^1	x^0	r

Interpretation

This says

$f(x) = (x-2)(x^4 - 3x^3 - 3x^2 + 5x + 9) + 19$