#10 Completing the square to manipulate an expression versus completing the square to solve a quadratic equation.

To manipulate:

$g(x) = 5x^2 + 4x + 17$

$= 5\left(x^2 + \frac{4}{5}x\right) + 17$

$= 5\left(x^2 + \frac{4}{5}x + \left(\frac{2}{5}\right)^2\right) + 17 - 5\left(\frac{4}{25}\right)$

$= 5\left(x + \frac{2}{5}\right)^2 + \frac{81}{5}$ → Done Manipulating

Scratch: $17 - 5\left(\frac{4}{25}\right)$

$= 17 - \frac{4}{5} = \frac{17}{1}\cdot\frac{5}{5} - \frac{4}{5}$

$= \frac{85}{5} - \frac{4}{5} = \frac{81}{5}$

Can go on to solve "$= 0$"

$5\left(x + \frac{2}{5}\right)^2 + \frac{81}{5} = 0$

$5\left(x + \frac{2}{5}\right)^2 = -\frac{81}{5}$

$\left(x + \frac{2}{5}\right)^2 = -\frac{81}{25}$

To solve:

$5x^2 + 4x + 17 = 0$

$x^2 + \frac{4}{5}x + \frac{17}{5} = 0$

$x^2 + \frac{4}{5}x = -\frac{17}{5}$

$x^2 + \frac{4}{5}x + \left(\frac{2}{5}\right)^2 = -\frac{17}{5} + \frac{4}{25}$

$\left(x + \frac{2}{5}\right)^2 = -\frac{81}{25}$

Scratch: $-\frac{17}{5}\cdot\frac{5}{5} + \frac{4}{25}$

$= \frac{-85 + 4}{25} = -\frac{81}{25}$

optimal = $\sqrt{}$

$\left|x + \frac{2}{5}\right| = \sqrt{-\frac{81}{25}} = \frac{9}{5}i$

$x + \frac{2}{5} = \pm\frac{9}{5}i$

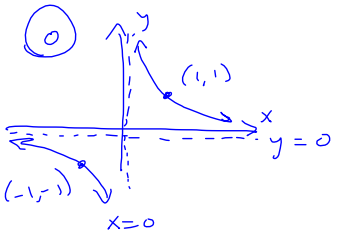
$x = \frac{-2 \pm 9i}{5}$

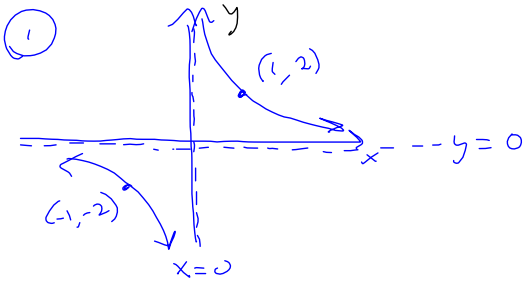
#1 $g(x) = \frac{2}{5x+15} + 7$

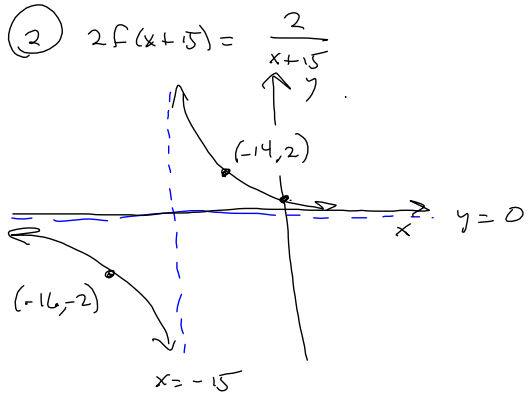
(0) $f(x) = \frac{1}{x}$

(1) $2f(x) = 2\left(\frac{1}{x}\right) = \frac{2}{x}$ $\quad (x,y) \longmapsto (x,2y)$

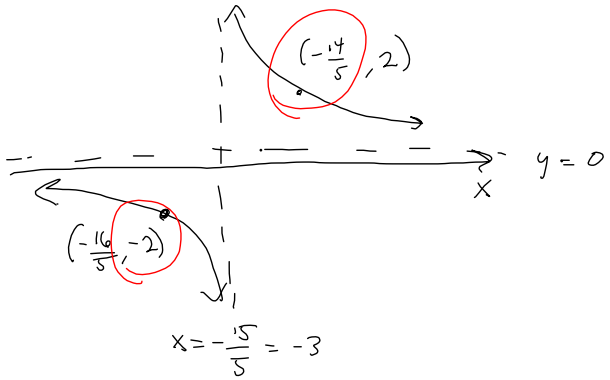
(M1) (2) $2f(x+15) = \frac{2}{x+15}$ $\quad (x,y) \longmapsto (x-15,y)$



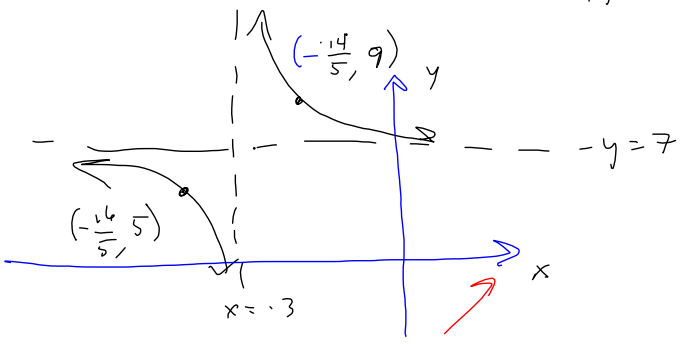




(3) $2f(5x+15) = \frac{2}{5x+15}$ $\quad (x,y) \longmapsto \left(\frac{x}{5},y\right)$



(4) $g(x) = 2f(5x+15)+7 = \frac{2}{5x+15} + 7$ $\quad (x,y) \longmapsto (x,y+7)$



$f(x) = \sqrt{x+7}$, $g(x) = x^2 - 5x - 20$

$\mathcal{D}(f) = [-7, \infty)$ $\mathcal{D}(g) = (-\infty, \infty)$

Need $x + 7 \geq 0$

$\Rightarrow x \geq -7$

Now, $g(x)$

$f \circ g = f(g(x)) = \sqrt{g(x) + 7} = \sqrt{x^2 - 5x - 20 + 7}$

$= \sqrt{x^2 - 5x - 13}$ → Need: $x^2 - 5x - 13 \geq 0$ INFORMAL.

$\mathcal{D}(f \circ g) = \{ x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f) \}$ FORMAL

$= \{ x \mid g(x) \geq -7 \}$

Scratch:

$g(x) \geq -7$

$x^2 - 5x - 20 \geq -7$

$x^2 - 5x - 13 \geq 0$

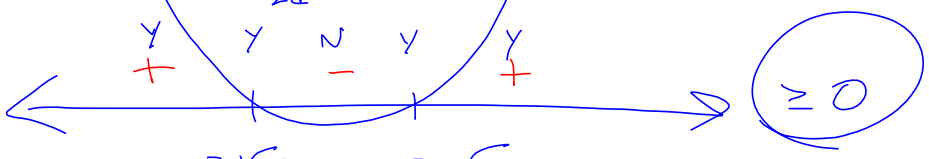
$a = 1, b = 5, c = -13$

$b^2 - 4ac = 5^2 - 4(1)(-13)$

$= 25 + 52 = 77$

$\sqrt{77} = \sqrt{7 \cdot 11}$ can't simplify

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{77}}{2}$



$\frac{-5 - \sqrt{77}}{2} = 0$ $\frac{-5 + \sqrt{77}}{2} = 0$

$= (-\infty, \frac{-5 - \sqrt{77}}{2}] \cup [\frac{-5 + \sqrt{77}}{2}, \infty) = \mathcal{D}(f \circ g)$